# Summary

# 8. SRM drive

## 8.1 Switched reluctance machine dynamic model

Consider the machine shown schematically in Figure 8-1.

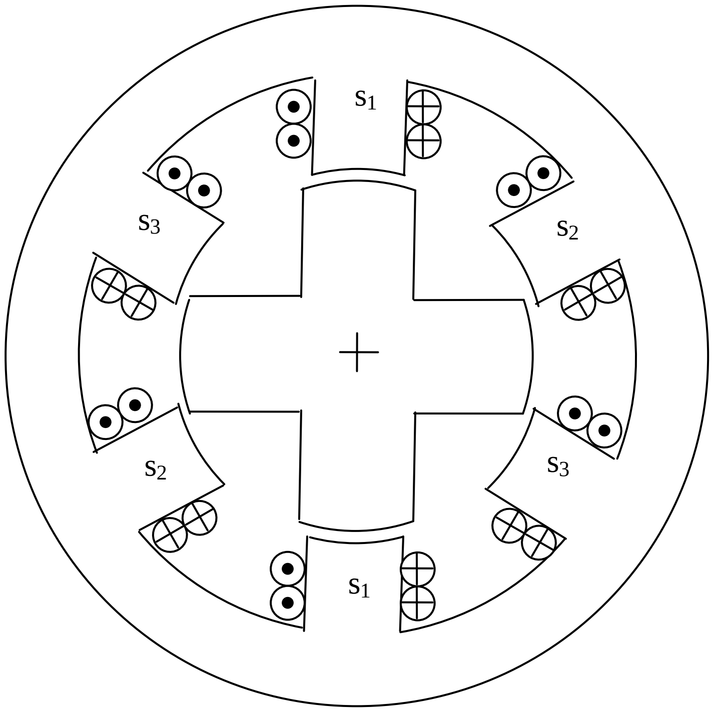



Figure 8-1 Schematic representation of an SRM 6/4

It is a machine with 6 stator poles and 4 rotor ones. In Figure 8-2 some typical quantities are shown:

$\beta_s$ stator pole angle

$\beta_r$ rotor pole angle

$\theta_m$ mechanical angle

$r_m$ average radius at the air gap

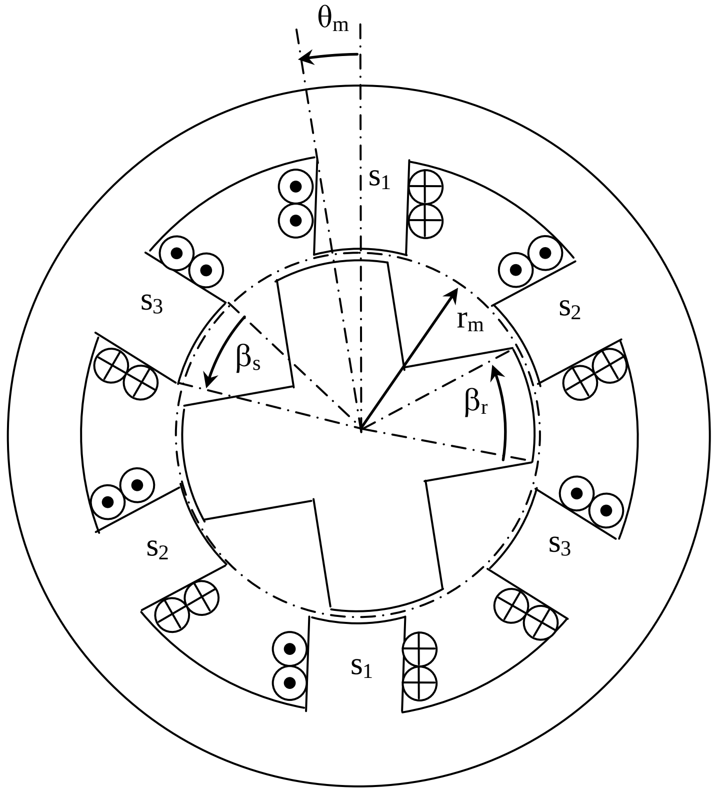



Figure 8-2: Typical quantities

Suppose to linearize the machine and consider that all distances are obtained from the product of the angle corresponding to the average radius (and thus all the distances may be represented as angles except for a scaling factor equal to $r_m$) the Figure 8-3 is obtained.

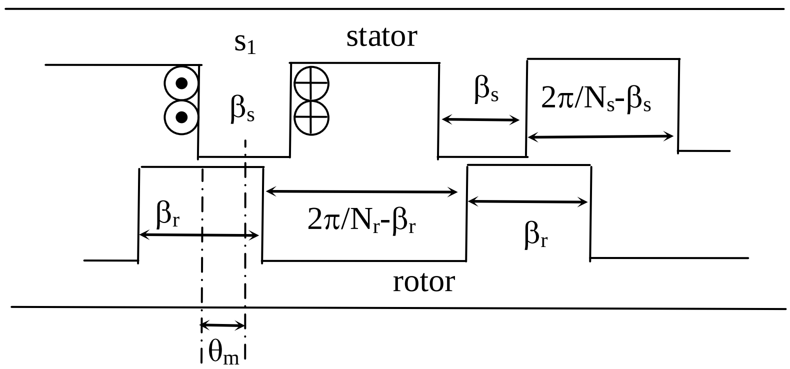



Figure 8-3: Linear extension of the machine air gap (scale factor $r_m$)

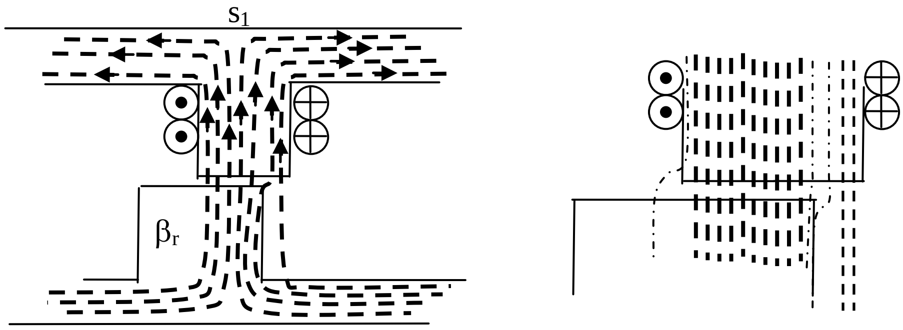



Figure 8-4: Field lines trajectories

The qualitative trajectories of the flux lines is shown in Figure 8-4. Suppose to neglect the fringe effects: the magnetic induction associated to the lines that pass through the low air gap have high values, vice versa for the lines through a large air gap. The area crossed by the magnetic flux varies linearly with the mechanical angle $\theta_m$.

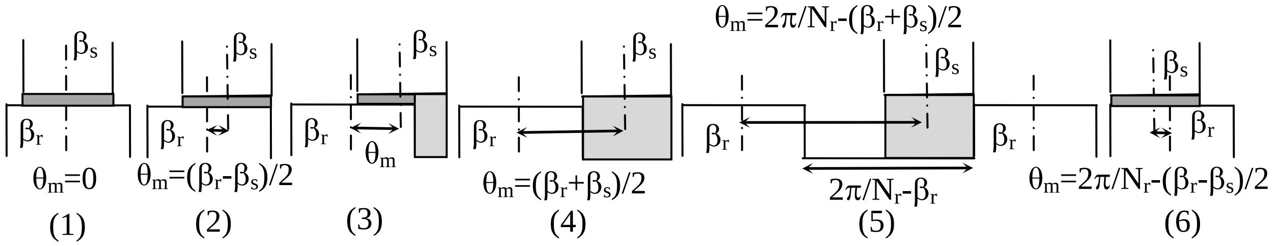



Figure 8-5: Variation of the cross-section of the magnetic flux (in the case of $\beta_s<\beta_r$)

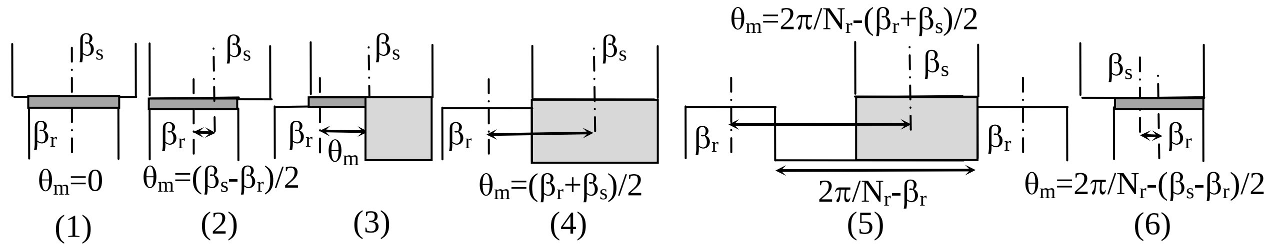



Figure 8-6: Variation of the cross-section of the magnetic flux (in the case of $\beta_s>\beta_r$)

Consider the case of $\beta_s<\beta_r$ (usually, more common due to $N_s>N_r$).

Having neglected the fringe effects, the cross-section of the flux between the position (1) and the position (2) remains constant. This means that the magnetic flux supported by the current $i_{s1}$ remains constant. Also the flux linked with the winding s1 will be constant. The self-inductance, defined as the ratio between the flux linked with the winding and the current in it, will therefore be constant. This period will be then (valid for both $\beta_s<\beta_r$ and $\beta_r<\beta_s$): $|\beta_s-\beta_r|/2$.

For angles greater than $|\beta_s-\beta_r|/2$ the cross-section of the magnetic flux through the low air gap decreases linearly with the angle itself while the cross-section of the magnetic flux, that passes through the high air gap, grows linearly (3). This linear variation affects both the magnetic flux, the flux linkage and the self-inductance. This behavior lasts till configuration (4) where the stator pole abandons the rotor pole. The distance is $\beta_s$ in the first case ($\beta_s<\beta_r$), while in the second case is to $\beta_r$: in general we can say that the length is the minimum of $\beta_s$ and $\beta_r$ [$\min(\beta_s,\beta_r)$].

From the configuration (4) and as long as the stator pole is inside the space between two rotor poles ($2\pi/N_r-\beta_r$) (5), the magnetic flux must pass through a high air gap, but the cross-section remains constant. During this period ($(2\pi/N_r-\beta_r)-\beta_s=2\pi/N_r-\beta_r-\beta_s$), the inductance is constant and equal to its minimum value.

Between the configuration (5) and (6) there is partial overlap between the stator pole and the rotor one (in a specular way to what occurred between the configuration (2) and (4)), so the inductance increases linearly; the distance is still $\min(\beta_s,\beta_r)$.

From the configuration (6) and till the end of the period $2\pi/N_r$ (which corresponds to the configuration (1)), the inductance is constant and equal to its maximum value; the distance is $|\beta_s-\beta_r|/2$.

The winding inductance profile is a function of the mechanical angle as shown in Figure 8-7.

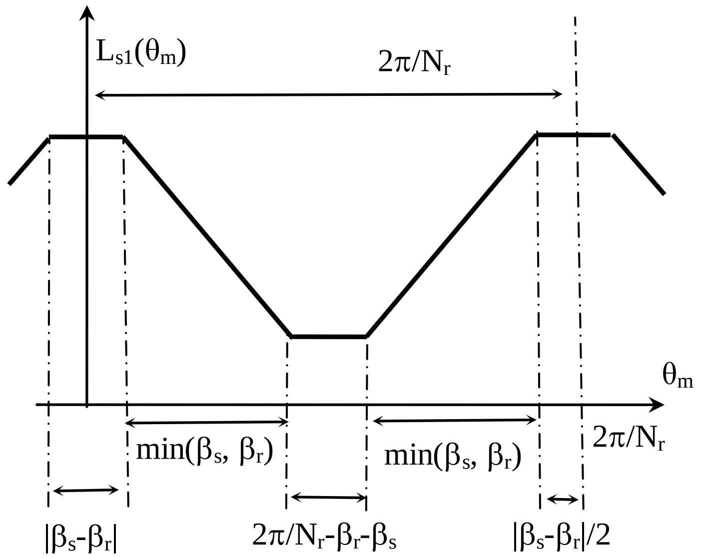



Figure 8-7: Self inductance profile as a function of the mechanical angle

As regards the other windings, the corresponding self inductance will have a similar trend but phase-shifted by an angle equal to the period of the self inductance ($2\pi/N_r$) divided by the number of windings (or phases q). The angle is, therefore, $\varepsilon=2\pi/(N_r\ q)$. In this case, q is equal to 3 and Nr=4, then the period ($2\pi/N_r$) corresponds to 90° (mechanical angle) while the phase shift $\varepsilon$ is 30° (mechanical).

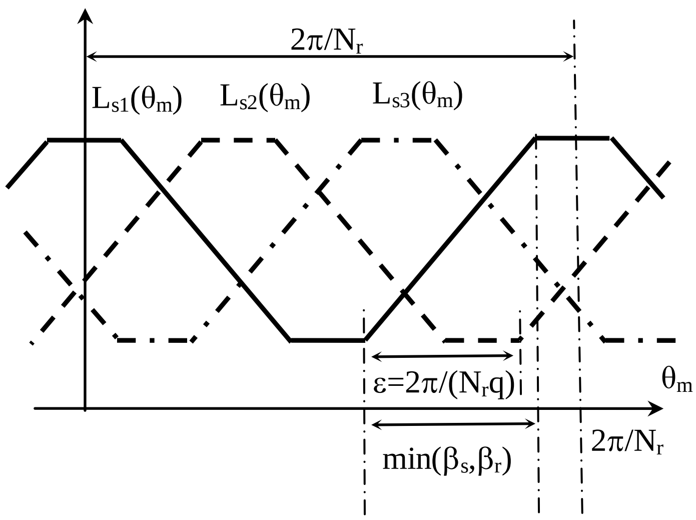



Figure 8-8: Self-inductances profiles as a function of the mechanical angle

## 8.2 Torque value

Consider to supply only one winding at once.

Relative to phase s1, the relationship voltage/current and flux/current will be:

$$v_{s1} = R_s i_{s1} + \frac{d\psi_{s1}}{dt}$$
$$\psi_{s1} = L_{s1}(\theta_m) i_{s1}$$

The energy balance gives:

$$v_{s1} i_{s1} = R_s i_{s1}^2 + i_{s1}\frac{d\psi_{s1}}{dt} = R_s i_{s1}^2 + i_{s1} L_{s1}(\theta_m)\frac{di_{s1}}{dt} + i_{s1}^2 \frac{dL_{s1}(\theta_m)}{dt} =$$
$$\ldots = R_s i_{s1}^2 + i_{s1} L_{s1}(\theta_m)\frac{di_{s1}}{dt} + i_{s1}^2 \frac{dL_{s1}(\theta_m)}{d\theta_m}\frac{d\theta_m}{dt}$$

To understand how the input power is divided into the different contributions (Joule losses, derivative of the internal magnetic energy of the system and mechanical power) it is better to consider the derivative of the energy stored in an inductor:

$$U = \frac{1}{2} L_{s1}(\theta_m) i_{s1}^2$$
$$\frac{dU}{dt} = \frac{1}{2} L_{s1}(\theta_m) 2 i_{s1}\frac{di_{s1}}{dt} + \frac{1}{2} i_{s1}^2 \frac{dL_{s1}(\theta_m)}{dt} = L_{s1}(\theta_m) i_{s1}\frac{di_{s1}}{dt} + \frac{1}{2} i_{s1}^2 \frac{dL_{s1}(\theta_m)}{d\theta_m}\frac{d\theta_m}{dt}$$

In the energy balance equation, the first term ($R_s i_{s1}^2$) is related to the Joule losses, while the second one and an half of the third one represents the change of internal magnetic energy. One half of the third term, then, represents the mechanical power.

$$P = \frac{1}{2} i_{s1}^2 \frac{dL_{s1}(\theta_m)}{d\theta_m}\frac{d\theta_m}{dt}$$

Called $n_p$ the number of pole pairs for each phase, the mechanical speed in the mechanical world is:

$$\Omega_m = \frac{1}{n_p}\frac{d\theta_m}{dt}$$

So the torque value is:

$$T_e = \frac{P}{\Omega_m} = \frac{n_p}{2} i_{s1}^2 \frac{dL_{s1}(\theta_m)}{d\theta_m}$$

It is clear that the torque is zero if there is no variation of the inductance and that the sign of the torque does not depend on the sign of the current but on the sign of the derivative of the inductance. This means that in the sector in which the inductance decreases, the machine is working as a generator and as a motor when it increases. If during the period ($\min(\beta_s, \beta_r)$), in which the derivative of the inductance is constant (this derivative is then indicated by $k_c$), the phase current is maintained constant (with a value equal to $I_d$) through appropriate power converter, the torque $T_e$ remains constant and equal to:

$$T_e = \frac{n_p}{2} I_d^2 k_c$$

In order to maintain constant the torque, throughout the period ($2\pi/N_r$), it is necessary that before the end of the constant slope period of the phase inductance, a new phase in which the inductance begins to have a constant slope has to be ready. Since the phase shift between two phases is equal to $\varepsilon=2\pi/(N_r\, q)$, for a constant torque it must be $\varepsilon\leq\min(\beta_s,\beta_r)$.

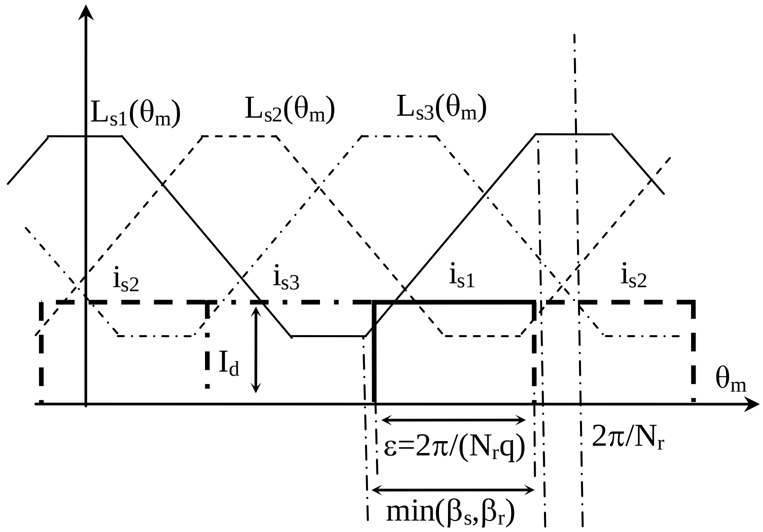



Figure 8-9: Current waveforms during operation as a motor

## 8.3 Design region

From the considerations made above, one can understand that there are limits on the values that $\beta_s$ e $\beta_r$ can assume. In particular, the fact that usually $N_s$ is greater than $N_r$ generally implies that $\beta_s<\beta_r$.
In addition, the torque continuity is ensured by the condition $\min(\beta_s,\beta_r)\geq\varepsilon$. There is another condition: the inductance has to be able to reach its minimum value. This may happen if the stator pole can stay inside the space between two rotor poles, i.e. if $\beta_s\leq 2\pi/N_r-\beta_r$ or better if $2\pi/N_r-\beta_s-\beta_r\geq 0$. If this condition is not satisfied, the period during which the derivative of the inductance is constant could be less than the theoretical $\min(\beta_s,\beta_r)$ and less than $\varepsilon$, with possible implications on the continuity of the torque.
The application of these conditions determines, in the plane $\beta_s$-$\beta_r$, an area of possible values that may be assumed by $\beta_s$ and $\beta_r$, in order to properly design the machine.

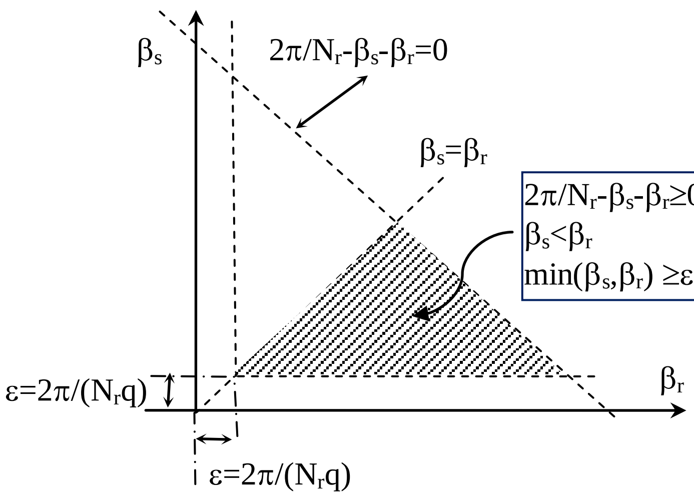



Figure 8-10: Design region

## 8.4 Control scheme

Recall the torque expression:

$$T_e = \frac{n_p}{2} I_d{}^2 k_c$$

and the voltage dynamic equation (referred to the first phase)

$$v_{s1} = R_s i_{s1} + p\psi_{s1} = R_s i_{s1} + L_{s1}(\theta_m)\frac{di_{s1}}{dt} + i_{s1}\frac{dL_{s1}(\theta_m)}{dt} =$$

$$... = R_s i_{s1} + L_{s1}(\theta_m)\frac{di_{s1}}{dt} + i_{s1}\frac{dL_{s1}(\theta_m)}{d\theta_m}\frac{d\theta_m}{dt} = R_s i_{s1} + L_{s1}(\theta_m)\frac{di_{s1}}{dt} + i_{s1}k_c n_p \Omega_m = R_s i_{s1} + L_{s1}(\theta_m)\frac{di_{s1}}{dt} + E$$

the control scheme of Figure 8-11is obtained ($i_{sx}$ refers to the generic phase "x", one among the "q" ones).

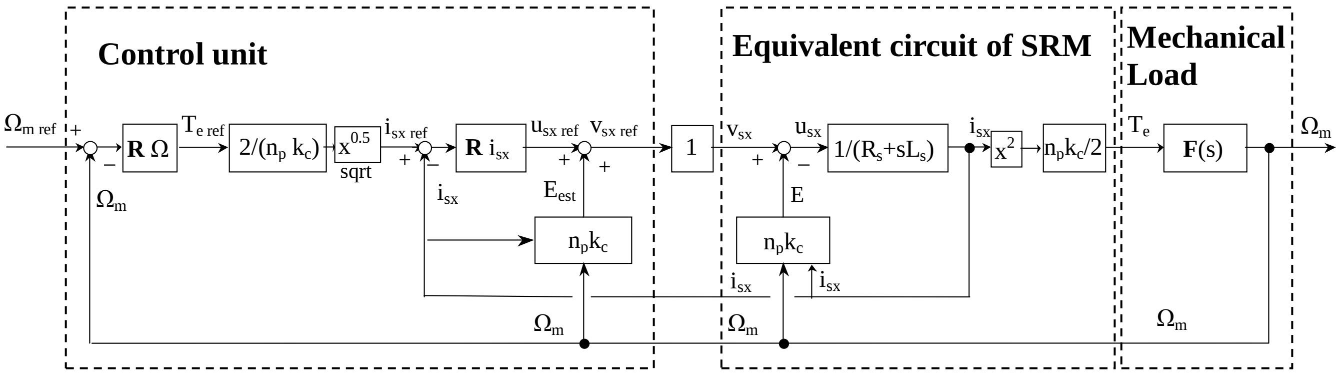



Figure 8-11: Power converter

The system is not linear, for the presence of the "square root". The current regulator "sees" a transfer function whose pole is variable with the mechanical position $\theta_m$: $1/(R_s+sL_s(\theta_m))$.

The control scheme is applied to each phase, for a period of $\varepsilon=2\pi/(N_r\ q)$, in phase with the self-inductance (centered with respect to the period in which the derivative of the inductance is constant). Within the overall period ($2\pi$) there are $N_r$ sub-periods characterized by q sectors. In total the sectors are $N_r \cdot q$ (in the case of a machine 6/4 sectors with q=3 are 12). It needs, therefore, a position sensor in order to define the right phase to be supplied.

## 8.5 Power converter

Each phase of the machine must be able to be supplied independently of the other. Furthermore, since the torque does not depend on the sign of the current, the converter (a dc-dc converter) can operate only on two quadrants (positive current, positive and negative voltage). In the case of a machine with q = 3, a possible converter is represented in Figure 8-12, which has nothing to do with an inverter (there are 3 two quadrants dc-dc converter, one for each phase).

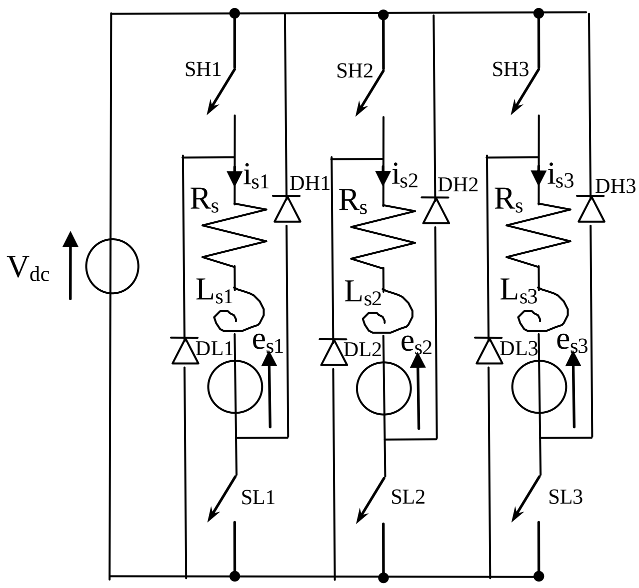



Figure 8-12: Power converter

Closing both the high side static switch (eg. SH1) and the low side switch (SL1), the applied voltage is equal to $V_{dc}$. Closing only SH1, the current flowing in the inductance $L_{s1}$, cannot be instantly reduced to zero, but continues to flow in SH1 and the freewheeling diode DH1. The applied voltage is zero. The same voltage can be achieved by closing the only switch SL1: the current will flow in SL1 and DL1. By opening both switches, the current that was flowing in $L_{s1}$, begins to flow in the two freewheeling diodes DL1 and DH1. The voltage applied to the winding, in this case, is $-V_{dc}$. The current, clearly, is always positive.

## 8.6 Operating regions

The control logic presented above, which expects to impose a constant current, equal to $I_d$, in each phase for a period ε, can operate only below a certain mechanical speed. In fact, the bemf E, with $i_{sx}=I_d$, is proportional to the speed.

$$E = I_d k_c n_p \Omega_m$$

As for all other machines, it is evident that the maximum value of voltage, that the power supply can provide, fixes limits on the maximum speed that can be achieved using this logic.

In particular, said $V_{dmax}$ the maximum power supply voltage (compatibly with the limitations due to the insulation of the windings of the machine and maintaining a certain margin for the dynamic control of the current), this maximum speed, also called base speed $\Omega_b$, is reached when $V_{dmax}=R_sI_d+I_dk_cn_p\Omega_b$ i.e. $\Omega_b=(V_{dmax}-R_sI_d)/(I_dk_cn_p)$. For speed higher than the base speed a new strategy must be required.

It will resume the dynamic equations of the stator winding

$$v_{s1} = R_s i_{s1} + L_{s1}(\theta_m)\frac{di_{s1}}{dt} + i_{s1}\frac{dL_{s1}(\theta_m)}{d\theta_m}\frac{d\theta_m}{dt} = R_s i_{s1} + L_{s1}(\theta_m)\frac{di_{s1}}{dt} + i_{s1}\frac{dL_{s1}(\theta_m)}{d\theta_m}n_p\Omega_m$$

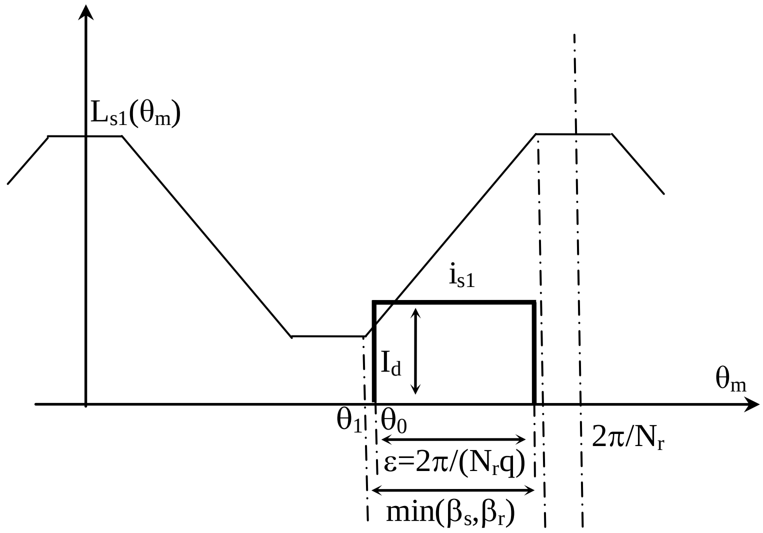



Figure 8-13: Current waveform when $\Omega_m<\Omega_b$

If you would anticipate the change of the sector before the theoretical angle ($\theta_0$ di Figure 8-13 ),, where the derivative of the inductance is zero ($\theta_1$) (for example at the angle $\theta_2$ di Figure 8-14), the term corresponding to the bemf would be zero (since the derivative is zero).

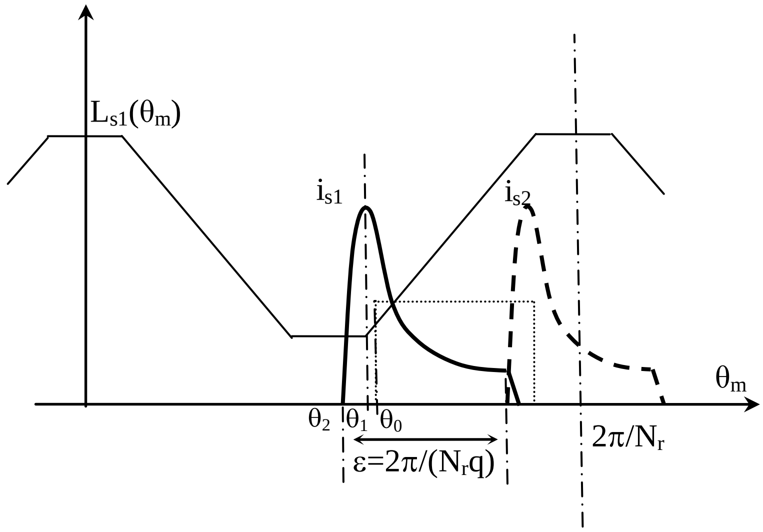



Figure 8-14: Current waveform when $\Omega_m>\Omega_b$

The forcing term of the dynamical system is $V_{dc}$ (obtained by closing both SH1 and SL1), while the bemf E=0. This voltage would grow very fast the current till the angle $\theta_1$. Since that time, the forcing term becomes $V_{dc}-i_{s1}k_cn_p\Omega_m$, which, at that speed and power, is surely negative. Then the current decreases until reaching a steady state value, depending on the actual speed and such as to satisfy the following equality (the derivative of the current in steady state conditions is zero): $V_{dc}=R_si_{steady\_state}+i_{steady\_state}k_cn_p\Omega_m$, then $i_{steady\_state}=V_{dc}/(R_s+k_cn_p\Omega_m)\cong V_{dc}/(k_cn_p\Omega_m)$. This value could not be reached because it is necessary that, after an angle $\varepsilon$ starting from $\theta_2$, a change of the sector has to take place and the control system has to stop supplying s1 and start to supply s2. The current of the phase s1 will go to zero quickly, forced by $-V_{dc}-i_{s1}k_cn_p\Omega_m$ (both switches SH1 and SL1 are opened).

The angle $\theta_2$ must not be too different from $\theta_1$ as the current increases very fast (not being limited by any bemf). Even in this case the thermal requirements must be met, limiting the rms value of the current (in addition to limiting the maximum current value to keep safe the static switches).

The torque will be different from zero only in the section in which the derivative of the inductance is different from zero. It will be no longer constant, but it will allow the machine to reach very high speeds.

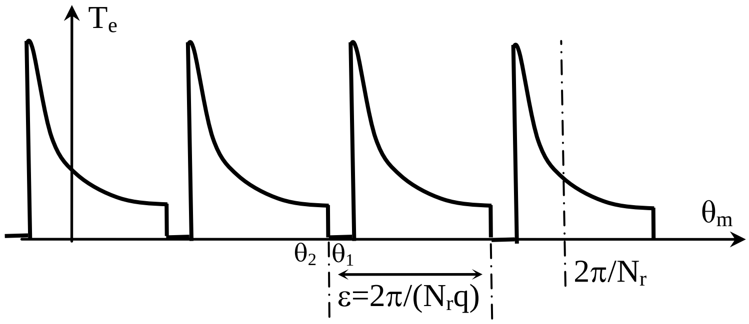
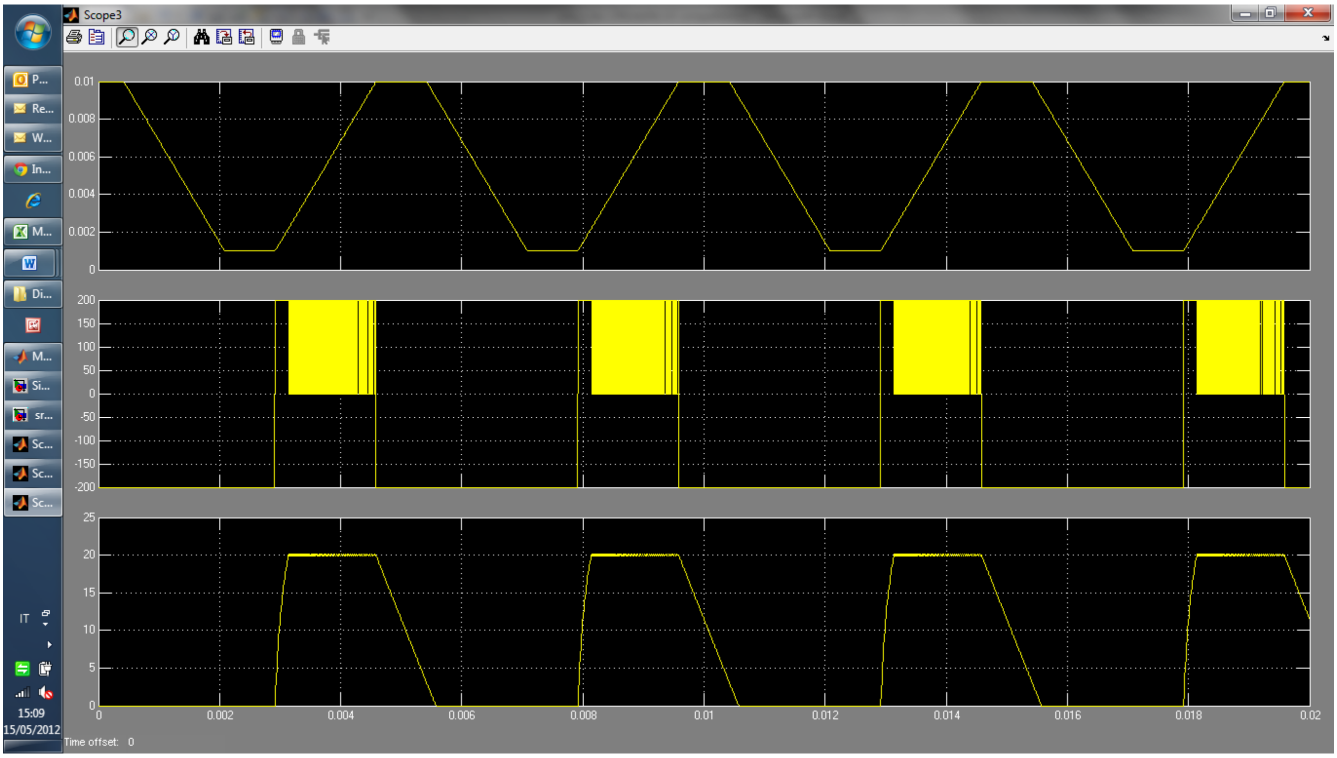



Figure 8-15: Electromagnetic toque waveforms for $\Omega_m > \Omega_b$

## 8.7 Example

$V_{dc}$=200V, $\beta_s=\pi/6$, $\beta_r=\pi/4$, $L_{min}$=1mH, $L_{max}$= 10mH, $N_r$=4, $N_s$=6, $n_p$=1; so $k_c=(L_{max}-L_{min})/\beta_s=0.0172$

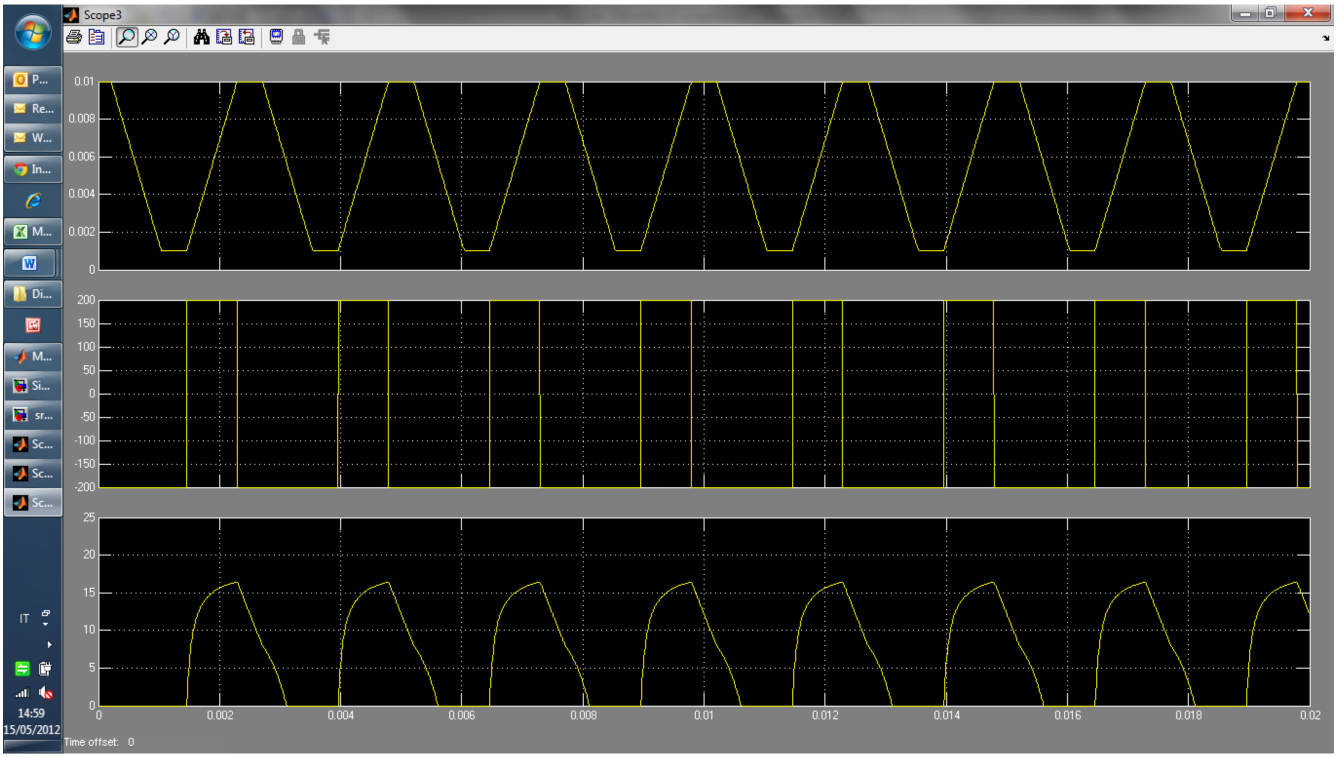
f=50Hz, Iref=20A



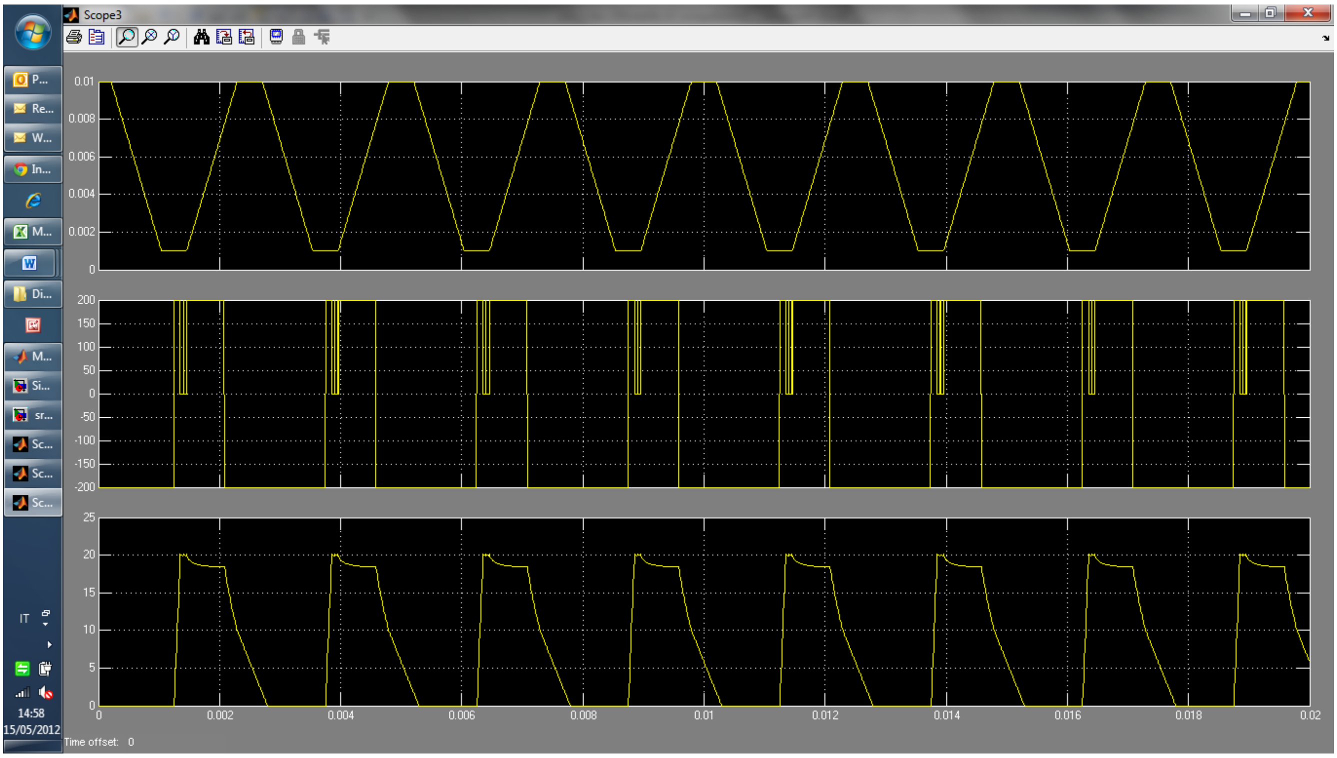

from top to bottom L(θ), va, ia

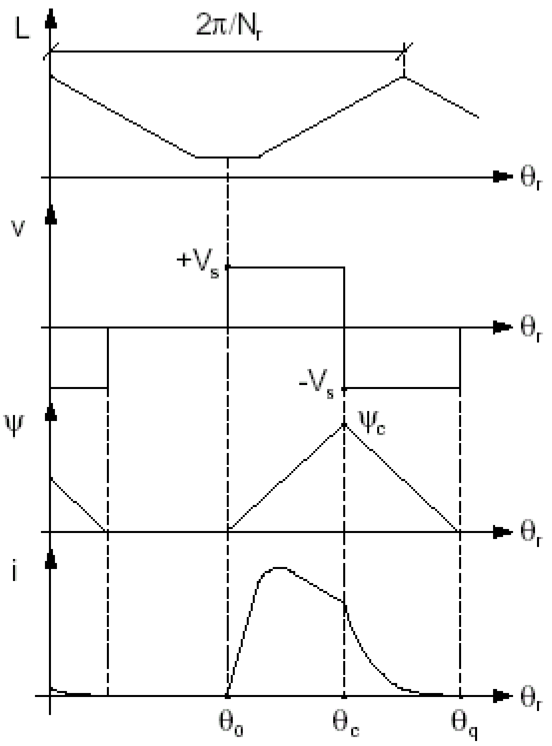
f=100Hz, Iref=20A, I steady state 18.2A

No leading




Leading of π/24





L
2π/Nr
θr
v
+Vs
θr
-Vs
ψ
ψc
θr
i
θr
θ0
θc
θq