Sensorless techniques for AC machines drives

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8. Induction machine drives

One of the most active areas of research in the field of electrical drives is the study of estimators of the state of the machine. The functioning of a vector control or direct torque control depends mainly on the precision with which module and position of the magnetic flux are known (rotor, stator or more, depending on the type of control implemented). From these values it is possible to estimate the electromagnetic torque produced by the machine. Assuming that we know the values of the currents and voltages (using sensors or by means of their references), the complete knowledge of the state of the machine still needs the availability of two variables: position and speed. In the market there are different sensor families, both of speed and position. The cost is relatively variable, but especially for small and medium power drives it affects decisively on the final cost of the product. Not only that, but for large powers, the problem of transferring the measurement of speed (or position) from the motor to the drive without problems can be solved only with complex wiring. In this sense it becomes necessary to remove the speed sensors. Drives where the control of the electromechanical system is not based on direct measurements of speed or position but on estimators (or observers) are called "sensorless". There will, therefore, more or less sophisticated systems from the point of view of control (scalar, vector, direct, indirect vector to direct torque control, ...) with more or less complex estimators of speed or position.

Figure 8-1 shows a classical vector control scheme based on an orientation with the rotor flux. The value of speed is necessary for the speed control loop and for the flux observer. This is just an example of a control with sensor, but the same signal is necessary in any other architecture in which there is a speed or position control loop.



Figure 8-1 Architecture of a classic rotor field oriented control



Figure 8-2 Architecture changes in a sensorless version of the control

It is well-known that the main problem of the sensorless control techniques is the dependence on the machine parameters that may depend on temperature, frequency, state of saturation, There are more or less complex methods to compensate for these nonlinearities, but it should be noted that the classical model of the machine provides very little information at low speeds. In addition, the voltage drop on the switches of the inverter, the dead time and the quantization errors of the modulator are much more influential around the zero speed. Some techniques that are based on the anisotropy of the rotor (out of roundness of the rotor itself or simply the presence of slots in a rotor cage) can provide further information on these conditions. New methodologies based on artificial intelligence (neural networks, fuzzy, neuro-fuzzy, expert systems, ...) should open new horizons.

These notes aim to present some solutions to the problem of estimating the speed in a sensorless drive for induction machines.

8.1 Volt/Hz

Simple solutions can be obtained as the evolution of the classic control schemes: the scalar control or V/f. Figure 8-3 presents the typical V/f control using a speed sensor.



Figure 8-3 Diagram of V/f control with a speed sensor

Figure 8-4, however, shows an open loop scheme, in which the error between the reference speed and actual speed is a function of slip and thus of the load.



Figure 8-4 Diagram of V/f control without a speed sensor

Figure 8-5 is an evolution of the proposed scalar control. Suppose that the reference frame will be fixed with the stator flux. If the resistive voltage drop is negligible, the stator voltage V_s is placed on the "q" axis and if the ratio V/f is maintained constant, ψ_s remains constant. With a constant ψ_s and for low value of the slip, the mechanical characteristic (torque vs speed) is linear and there is proportionality between the torque and the slip (x ω or ω_{slip}). On the other hand, the torque at constant flux is proportional to the quadrature component of the stator current i_{sq} ($T_e=n_p \cdot \psi_s \cdot i_{sq}$).

So, in a V/f control scheme and with a reference frame fixed to the stator flux, the slip frequency is proportional to the quadrature component of stator current.



Figure 8-5 Diagram of V/f control with a slip compensation

8.2 Open loop methods based on stator voltages and currents

The methods presented in the previous chapter are certainly easy to implement in the hardware of limited resources. They also have the advantage of not depending too much on the machine parameters. Conversely they have the serious defect of having slow responses to transient as they are based on the steady state model of the machine. There are schemes that offer excellent performance even in open loop. They are based on dynamic model of the machine (4 or 5 parameters) and knowledge (direct or indirect) of the stator voltages and currents. Their characteristic of operating in open loop implies that their behavior depends strongly on the machine parameters. The nonlinearity in the system and the imperfect knowledge of the values of the parameters during the transient will void the performance of these observers, especially at low speed. The main reasons for this downgrade include: saturation of ferromagnetic material, dependence of the resistance and inductance on the frequency, variation of the resistance with temperature, non-ideal inverter, problems with quantization of signals. ... Being able to compensate for these phenomena can be achieved very good performance even at low speeds.

The methods are based mainly on the dynamic equation of rotor, properly manipulated to obtain a dependence on only stator currents and stator flux (directly linked to the stator voltage).

$$0 = R_r \cdot \overline{i_r} + \frac{d\overline{\psi_r}}{dt} + j \cdot \omega_r \cdot \overline{\psi_r}$$

where ω_r is the generic speed of the reference axes respect the axes fixed with the rotor.

The first method is based on writing the equation of dynamic rotor axis in a reference fixed to the stator (stationary reference). In this case it results $\omega_r = -\omega_m$. In particular the equation on the "d" axis is:

$$0 = R_r \cdot i_{rd} + \frac{d\psi_{rd}}{dt} + \omega_m \cdot \psi_{rq}$$
$$\psi_{rd} = M \cdot (i_{rd} + i_{sd}) \Longrightarrow i_{rd} = \frac{\psi_{rd}}{M} - i_{sd}$$
$$\omega_m = \frac{-\frac{d\psi_{rd}}{dt} - \frac{\psi_{rd}}{\tau_r} + R_r \cdot i_{sd}}{\psi_{rq}}$$

where $\tau_r = M/R_r$ is the rotor time constant.

The problem now is to find the most appropriate relationship between the rotor fluxes and the stator voltages and currents.

But the rotor flux is related to the stator flux through the term $L_{ks} \cdot i_s$ and a suitable transformation gain:

$$\overline{\psi_r} = \overline{\psi_s} - L_{ks} \cdot \overline{i_s}$$

while the stator flux is calculated as an integral (pure or with low-pass filter) of the stator voltage minus the resistive voltage drop.

In conclusion, we obtain:

$$\psi_r = \psi_s - L_{ks} \cdot i_s$$

$$\frac{d\overline{\psi_r}}{dt} = \frac{d\overline{\psi_s}}{dt} - L_{ks} \cdot \frac{d\overline{i_s}}{dt}$$

$$\frac{d\overline{\psi_s}}{dt} = \overline{\psi_s} - R_s \cdot \overline{i_s}$$

$$\overline{\psi_s} = \int (\overline{\psi_s} - R_s \cdot \overline{i_s}) dt$$

...

The parameters are:

- R_s may be influenced by temperature, though in a lower way than the rotor resistance; it is not much influenced by the frequency, especially for small-medium machine
- τ_r depends on temperature, saturation and frequency (especially for deep slots)
- L_{ks} depends very little on saturation (it takes into account the leakage flux in the air-gap)

There is only the difficulty of calculating the derivative of the stator current as the signals from the sensors are generally very noisy. The stator voltage, however, can be accessed directly or estimated from a given voltage of the dc bus and the configuration of the inverter.

Another formulation of the mechanical speed, very nice, is as follows:

$$\overline{\psi'_s} = \overline{\psi_s} - (L_{ks} + M) \cdot \overline{i_s} \left(= M \cdot \overline{i_r}\right)$$
$$\omega_m = \frac{\operatorname{Im}(\frac{d\overline{\psi_r}}{dt} \cdot \underline{\psi'_s})}{\operatorname{Re}(\overline{\psi_r} \cdot \psi'_s)}$$

where Im() means the imaginary part, Re() the real part and the underscore refers to the complex conjugate.

Again we need to find the most appropriate link between the rotor flux and the stator voltages and currents.

8.3 Adaptive methods (MRAS)

The open-loop methods have the advantage of easy implementation. But they are definitely influenced by the uncertainty on the values of the machine parameters and on the measurement errors. The closed-loop methods provide better accuracy at the expense of increased algorithmic complexity.

Among these, the method MRAS (Model Reference Adaptive System) provides good results. The operating principle is based on the existence of two estimators: one based on a reference model and the other on an "adaptive" model. The first doesn't depend on the parameters or variables to be identified, while the second is based on a model where these parameters appear. The error between the two estimations is used to obtain the unknown quantities (in our case the mechanical speed).



Figure 8-6 Diagram of a MRAS observer

A possible implementation of a MRAS observer will be presented, now.

The equations of the reference model are referred to a stationary reference frame. The stator flux is obtained from the integration of the voltage after the stator resistance, while the rotor flux is calculated from the stator flux and the stator current. The inputs of the estimator are, therefore, the stator currents and voltages on axes fixed with stator.

The equations become:

$$\psi_{s\alpha} = \int (v_{s\alpha} - R_s i_{s\alpha}) dt$$
$$\psi_{s\beta} = \int (v_{s\beta} - R_s i_{s\beta}) dt$$
$$\psi_{r\alpha} = \psi_{s\alpha} - L_{ks} i_{s\alpha}$$
$$\psi_{r\beta} = \psi_{s\beta} - L_{ks} i_{s\beta}$$

The adaptive model is based on the classic estimator I Ω (current and speed) where the mechanical speed is the estimated one.

The equations are:

$$p\hat{\psi}_{r\alpha} = -\frac{R_r}{M}\hat{\psi}_{r\alpha} - \hat{\omega}_m\hat{\psi}_{r\beta} + R_r i_{s\alpha}$$
$$p\hat{\psi}_{r\beta} = -\frac{R_r}{M}\hat{\psi}_{r\beta} + \hat{\omega}_m\hat{\psi}_{r\alpha} + R_r i_{s\beta}$$

The inputs to the estimator, therefore, are stator currents on stationary axes and the estimated mechanical speed.

The estimated mechanical speed is obtained as the output of a classical PI controller with proportional and integral action. The input to the PI is somehow an error. For example, it can be chosen the following value:

$$e = \operatorname{Im}(\overline{\psi}_r \cdot \hat{\psi}_r)$$

which is a function of the product of the fluxes modules and of the sine of the angle between them.



Figure 8-7 An example of a MRAS observer

8.4 Observers

A closed-loop estimator is called observer. In addition to the MRAS techniques presented in the previous paragraph in the literature there are two large groups of observers: deterministic or stochastic. The most famous observer of the first family is the Luenberger observer, applicable to linear time invariant systems. For nonlinear deterministic systems with time-varying there is an extended version of that observer (ELO).

A famous stochastic observer is the Kalman filter for linear systems and the extended Kalman filter (EKF) for nonlinear stochastic systems.

AC machine are non linear system, so that only extended versions will be introduced now.

8.4.1 Extended Luenberger Observer (ELO)

The classic Luenberger observer can estimate the state of linear time invariant systems described by a standard format:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

The observer is described by the following system of differential equations:

$$\hat{\dot{x}} = A\hat{x} + B\hat{u} + G(y - \hat{y})$$
$$\hat{y} = C\hat{x}$$

The observer scheme is presented in Figure 8-8.



Figure 8-8 Scheme of an Extended Luenberger Observer (ELO)

It may be noted that the structure is identical to that of a Kalman filter. The method for the calculation of the matrix of gains G is different.

The extended version of the observer applies to systems with nonlinear and time variations. It derives from the basic version, but the model must be linearized around a point on the trajectory of the state.

The vector of parameters to be estimated (x_p) must be added to the state variables chosen to describe the model (x_n) . The system model becomes:

$$\dot{x} = f(x) + Bu = [f_n(x_n, x_p), f_p(x_n, x_p)]^T + Bu$$
$$y = Cx = C[x_n, x_n]^T$$

A possible representation of the model is as follows:

The matrix G is the matrix of gains and is not constant but depends on the point around which the system is linearized. At each integration step, the matrix G must be recalculated on the basis of the observer poles required (i.e. the speed of its response).

Choosing a constant G simplifies the method, but the system has a dynamic response which depends on the operating point.

8.4.2 Extended Kalman Filter (EKF)

The Kalman filter has the same structure already presented in Figure 8-8, but the logic is completely different. In particular, in a deterministic observer the noise, the measurement and model errors are considered disturbances and the observer must be robust enough to be able to converge in any case. The Kalman filter, however, could not operate on a system without noise. In particular, the noise must be white and there must be correlation between the model errors (noise) and measurement errors (noise).

The main difference between an observer like ELO and EKF is the way of calculation of the matrix G (or K in the Kalman filter). In the first case, the matrix is obtained using a deterministic method, defining the poles of the observer. In the second case it is obtained as a linear combination of three sub-matrices of covariance: P refers to the state vector of the system, Q the vector of model noise, and R to the noise in the measurements. The difficulty lies in determining such matrices as parameters depend on the characteristics of noise, generally not known a priori. Often, therefore, they are realized after repeated attempts at convergence.

The system model can be represented by any system of independent state variables. The mechanical speed should also be present.

The Extended Kalman Filter can be used also to reconstruct, in addition to non-measurable state variables, the value of some parameter that varies during the transient.

The main problems of this method are:

- it is not true that the system noise is white because it is linked to the modulation technique
- the value of parameters has to be known very well in order to increase the speed and precision of the method
- at low speed, the information about the voltages are very attenuated
- the method is recursive and requires high computational power.

8.5 Methods based on the anisotropy of the magnetic circuit

To make better use of iron, the present tendency of designers of induction machines is to put the rated operating point in the saturation zone. This leads to the birth of a third harmonic component, in the voltage between phase and star point of the windings. It corresponds to a third harmonic magnetizing flux, in phase with the magnetizing flux at the fundamental frequency; so there is a correspondence between the maximum of the two fluxes. The third harmonic flux does not depend on the loading conditions and it is virtually free of noise.

In the induction machine, there are two phenomena of saturation: the saturation due to the teeth and that present in the iron of the stator yoke. In the first, the saturated stator teeth are those in the direction of the magnetizing flux. In this way the induction in the gap shows a reduction of its maximum value. The third harmonic has a minimum at this point.



Figure 8-9 air gap induction waveform due to the teeth saturation (solid line: real trend; light stroke: first harmonic; dotted line: the third harmonic)

The stator yoke saturation is due to the magnetomotive force distribution along the air-gap. In correspondence to the interpole in the yoke, the flux is maximum and can send the iron in its saturation zone. In this case, the induction waveform in the gap is completely different from the previous case: at the maximum value of the magnetizing flux, the third harmonic has its maximum. In this work we assume that the saturation of the first type is predominant.



Figure 8-10 air gap induction waveform due to the yoke saturation (solid line: real trend; light stroke: first harmonic; dotted line: the third harmonic)

It can be shown that the sum of the three-phase voltages with respect to the star point of the stator windings is three times the third harmonic voltage (having neglected the higher harmonics due mainly to the presence of the slots).

$$v_{sa} = R_s \cdot i_{sa} + L_s \cdot \frac{di_{sa}}{dt} + e_{sa}$$
$$v_{sb} = R_s \cdot i_{sb} + L_s \cdot \frac{di_{sb}}{dt} + e_{sb}$$
$$v_{sc} = R_s \cdot i_{sc} + L_s \cdot \frac{di_{sc}}{dt} + e_{sc}$$

But every emf e_{sn} can be expressed as the sum of different harmonics. Stopping at the first and third, we have:

$$e_{sa} = e_{s1} \cdot \cos(\omega_s t) + e_{s3} \cdot \cos(3\omega_s t) + \dots$$
$$e_{sb} = e_{s1} \cdot \cos(\omega_s t - \frac{2}{3}\pi) + e_{s3} \cdot \cos[3(\omega_s t - \frac{2}{3}\pi)] + \dots$$
$$e_{sc} = e_{s1} \cdot \cos(\omega_s t + \frac{2}{3}\pi) + e_{s3} \cdot \cos[3(\omega_s t + \frac{2}{3}\pi)] + \dots$$

So because the system has three wires

$$i_{sa} + i_{sb} + i_{sc} = 0$$
$$\frac{di_{sa}}{dt} + \frac{di_{sb}}{dt} + \frac{di_{sc}}{dt} = 0$$

it results:

 $v_{sa} + v_{sb} + v_{sc} = 3 \cdot e_{s3} \cdot \cos(3\omega_s t)$

By integrating the total voltage, you obtain the third harmonic magnetizing flux which is in phase with the main flux. Moreover, its value is independent of load conditions and speed. It is also virtually free from the noise of the modulator. So there will be a law that links the two modules (the third harmonic flux and the main flux). This law is the same which links the third and first harmonic of the voltage. Using a classical no-load test, that function can be built easily. Knowing the stator leakage inductance it is possible to calculate the stator flux:

$$\overline{\psi_s} = L_{ks} \cdot \overline{i_s} + \overline{\psi_r}$$

and the electromagnetic torque

 $T_e = n_p \cdot \operatorname{Im}(\overline{i_s} \cdot \psi_r)$

However, there are two problems:

- at low speed the voltage becomes evanescent
- the star point of the stator windings has to be accessible in order to measure the threephase voltages.

For the measurements of phase voltages three sensors are needed. A low-cost solution can be obtained by using three single phase transformers. If the primaries are connected in parallel to each stator windings (between a terminal and the star point) and the secondaries are connected in series, the voltage measured across the series is equal to the sum of the three phase voltages, i.e. three times the third harmonic voltage. This value may be used directly by the control system, as the galvanic insulation form the power is provided by the transformers themselves.

Another technique uses the saliencies of the machine (due to the slots) to estimate the mechanical speed. In a machine, the presence of slots/teeth implies a variable reluctance in the air-gap with a frequency proportional to the number of the slots. Given the voltage, the stator current have a spectrum that contains different harmonics: some related to the supply frequency, others to the switching frequency and, finally, some related to the number of rotor slots. The switching frequency can be eliminated by appropriate low pass filters or by careful choice of the acquisition instant (in a space vector modulator, the sampling carried out in the middle of the period ensures the absence of switching frequency harmonics in the current).

The supply frequency (and multiples of it) is known if you know the position (and frequency) of the stator flux. The frequency due to the slots on the other hand, has the following expression:

 $f_{slots} = N_s \cdot f_s \pm k \cdot f$

where N_s is the number of slots per pole pair, f is the supply frequency, $f_s=(1-x)f$, where x is the slip, k is an integer.

The number of slots may be acquired by means of a no-load test (in this case $f_{slots} = (N_s + 1) f_s$)

From the above expression follows that the mechanical speed is:

$$\omega_s = 2\pi f_s = 2\pi \frac{f_{slots} \pm f}{N_s}$$

With appropriate filtering techniques f_{slots} can be recognized.

It should be noted that the useful frequency range within the spectrum is not very large. The slots frequency range, in fact, varies between the no-load condition where it is $(N_s+1) f$, and the condition in which the slip is maximum $[N_s (1-x_{max})+1] f$.

A final approach exploits the effect of ovality (natural or forced) of the rotor. In this situation, the reluctance in the gap shows a periodic trend (a mean value different from zero) with a period equal to the pole pitch. Stressing the system with appropriate forcing quantities, produces a modulated response of the periodic pattern.

9. PM synchronous machines

The vector control of synchronous machine, with good performance, requires the knowledge of the position of the magnetic flux of rotor (fixed with the rotor itself). The better exploitation of the machine is, in fact, when the stator current space vector is imposed perpendicular to the rotor flux (in this case the stator current only operates on torque and not on flux).

On the other hand, a mechanical speed signal is necessary if a speed control loop is desired. Such objectives are generally reached by a sensor, integral with the mechanical shaft. In a "axis" drive absolute sensors are preferably used (resolver, absolute encoder). For "spindles" drives incremental encoder are preferred, because they are easier to manage.

If conditions are unfavorable or costs are too high, the removal of these sensors is auspicious. This would involve:

- reduced system complexity
- decrease the final cost of the product
- increase the robustness and reliability of the drive
- reduction of maintenance
- do not increase the inertia of the motor
- improved noise immunity.

The solutions are different and can be combined with each other in order to be valid despite the variable operating conditions. This note proposes a few solutions, but based on different principles:

- open loop estimators requiring stator voltages and currents
- position estimators based on the third harmonic
- position estimators based on the electromotive force
- observers (ELO, EKF)
- position estimators using variable inductance and/or saturation
- estimators based on artificial intelligence techniques

Not all solutions are suitable for high performance and there is no "ideal" solution. All those proposals have strengths and weaknesses. The drive designer will decide, depending on the application, which take and whether to use more than one (depending, for example, on the value of speed). Some techniques are better suited to drives with permanent magnet synchronous machine with sinusoidal waveform (the so-called AC brushless), others for the one with trapezoidal electromotive force (DC brushless).

Many of the proposed methods require suitable measures to achieve the starting of the machine. Some indication will be given later. Not only: the methods that rely on the voltage (to estimate the stator flux or to measure the electromotive force) does not work well at low speeds because the voltage is, in some way, proportional to the speed itself. Methods based on the anisotropy of the machine, however, are especially effective at low speeds.

A special case is represented by methods based on artificial intelligence techniques. Neural networks or neuro-fuzzy networks are elegant and efficient methods to reconstruct the behavior of a system without knowing the model. Through an appropriate stage of learning, a neural network can be trained to behave like the real system. The larger is the network, the greater are the degrees of freedom. The advent of chip that integrates neural networks solves the problem of complexity of network management and will reduce the computation time at a reasonable cost.

9.1 Open loop methods based on stator voltages and currents

These methods are based on the flux estimation starting from measurements of stator voltages and currents. In steady state or quasi-stationary the speed at which the stator flux vector is moving coincides with the mechanical speed except for the number of pole pairs n_p (during transient this is no longer true). This is especially true if the torque change is limited in time.

The control method with a unity power factor is based on this principle. In fact, given a value of stator current and stator flux, torque is maximum if i_s and ψ_s are orthogonal to each other. But if you neglect the resistive drop voltage, the stator voltage v_s is the derivative of ψ_s then the two space vectors are orthogonal.

$$T_e = n_p \operatorname{Im}(i_s \cdot \psi_s)$$

So the maximum torque is achieved if v_s is in phase with i_s (as in these conditions v_s - $R_s i_s$ is still in phase with v_s), that is if the power factor is unity.



Figure 9-1 Vector diagram with unity power factor

The stator flux can be obtained by integration of the voltage after the resistive drop (reference frame fixed to the stator):

$$\frac{d\psi_s}{dt} = \overline{v_s} - R_s \cdot \overline{i_s}$$
$$\overline{\psi_s} = \int \left(\overline{v_s} - R_s \cdot \overline{i_s}\right) dt$$

And the flux position is obtained as

$$\rho_s = \arg(\psi_s) = \operatorname{atan}(\psi_{s\beta}/\psi_{s\alpha})$$

Then:

$$\omega_{s} = \frac{d\rho_{s}}{dt} \approx \omega_{n}$$

or it can be shown that:

$$\omega_{s} = \frac{\operatorname{Im}(\frac{d\psi_{s}}{dt} \cdot \underline{\psi_{s}})}{\overline{\psi_{s}} \cdot \underline{\psi_{s}}} \approx \omega_{n}$$

The value of the stator voltage can be obtained by using two or three voltage sensors at the terminals of the machine, or can be reconstructed by means of the dc bus voltage value (measured or estimated) and the switching functions. It is clear that good performance can be

achieved only if the values of voltages, currents and stator resistance (which depends on the temperature) are accurate. In particular, the voltage at low speed takes values comparable with the forward voltage drop of the switches and the effect of inverter dead time. In these circumstances, also the quantization errors can influence the calculation. The current can be affected by errors due to sensors, like: phase displacement, gain error, offset.

The choice of the method of integration is also important. To calculate the stator flux, the pure integral may be replaced by a low-pass filter.



Figure 9-2 Pure integral



Figure 9-3 Stator flux estimation using a reference system fixed with the flux itself



Figure 9-4 Low-pass filter instead of pure integral

In this way you can stop the drift due to any offset present in the measurements.

9.1.1 An original scheme with current-controlled VSI

Having a current-controlled voltage source inverter (VSI) (eg hysteresis control), a possible control scheme would be the one shown in Figure 9-5. The methodology is based on the technique with unity power factor, but can be improved by using the method presented in the previous paragraph.



Figure 9-5 Unity power factor control

This type of control, like many others (unfortunately) is not able to start the machine. It is therefore necessary to adopt alternative methods for the machine starting. The simplest is to impose a smooth acceleration ramp in an open loop control scheme. In this way, if the initial value of rotor position is properly chosen, you get a soft start.

9.2 Methods based on the third harmonic

In a DC brushless machine the performance of the three back EMF is trapezoidal with a 120° phase displacement. So a third harmonic component is present, among higher frequency harmonics. These components, for each phase, are clearly in phase with each other $(3*120^{\circ}=360^{\circ})$ and are synchronous with to the rotor. It can be shown that the sum of the three phase voltages with respect to the star point of the stator windings is three times the third harmonic voltage.

$$v_{sa} = R_s \cdot i_{sa} + L_s \cdot \frac{di_{sa}}{dt} + e_{sa}$$
$$v_{sb} = R_s \cdot i_{sb} + L_s \cdot \frac{di_{sb}}{dt} + e_{sb}$$
$$v_{sc} = R_s \cdot i_{sc} + L_s \cdot \frac{di_{sc}}{dt} + e_{sc}$$

But every emf e_{sn} can be expressed as the sum of different harmonics. Stopping at the first and third, we have:

$$e_{sa} = e_{s1} \cdot \cos(\omega_m t) + e_{s3} \cdot \cos(3\omega_m t) + \dots$$
$$e_{sb} = e_{s1} \cdot \cos(\omega_m t - \frac{2}{3}\pi) + e_{s3} \cdot \cos[3(\omega_m t - \frac{2}{3}\pi)] + \dots$$
$$e_{sc} = e_{s1} \cdot \cos(\omega_m t + \frac{2}{3}\pi) + e_{s3} \cdot \cos[3(\omega_m t + \frac{2}{3}\pi)] + \dots$$

So, because the system has three wires

$$i_{sa} + i_{sb} + i_{sc} = 0$$
$$\frac{di_{sa}}{dt} + \frac{di_{sb}}{dt} + \frac{di_{sc}}{dt} = 0$$

it results

 $v_{sa} + v_{sb} + v_{sc} = 3 \cdot e_{s3} \cdot \cos(3\omega_m t)$

Studying the appropriate zero crossings of this voltage it is possible to reconstruct the time required for switching the inverter from one phase to another (like in the switches control logic of a DC brushless drive).

In addition, the integral of the third harmonic voltage corresponds to the third harmonic rotor flux. It is almost free from the noise of the modulator and its value is independent of load and speed. The analysis of this flux may, therefore, allows the reconstruction of the rotor position. However, there are two problems:

- at low speed the voltage becomes evanescent
- the star point of the stator windings has to be accessible and the three phase voltages has to be measured

The measurements of phase voltages need three sensors. A low-cost solution can be obtained by using three single phase transformers. If the primaries are connected in parallel to the stator windings (between a terminal and star point) and the secondaries are in series, the voltage measured across the series is equal to the sum of the three phase voltages, ie three times the third harmonic voltage. The galvanic isolation is provided by the transformers themselves.

9.3 Methods based on the electromotive force

It is a valid method for DC brushless machines. It is based on the fact that every phase winding is disconnected from the inverter for 120 electrical degrees. The winding voltage, because the current is zero, is equal to the trapezoidal emf.

One possible method is based on the fact that when the emf of the disconnected phase passes through zero, the phase voltage referred to the negative terminal ("N") of the dc bus is equal to the voltage between the star point (n) and the same terminal N.

$$v_{saN} = R_s \cdot i_{sa} + L_s \cdot \frac{di_{sa}}{dt} + e_{sa} + v_{nN}$$
$$i_{sa} = 0 \quad e_{sa} = 0$$
$$v_{saN} = v_{nN}$$

But at the same time, if the phase b is connected to the positive terminal of the dc bus and phase c is connected to the negative, we have:

 e_{sc}

$$v_{sb} = R_s \cdot i_{sb} + L_s \cdot \frac{di_{sb}}{dt} + e_{sb}$$

$$v_{sc} = R_s \cdot i_{sc} + L_s \cdot \frac{di_{sc}}{dt} + e_{sc}$$

$$i_{sb} = -i_{sc}$$

$$v_{sb} - v_{sc} = V_{dc}$$

$$V_{dc} = 2 \cdot R_s \cdot i_{sb} + 2 \cdot L_s \cdot \frac{di_{sb}}{dt} + e_{sb} -$$

With reference to the negative terminal of the dc bus:

$$\begin{aligned} v_{sbN} &= V_{dc} = R_s \cdot i_{sb} + L_s \cdot \frac{di_{sb}}{dt} + e_{sb} + v_{nN} \\ V_{dc} &= 2 \cdot (V_{dc} - e_{sb} - v_{nN}) + e_{sb} - e_{sc} \\ e_{sb} + e_{sc} &= 0 = V_{dc} - 2 \cdot v_{nN} \\ v_{nN} &= \frac{V_{dc}}{2} \end{aligned}$$

We reach the same result if the phase b is connected to the negative terminal of the dc bus and phase c is connected to the positive one.

Then monitoring the voltage between the disconnected phase and the negative terminal of the dc bus and comparing it to $V_{dc}/2$ are the adequate activities. When the comparison occurred, it means that the corresponding emf went to zero. In light of this and with appropriate delays, you can choose the instant at which the configuration of the inverter has to be changed.

This method suffers from problems typical at low speed (emf is too low), it is not able to start the machine and is subject to modulation interference.

Another method uses a suitable phase-locking with the voltage between the disconnected phase and the negative terminal of the dc bus. This makes it possible to reconstruct the value of the position of the rotor.

9.4 Observers

The knowledge of phase voltages and currents allows the use of methods based on observers such as the extended Kalman filter (EKF) or the extended Luenberger observer (ELO).

The method is similar to that shown for an induction machine (paragraph 8.4).

The model changes in this way:

 $x = [i_{sd}, i_{sq}, \omega_m, \theta_m]^{T}$ $y = [i_{scs}, i_{s\beta}]^{T}$ $u = [v_{scs}, v_{s\beta}, e_F]^{T}$ where $e_F = \omega_m \psi_{pm}$ is considered constant within the filter iteration.

9.5 Methods based on the anisotropy of the machine

In a control of synchronous machine (both AC and DC), the estimation of the rotor position is possible by exploiting the nonlinearity of the magnetic circuit: saturation and anisotropy. Some schemes are now proposed. In general, these methods are effective even at zero speed.

9.5.1 Saturation

In synchronous machines with surface mounted permanent magnets, the rotor magnetic circuit is slightly saturated in the flux direction. Assuming a reference frame fixed with the rotor flux, the inductance L_d along the axis "d" is lower than the inductance L_q along the quadrature axis "q". The reluctance in the air-gap is a function of the mechanical position and is given by the sum of a constant and an AC signal with a period equal to the pole pitch (in a machine with two poles, then, the reluctance in the air-gap has two maxima and two minima). When the rotor is stopped, it is possible to estimate the mechanical position by feeding the machine with a rotating voltage vector. The current response (for a given position θ_m) is described by the following equation (we neglect the resistive drop):

$$\frac{di_s}{dt} = \frac{\overline{v_s(\theta_m, t)}}{L(\theta_m)}$$

from which you can calculate $L(\theta_m)$. The direction in which L is at its minimum value is the direction of rotor flux (save the sign)

9.5.2 Anisotropy

In an interior permanent magnet synchronous machine (IPM), the structure of the magnetic circuit is not symmetrical in all directions. In fact, in correspondence of the magnet (which has a permeability similar to that of air), the equivalent air gap is larger.

The inductances of the three stator windings will have the following behavior (θ_m in electrical radians) (the harmonics higher to the second are neglected in the series expansion):

$$L_{sa}(\theta_m) = L_{so} + L_{s2} \cdot \cos(2 \cdot \theta_m)$$
$$L_{sa}(\theta_m) = L_{sa} + L_{s2} \cdot \cos(2 \cdot \theta_m) + \frac{2}{\pi}$$

$$L_{sc}(\theta_{m}) = L_{so} + L_{s2} \cdot \cos(2 \cdot \theta_{m} - \frac{2}{3}\pi)$$

$$L_{sc}(\theta_{m}) = L_{so} + L_{s2} \cdot \cos(2 \cdot \theta_{m} - \frac{2}{3}\pi)$$

The mutual inductances between stator windings have similar trends.

Therefore, if you place the reference fixed with the rotor flux, it will still $L_d < L_q$.

The value of the inductance $L_{sa}(\theta_m)$ can be obtained using the differential equation of the phase "a":

$$v_{sa} = R_s \cdot i_{sa} + L_{sa}(\theta_m) \cdot \frac{di_{sa}}{dt} + e_{sa}$$

with e_{sa} is proportional to the mechanical speed.

Another technique is to add a high frequency voltage to the voltage reference (required by the regulator). As a result, this voltage imposes on the air gap (the resistive drop is certainly negligible at these frequencies) a rotating magnetic field at a speed much higher than the mechanical speed. This field will find a magnetic structure, variable with the rotor position. The motor current will be equal to the sum of the current at the rated frequency plus high frequency component modulated by the effective air gap (in particular by a function $\cos(2\theta_m)$). In a reference fixed with the rotor (located at an estimated angle θ_{m_est} respect to the stator), this current is modulated with a function like $\cos(2\theta_m-2\theta_{m_est})$. After applying appropriate demodulation techniques, the result can be considered an error. A PI controller can receive this error as input and give the estimated mechanical speed as output. θ_{m_est} is the integral of that speed.

10. References

- P. Vas: "Parameter Estimation, Condition Monitoring, and Diagnosis of Electrical Machines", Oxford University Press, 1993
- P. Vas: "Sensorless Vector and Direct Torque Control", Oxford University Press, 1998