Summary

5. Dynamic model of an induction machine

The AC machine models, necessary for the development of control systems, are obtained using prerequisite mathematics in relation to what is called the unified theory of electrical machines. The final goal of this theory is that you can transfer the control technique of a DC machine to the AC one.

At this stage there is still time to go in deep into control aspects, but now we want to introduce a dynamic model of an induction machine that allows us to study the transient behavior.

Consider the structure of a three-phase induction machine.

Suppose we have a three-phase winding either in the stator and in the rotor (consisting of three windings physically displaced by 120 mechanical degrees); in Figure 5-1 the magnetic axes of the windings are reported.

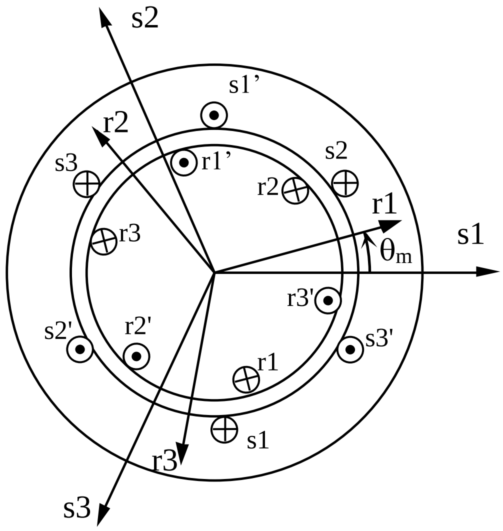

Figure 5-1: Induction machine structure

Assume that the electromagnetic structure is equipped with only two poles (as shown in Figure 5-1) and then consider valid the hypothesis that the iron permeability is very high, infinite compared to the air.

If the stator windings are supplied by a symmetrical system of voltages, as seen in the classical theory, a rotating field, with a sinusoidal distribution along the air-gap, is generated.

Suppose that the machine is rotating at a speed $\boldsymbol{\Omega}_m$; the behavior can be studied using the mutually coupled circuits theory.

This brings to a system of equations like:

$$v_k = R_k \cdot i_k + p\psi_k$$

where k refers to one of the six windings, R_k is the resistance of the k-th winding, p is the time derivative operator and ψ_k is the flux linked with the k-th winding.

We must then consider the relationship between fluxes and currents:

$$\psi_k = \sum_i L_{ki}(\theta_m) \cdot i_i$$

where the mutual inductance $L_{ki}(\theta_m)$ is a function of the mechanical angle (θ_m) between the magnetic axes of the rotor and the stator.

For example, the mutual inductance between the winding "r1" and the winding "s1" is the highest when they are aligned, while it is zero when they are at 90 degrees.

Now, only the mechanical energy balance equation has to be added:

$$T_e - T_r = J \cdot p\Omega_m$$

Now consider a reference frame fixed with the stator (stationary frame, superscript "s") applying the space phasor formula to the three stator equations. A new machine is obtained, with only two stator windings, fixed with the stator itself and orthogonal each other, passed through two currents: the real and imaginary parts of the stator current space phasor. You get:

$$\bar{v}_s^{\,s} = R_s \cdot \bar{i}_s^{\,s} + p\overline{\psi}_s^{\,s}$$

Similarly, for the rotor quantities, referred to a reference frame fixed with the rotor (a frame fixed with the rotor, superscript "r"): applying the space phasor formula to the rotor windings we obtain two rotor windings, fixed with the rotor and orthogonal each other. It results:

$$\bar{v}_r^{\,r} = R_r \cdot \bar{i}_r^{\,r} + p\overline{\psi_r}^{\,r}$$

But the two reference frames are not the same, so there is no way to compare the two quantities. It's necessary to have a unique reference frame from which to look at the variables. There are many possibilities. Among them, a reference frame fixed with the stator (stationary reference frame) is chosen. Recalling the expression of a space phasor as a function of the reference frame, we have that the rotor quantities "seen" in a stator reference frame becomes (remember that $p\left(e^{-j\theta_m}\right) = -j\dot{\theta}_m \cdot e^{-j\theta_m}$):

$$\bar{v}_r^{\,s} = R_r \cdot \bar{i}_r^{\,s} + p\overline{\psi_r}^{\,s} - j\dot{\theta}_m \overline{\psi_r}^{\,s}$$

where $-\theta_m$ is the angle between the stationary frame and the rotor winding.

In these conditions, the two magnetic axes of the stator (orthogonal to each other) are always facing the corresponding magnetic axis of the rotor, for which the mutual couplings are no longer dependent on the position between the rotor and the stator (θ_m).

The relationship fluxes/currents is expressed as:

$$\overline{\psi}_s^{\,s} = L_s \cdot \bar{i}_s^{\,s} + M \cdot \bar{i}_r^{\,s}$$

$$\overline{\psi_r}^{\,s} = L_r \cdot \bar{i}_r^{\,s} + M \cdot \bar{i}_s^{\,s}$$

This relationship is independent of the reference frame used. The important result is that it is the same for the rotor and the stator (θ_m).

The model now presented is called 5 parameters dynamic model of the asynchronous machine (R_s, R_r, L_s, L_r, M)

The expression of the torque is achieved from the energy balance.

The total instantaneous power entering the system from the stator terminals is the sum of the incoming power from the terminals of the sα winding and from sβ winding terminals. This sum can be represented using the space phasors approach as the real part of the product between the voltage and the conjugate of the current.

The same reasoning for the rotor terminals.

$$v_{s\alpha} i_{s\alpha} + v_{s\beta} i_{s\beta} + v_{r\alpha} i_{r\alpha} + v_{r\beta} i_{r\beta} = \mathrm{Re}(\bar{v}_s^{\,s} \cdot \underline{i}_s^{\,s}) + \mathrm{Re}(\bar{v}_r^{\,s} \cdot \underline{i}_r^{\,s}) =$$
$$= R_s \cdot i_s^{s\,2} + \mathrm{Re}(p\overline{\psi}_s^{\,s} \cdot \underline{i}_s^{\,s}) + R_r \cdot i_r^{s\,2} + \mathrm{Re}(p\overline{\psi}_r^{\,s} \cdot \underline{i}_r^{\,s}) + \mathrm{Re}(-j\dot{\theta}_m \overline{\psi}_r^{\,s} \cdot \underline{i}_r^{\,s})$$

The total input power is: dissipated into heat by the Joule effect (terms Ri^2); used to change the magnetic energy stored in the inductors; transformed into mechanical power. In effect, the terms:

$$\mathrm{Re}(p\overline{\psi}_s^{\,s} \cdot \underline{i}_s^{\,s}) + \mathrm{Re}(p\overline{\psi}_r^{\,s} \cdot \underline{i}_r^{\,s}) = i_{s\alpha} p\psi_{s\alpha} + i_{s\beta} p\psi_{s\beta} + i_{r\alpha} p\psi_{r\alpha} + i_{r\beta} p\psi_{r\beta}$$

represent precisely the variation of internal energy of the four equivalent inductors (two on the rotor and two on the stator, isotropic machine).

So the mechanical power is:

$$P_m = \mathrm{Re}(-j\dot{\theta}_m \overline{\psi}_r^{\,s} \cdot \underline{i}_r^{\,s}) = \mathrm{Im}(\dot{\theta}_m \overline{\psi}_r^{\,s} \cdot \underline{i}_r^{\,s}) = \dot{\theta}_m \,\mathrm{Im}(\overline{\psi}_r^{\,s} \cdot \underline{i}_r^{\,s})$$

But since there are the following relations:

$$P_m = \Omega_m \cdot T_e$$

and

$$\dot{\theta}_m = n_p \Omega_m$$

with n_p number of pole pairs, it results

$$T_e = n_p \cdot \mathrm{Im}(\overline{\psi}_r^{\,s}\, \underline{i}_r^{\,s})$$

Using the relationship flux/current it is possible to obtain different expressions of the torque.

$$T_e = n_p \cdot \mathrm{Im}(\overline{\psi}_r \underline{i}_r) = n_p \cdot \frac{M}{L_r} \mathrm{Im}(\bar{i}_s \underline{\psi}_r) = n_p \cdot \mathrm{Im}(\bar{i}_s \underline{\psi}_s) = n_p \cdot \frac{M}{L_r (L_s - M^2 / L_r)} \mathrm{Im}(\overline{\psi}_s \underline{\psi}_r)$$

Let us consider now a new generic reference frame *dq*, rotated by θ_s with respect to the stator (Figure 5-2). The dynamic model of the induction machine, in this new frame, is:

$$\bar{v}_s = R_s \cdot \bar{i}_s + p\overline{\psi_s} + j\dot{\theta}_s \overline{\psi_s}$$
$$\bar{v}_r = R_r \cdot \bar{i}_r + p\overline{\psi_r} + j\dot{\theta}_r \overline{\psi_r}$$
$$T_e - T_r = J \cdot p\Omega_m$$

with $\theta_r = \theta_s - \theta_m$

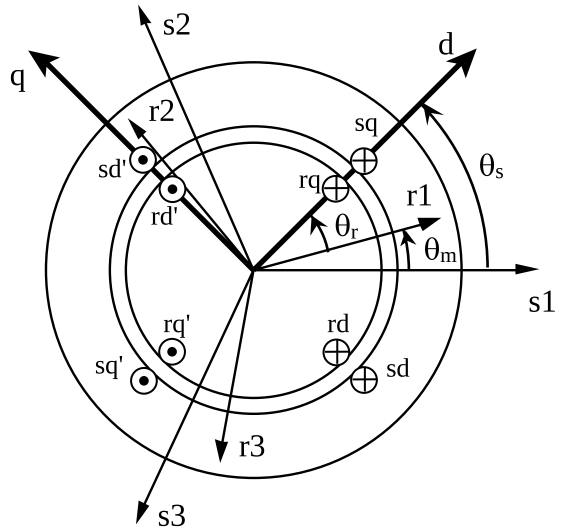

Figure 5-2: The machine and the new *d* and *q* axes

A final observation is necessary to show how the use of space phasors and the adoption of a single reference frame for the stator to the rotor has eliminated the dependence of the mutual inductances by the angle θ_m between stator and rotor. In this way the parameters needed to describe the machine are constant and identical to those in the classical theory (4 or 5 parameters).

5.1 Equivalent circuit of an induction machine

Substituting the relationship fluxes/currents in the stator and rotor dynamic equations on a generic axes, but maintaining unaltered the motional terms, we obtain:

$$\bar{v}_s = R_s \cdot \bar{i}_s + L_s p\overline{i_s} + Mp\overline{i_r} + j\dot{\theta}_s \overline{\psi_s}$$
$$\bar{v}_r = R_r \cdot \bar{i}_r + L_r p\overline{i_r} + Mp\overline{i_s} + j\dot{\theta}_r \overline{\psi_r}$$

Adding and subtracting Mpi_s in the first equation and Mpi_r in the second one, we obtain:

$$\bar{v}_s = R_s \cdot \bar{i}_s + L_s p\overline{i_s} + Mp\overline{i_r} + j\dot{\theta}_s \overline{\psi_s} + Mp\overline{i_s} - Mp\overline{i_s} = R_s \cdot \bar{i}_s + (L_s - M) p\overline{i_s} + Mp(\overline{i_s} + \overline{i_r}) + j\dot{\theta}_s \overline{\psi_s}$$
$$\bar{v}_r = R_r \cdot \bar{i}_r + L_r p\overline{i_r} + Mp\overline{i_s} + j\dot{\theta}_r \overline{\psi_r} + Mp\overline{i_r} - Mp\overline{i_r} = R_r \cdot \bar{i}_r + (L_r - M) p\overline{i_r} + Mp(\overline{i_s} + \overline{i_r}) + j\dot{\theta}_r \overline{\psi_r}$$

These equations represent the dynamic equivalent circuit of Figure 5-3.

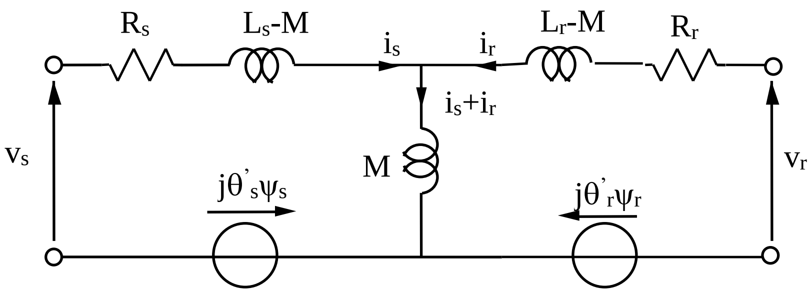

Figure 5-3: Dynamic equivalent circuit

In the case in which the stator voltage corresponds to a symmetrical three-phase system, the corresponding spatial phasor, obtained by applying the classic formula $\bar{v}_s = \sqrt{\frac{2}{3}}\left[v1(t) + \bar{\alpha} \cdot v2(t) + \bar{\alpha}^2 \cdot v3(t)\right]$, it would appear, on fixed axes, equal to:

$$\bar{v}_s = V \cdot e^{j\omega t}$$

with V equal to the rms value of the line to line voltage.
At steady state, all the quantities assume the same shape as the input voltage:

$$\bar{i}_s = I \cdot e^{j(\omega t + \varphi_i)}$$

$$\overline{\psi}_s = \Psi \cdot e^{j(\omega t + \varphi_\psi)}$$

In a similar way for the rotor quantities.
Suppose that the rotor is short-circuited (v_r=0)
Let us introduce the slip x:

$$x = \frac{\omega - \omega_m}{\omega}$$

and recalling that on fixed axes (θ_s=0), it follows that $\theta'_r = -\omega_m$ and the rotor equation, at steady state, becomes:

$$0 = R_r \cdot \bar{i}_r^{\,s} + j\omega\overline{\psi_r}^{\,s} - j\omega_m\overline{\psi_r}^{\,s} = R_r \cdot \bar{i}_r^{\,s} + jx\omega\overline{\psi_r}^{\,s}$$

where the derivative of the phasor (rotating) is jω times the phasor itself.

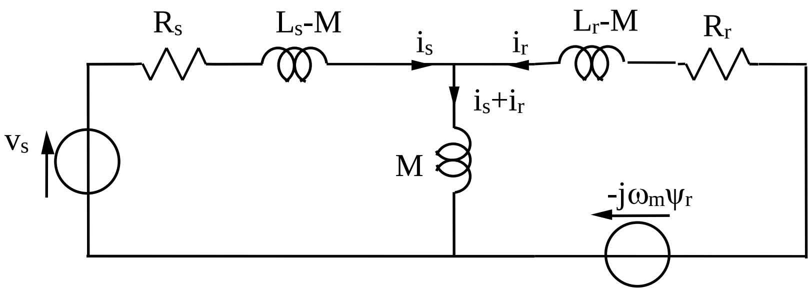

Figure 5-4: Dynamic equivalent circuit (stationary frame) , with a shorted rotor

Dividing by x and multiplying by (1-x) we have that

$$0 = \frac{1-x}{x} R_r \cdot \bar{i}_r^{\,s} + j(1-x)\omega\overline{\psi_r}^{\,s} = \frac{1-x}{x} R_r \cdot \bar{i}_r^{\,s} + j\omega_m\overline{\psi_r}^{\,s}$$

This means that the motional term (rotor side) is equivalent to a suitable value of a resistance, which is in series with the resistance of the rotor itself. In this way, the equivalent circuit scheme of the induction machine is obtained (classical theory).

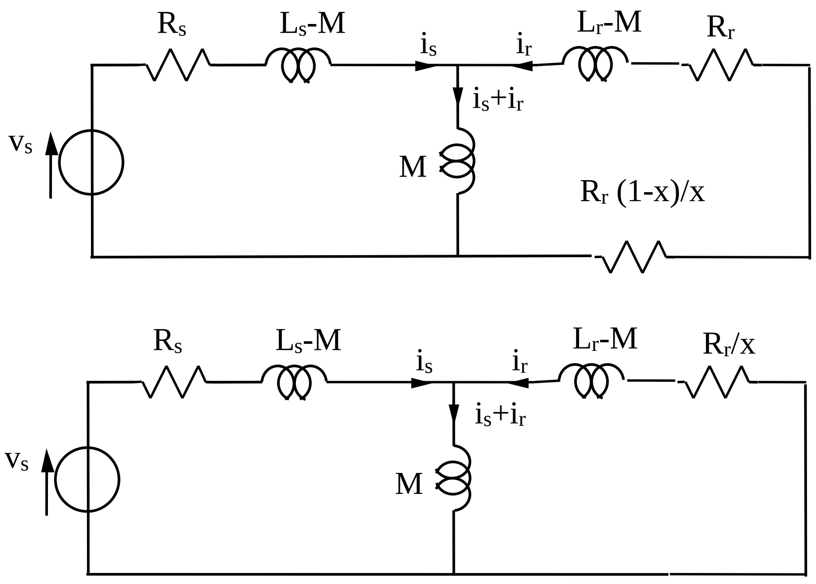

Figure 5-5: Steady state equivalent circuit: classical theory

It can be said that the classical theory is a special case of the dynamic theory.

5.2 Four parameters dynamic model

Suppose you connect to the rotor terminals an ideal transformer with transformation ratio k, maintaining the same quantities at the rotor terminals.
The rotor quantities, at the left of the transformer, will be linked to the original variables by the following relations:

$$\frac{\bar{v}_r}{\bar{v}'_r} = \frac{\overline{\psi_r}}{\overline{\psi'_r}} = \frac{\bar{i}'_r}{\bar{i}_r} = \frac{1}{k}$$

The dynamic equations become

$$\bar{v}_s = R_s \cdot \bar{i}_s + L_s p\overline{i_s} + Mpk\overline{i'_r} + j\dot{\theta}_s\overline{\psi_s}$$

$$\frac{\bar{v}'_r}{k} = R_r \cdot k\bar{i}'_r + L_r pk\overline{i'_r} + Mp\overline{i_s} + j\dot{\theta}_r\frac{\overline{\psi'_r}}{k}$$

therefore

$$\bar{v}_s = R_s \cdot \bar{i}_s + L_s p\overline{i_s} + kMp\overline{i'_r} + j\dot{\theta}_s\overline{\psi_s}$$

$$\bar{v}'_r = k^2 R_r \cdot \bar{i}'_r + k^2 L_r p\overline{i'_r} + kMp\overline{i_s} + j\dot{\theta}_r\overline{\psi'_r}$$

that corresponds to the equivalent circuit of Figure 5-6 (with a procedure similar to that introduced above).

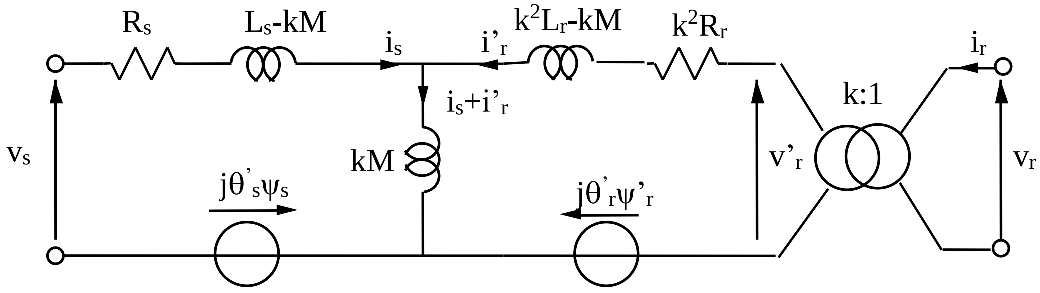

Figure 5-6: Addition of an ideal transformer

Choose k so as to cancel the series inductance rotor side

$$k^2 L_r - kM = 0 \Rightarrow k = \frac{M}{L_r}$$

we obtain the four parameters equivalent circuit ((L_{ks}=L_s-M^2/L_r : inductance of short circuit):

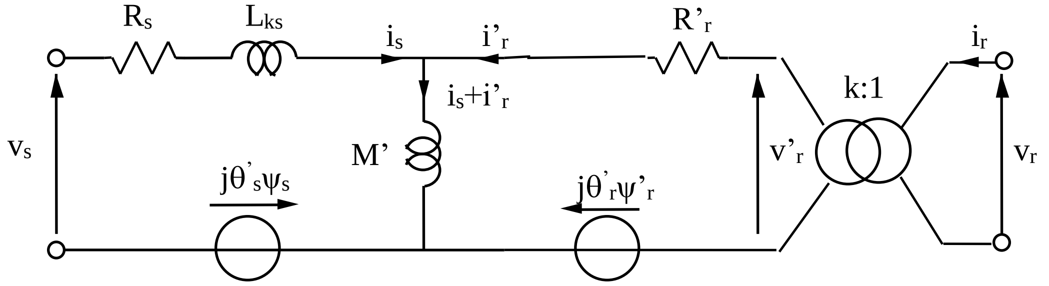

Figure 5-7: Choice of a suitable transformation ratio k=M/L_r

The four parameters dynamic model of an induction machine becomes, then:

$$\bar{v}_s = R_s \cdot \bar{i}_s + p\overline{\psi_s} + j\dot{\theta}_s \overline{\psi_s}$$
$$\bar{v}'_r = R'_r \cdot \bar{i}'_r + p\overline{\psi'_r} + j\dot{\theta}_r \overline{\psi'_r}$$
$$\overline{\psi_s} = L_{ks}\bar{i}_s + M'(\bar{i}_s + \bar{i}'_r) = L_{ks}\bar{i}_s + \overline{\psi'_r}$$
$$\overline{\psi'_r} = M'(\bar{i}_s + \bar{i}'_r)$$

The torque is

$$T_e = n_p \cdot \mathrm{Im}(\overline{\psi}_r \underline{i}_r) = n_p \cdot \mathrm{Im}(\overline{\psi}'_r \underline{i}'_r)$$

and basing on the rotor flux expression

$$T_e = n_p \cdot \mathrm{Im}(\overline{\psi}'_r \underline{i}'_r) = n_p \cdot \mathrm{Im}(\overline{\psi}'_r (\frac{\underline{\psi}'_r}{M'} - \underline{i}_s)) = -n_p \cdot \mathrm{Im}(\overline{\psi}'_r \underline{i}_s) = n_p \cdot \mathrm{Im}(\bar{i}_s \underline{\psi}'_r)$$

on the other hand, using the expression of the stator flux

$$T_e = n_p \cdot \mathrm{Im}(\bar{i}_s \underline{\psi}'_r) = n_p \cdot \mathrm{Im}(\bar{i}_s(\underline{\psi}_s - L_{ks}\underline{i}_s)) = n_p \cdot \mathrm{Im}(\bar{i}_s \underline{\psi}_s)$$

This expression is also valid for the 5 parameters model.
So, for a 4 parameters model it is:

$$T_e = n_p \cdot \mathrm{Im}(\overline{\psi}'_r \underline{i}'_r) = n_p \cdot \mathrm{Im}(\bar{i}_s \underline{\psi}'_r) = n_p \cdot \mathrm{Im}(\bar{i}_s \underline{\psi}_s) = n_p \cdot \frac{1}{L_{ks}} \mathrm{Im}(\overline{\psi}_s \underline{\psi}'_r)$$

The voltage v'$_r$ is linked to the real voltage v_r, but since normally the rotor of an induction machine is short-circuited, this voltage is zero. On the other hand, the rotor current is not measurable in the case in which the rotor is constructed by a squirrel cage structure and therefore it is not necessary to know the transformation ratio k.
From these considerations, it results that it is not necessary to know the transformation ratio k, because of no use. It is important to remember that the introduction in an ideal transformer has enabled us to introduce a degree of freedom, while maintaining the equivalence to the external effects.
This degree of freedom was used, in this case, to cancel the series inductance rotor side. Nothing prevents to take a different value of k (k=L_s/M) to cancel the series inductance stator (some applications in the wind, with power from the rotor, are easily analyzed by adopting such a model), or define a value k such that the two series inductance are equal (L_s-kM=k^2L_r-kM -> k=sqrt(L_s/L_r))..
Usually when a 4-parameter model is used, the superscript ' is not shown. It should be recalled that the two rotor resistors (4-parameter and 5-parameter) are different as the two inductances M are.

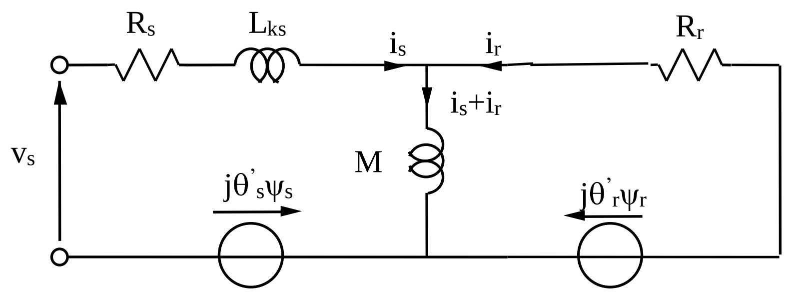

Figure 5-8: 4-parameter equivalent circuit with a shorted rotor

5.3 Theory of Vector Control of Electric Machines

5.3.1 Introduction

The gradual emergence of AC drives compared to solutions based on DC motors is undoubtedly a fact, today. A question can come first and foremost: why the industries have abandoned the old for the new solution?
It is universally known that this transition has led to a significant complication of the control structure by introducing new issues that still can be considered partially unresolved.
Before going into the mathematics of vector control, a comparison between old and new solution will be introduced.

5.3.2 Comparison between DC and AC drives

Whenever we look at this comparison, it is just natural to refer to those that may be considered the two weaknesses of the dc machine, ie:
a) commutator
b) brushes
As it is well known, both these elements are not present on both squirrel cage induction and synchronous permanent magnet machines, and this is a good reason to choose an AC solution:

- higher speed capability
- higher voltage levels because you are no longer bound by the maximum tolerable voltage between two copper segments

However, these are not the only constraints that are placed by the commutator. Most interesting fact, from the point of view of control, can be the limits that are imposed on the dynamics of the current.
A typical limit for a DC machines is the value of the derivative of armature current which is around 30 times the rated current per second. Although the most recent achievements in the DC motor design has come up to about 200 p.u./s, it should be remembered that there are no such limits for the AC machines.
However, additional parameters allow more accurate analysis of maximum performance achievable with the different solutions.
Power and speed range:
Due to the presence of the commutator, the speed of a DC machine is inherently limited. In general, with powerful drives, it is difficult to design a machine with a product speed-power higher than 2.6 10^6. For example, it is difficult to find 1350 kW DC motors with a rated speed of 1500 rpm. There are no back problems to overcome these limitations in the design of an AC motor.
Performance

The efficiency of an induction machine is generally comparable and in some cases greater than that of a DC machine even if the presence of the cage rotor introduces an additional term in the calculation of losses. In the case of motors for variable speed drives, in fact, the cage has not to limit the current at the starting of the machine.

Power factor

Considering the connection to a grid, typical industrial DC drives work with a power factor of around 0.9, in rated condition. Higher values are not possible because a margin is necessary in dynamic regime against the risk of loss of control. In the case of induction motor drives the situation does not vary much due to the presence of the DC-side filter capacitor and to the action of the voltage source inverter (VSI). It is also noted that modern AC/DC power converter, however, provides a unity power factor, with a limited harmonic content of the current.

Inertia

A machine for AC drives has generally, at the same power, less inertia than a DC machine. In a DC machine, in fact, we are often forced to increase the diameter of the machine as a result of constraints imposed by the commutator. So the machine tends to have higher inertia and thus lower dynamic performance for the same produced torque.

Protections

The protection of an induction machine is very easy for two main reasons:

1) the machine has excellent overload capability in consequence of the structure of rotor
2) the protection devices for DC machine, as we grow with the power, tend to become extremely expensive even by an economic point of view.

Maintenance

An induction motor is virtually maintenance-free if you neglect the shaft bearings. The DC motors, on the other hand, require cyclic maintenance in order to replace the brushes and to check the surfaces of the commutator.

Strength

The extremely simple structure of the induction machine is a clear indication of its strength as also demonstrated by using the machine in hostile environments even before the introduction of variable speed drives.

Zero speed operation

A DC motor has problems over long periods to provide high torque at zero speed due to the fact that always the same coils and copper segment of the commutator are stressed. The ability to operate in these conditions usually results in unnecessary oversizing of the DC machine.

Size and weight

Because of the commutator and the need to provide easy access for maintenance, the size of a DC machine are usually superior to an induction machine, at the same value of the torque.

Comparison between induction and synchronous machine solution

The traditional synchronous machine loses many of the advantages typical of alternating current solution as a result of the presence of brushes and rings for the transfer of excitation current to the rotor. It clearly except in the case of permanent magnet synchronous machine.

On the other hand, the synchronous machines can operate at unity power factor and with a leading current, but all this involves a loss of dynamic capability. Dynamic capacity which is already lower than normal as a result of the high value of synchronous inductance which limits the current transients.

The application of synchronous motor, however, becomes significant for high power, where the possibility of large converters based totally on thyristors becomes economically significant.

Vice versa, the application of permanent magnets, for the moment, are mainly confined to the low power partly as a result of the high cost of materials. In these last years, PM synchronous motor with high value of pole pairs and high power are also used as wind generators.

5.4 From the Unified Theory of the Electrical Machines to the Vector Control

If a practical point of view has demonstrated, in the previous paragraph, the superiority of the solution based on induction motors, from the control point of view the DC machine is still a point of reference.
It can be said that the focus of the theory of AC motor control is to try to recreate the same electromechanical situation of DC machine, by introducing the control of a fictitious machine, equivalent to the original one through appropriate mathematical transformations.
The strong point of the DC machine is the presence of a completely decoupled situation, from a magnetic point of view, so that you can define with precision:

- a polar axis in the same direction of the magnetization flux, generated by the excitation current
- an interpolar axis on which the armature current acts.

The particularly favorable situation is the fact that we can identify two well-defined electrical ports each capable of acting on only one of the two terms that define the electromagnetic torque.
In fact we have a first port, which can be called direct or *d*, on which the excitation current acts and a second port - quadrature or *q* - on which the armature current operates. The fact that the two ports have to work on two magnetic circuits at 90 degrees to each other, ensures complete decoupling between the two actions.

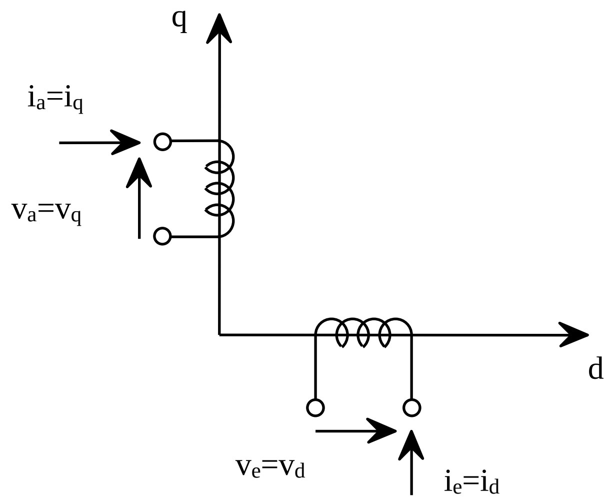

Figure 5-9: Two axes representation of a DC machine

Now consider a machine supplied by alternating current, whether induction or permanent magnet synchronous. In this case we are dealing with a three terminals single "power port": it follows that, in this case, direct and quadrature actions are not "naturally" decoupled.
The fundamental purpose of vector control is to make a mathematical transformation on the variables of the machine so as to highlight two electrical ports on the "fictitious machine" such that each of them acts on a decoupled magnetic circuit exactly as in the DC machine.
It results that:

- <u>direct axis</u>: fictitious circuit for the creation of the field. This circuit will be powered by a suitable current in the case of an induction machine to create the field in the gap, and will be an open circuit in the case of the machine with permanent magnets as the field is already present by the action of the magnets.
- <u>quadrature axis</u>: a fictitious circuit in which a current is proportional to the desired torque. This circuit therefore plays the same role played by the armature circuit of the DC machine.

The practical result is that you get a complete decoupling of the dynamics of the slow transient (ie those related to the magnetization of the machine) by the fast transient (ie, those related to the

electromechanical torque control). Therefore, it is possible to obtain, for the faster dynamics, the maximum bandwidth allowed by the machine, as having only limited by the time constant of the electric circuit seen by the converter.
With this in mind you can find a logical scheme that can be considered valid for vector control in general, regardless of the type of machine:

- the real electrical quantities are measured
- the real electrical quantities of the machines are transformed in the fictitious machine quantities
- the regulation, based on the theory of the DC machine, is applied on the fictitious variables
- the fictitious commands, to be applied to the machine, are converted into the appropriate commands to the real machine

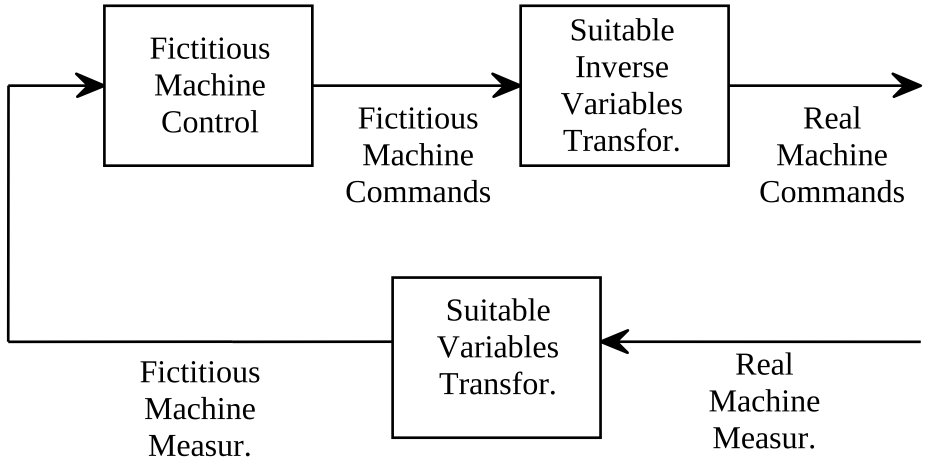

Figure 5-10: Vector control logical scheme

Then, it is easy to show what are the limitations and practical problems of this approach:

- retrieve the information needed to determine the state of the "fictitious machine"; that determines the conditions to be imposed on current and voltage of the real machine to turn them into the state variables of the "fictitious machine"
- determine the strategies to "map" the conditions of the "fictitious machine" on the real machine

In following, the principles expressed in general terms here will be applied to the design of permanent magnet synchronous and induction machine control. You will see that the appropriate transformation of variables is nothing but a Park transformation or a Space Phasor formula application, calculated with an angle of transformation according to criteria that are related to the machine physics.

5.5 Vector control of an induction machine

The vector control of an induction machine, as mentioned above, is based on a suitable choice of reference axes used by the controller so that a component of the stator current space phasor acts only on the flux, while the other one on the electromagnetic torque.
In this way, the induction machine is controlled like a DC machine where we act separately on the excitation and the armature current.
This approach to regulation has been known for ages, but its practical use has been achieved only in recent years due to the development of digital control systems and of power semiconductor switches.
To illustrate the principle of field-oriented control, the induction machine dynamic model has to be recalled.

Consider the dynamic equation of the induction machine, referred to a rotating reference frame *d*, *q*:

$$
\begin{aligned}
\overline{v_s} &= R_s \cdot \overline{i_s} + p\overline{\psi_s} + j\dot{\theta}_s \cdot \overline{\psi_s} \\
\overline{v_r} &= 0 = R_r \cdot \overline{i_r} + p\overline{\psi_r} + j\dot{\theta}_r \cdot \overline{\psi_r} \qquad \text{(5-1)} \\
p\omega_m &= \frac{n_p (T_e - T_r)}{J}
\end{aligned}
$$

where:

$T_e = n_p \cdot \mathrm{Im}(\overline{i_s} \cdot \underline{\psi_s})$ electromagnetic torque,

v_s = stator voltage;
v_r = rotor voltage;
i_s = stator current;
i_r = rotor current;
ψ_s = stator flux linkage;
ψ_r = rotor flux linkage;
$\dot{\theta}_s = \omega_s$ = speed of the rotating reference frame respect to a stationary frame (fixed with stator);
ω_m = mechanical speed (electrical degree per second);
$\dot{\theta}_r = \omega_r = \omega_s - \omega_m$ = speed of the rotating reference frame respect to a frame fixed with rotor;
R_s, R_r = stator and rotor resistances;
T_r = load torque;
n_p = number of pole pairs;
J = moment of inertia.

There is also the relationship between fluxes and currents (four parameters model):

$$
\begin{aligned}
\overline{\psi_s} &= L_{ks}\overline{i_s} + \overline{\psi_r} \qquad \text{(5-2)} \\
\overline{\psi_r} &= M \cdot (\overline{i_s} + \overline{i_r})
\end{aligned}
$$

from eq. (5-2) it results, also:

$$\overline{i_r} = \frac{\overline{\psi_r}}{M} - \overline{i_s} \qquad \text{(5-3)}$$

where:

L_{ks} short circuit stator inductance;
M magnetizing inductance

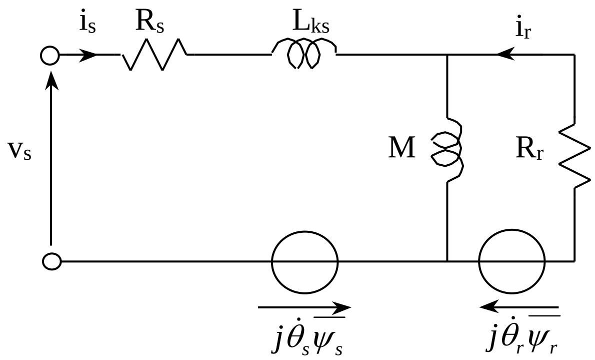

Figure 5-11 Dynamic equivalent circuit

Substituting eq. (5-2) and(5-3) in (5-1) we can get the system of equations expressed in the electrical state variables i_s and ψ_r:

$$(5\text{-}4)\qquad \begin{aligned} \overline{v_s} &= R_s \cdot \overline{i_s} + L_{ks} p\overline{i_s} + p\overline{\psi_r} + j\dot{\theta}_s \cdot L_{ks} \cdot \overline{i_s} + j\dot{\theta}_s \overline{\psi_r} \\ 0 &= \frac{R_r}{M} \cdot \overline{\psi_r} - R_r \cdot \overline{i_s} + p\overline{\psi_r} + j\dot{\theta}_r \cdot \overline{\psi_r} \end{aligned}$$

where the rotor voltages are zero, since this is an induction machine that has a short-circuited rotor.

From the rotor equations of (5-4) it results:

$$(5\text{-}5)\qquad p\overline{\psi_r} = R_r \cdot \overline{i_s} - \frac{R_r}{M} \cdot \overline{\psi_r} - j\dot{\theta}_r \cdot \overline{\psi_r}$$

Substituting equation (5-5) in the stator equations of (5-4) gives:

$$(5\text{-}6)\qquad \overline{v_s} = R_s \cdot \overline{i_s} + L_{ks} p\overline{i_s} + \left(R_r \cdot \overline{i_s} - \frac{R_r}{M} \cdot \overline{\psi_r} - j\dot{\theta}_r \cdot \overline{\psi_r} \right) + j\dot{\theta}_s \cdot L_{ks} \cdot \overline{i_s} + j\dot{\theta}_s \overline{\psi_r}$$

Rewriting (5-6) by putting $\dot{\theta}_r = \dot{\theta}_s - \dot{\theta}_m$, joining to (5-5) it is obtained

$$\overline{v_s} = (R_s + R_r) \cdot \overline{i_s} + L_{ks} p\overline{i_s} - \frac{R_r}{M} \cdot \overline{\psi_r} + j\dot{\theta}_s \cdot L_{ks} \cdot \overline{i_s} + j\dot{\theta}_m \overline{\psi_r}$$

Introducing the term $R_{ks}=R_s+R_r$ we have:

$$(5\text{-}7)\qquad \overline{v_s} = R_{ks} \cdot \overline{i_s} + L_{ks} p\overline{i_s} - \frac{R_r}{M} \cdot \overline{\psi_r} + j\dot{\theta}_s \cdot L_{ks} \cdot \overline{i_s} + j\dot{\theta}_m \overline{\psi_r}$$

Substituting the relationship of eq. (5-2) in the expression of electromechanical torque we have:

$$T_e = n_p \operatorname{Im}\left(\overline{i_s} \underline{\psi_r}\right)$$

At this point we choose the axes "*d*" and "*q*" so that the direction of the "*d*" has to be always coincident with the rotor flux space phasor; with this choice the quadrature component of the rotor flux is always zero, that is:

$$\psi_{rd} = \psi_r \qquad \text{and} \qquad \psi_{rq} = 0.$$

Therefore, eq. (5-7), on the axes d and q, becomes:

$$(5\text{-}8)\qquad \begin{aligned} v_{sd} &= R_{ks} \cdot i_{sd} + L_{ks} p i_{sd} - \frac{R_r}{M} \cdot \psi_r - \dot{\theta}_s \cdot L_{ks} \cdot i_{sq} \\ v_{sq} &= R_{ks} \cdot i_{sq} + L_{ks} p i_{sq} + \dot{\theta}_m \cdot \psi_r + \dot{\theta}_s \cdot L_{ks} \cdot i_{sd} \\ p\psi_r &= R_r \cdot i_{sd} - \frac{R_r}{M} \cdot \psi_r \\ 0 &= R_r \cdot i_{sq} - \dot{\theta}_r \cdot \psi_r \\ p\omega_m &= \frac{n_p}{J}\left(T_e - T_r\right) \\ T_e &= n_p \cdot \psi_r \cdot i_{sq} \end{aligned}$$

where $\dot{\theta}_s$ and $\dot{\theta}_r$ means respectively the speed of the reference frame (fixed with the rotor flux) respect the stator and rotor windings.

The (5-8) becomes, once expressed in canonical form:

$$pi_{sd} = \frac{1}{L_{ks}}\left[v_{sd} - R_{ks}\cdot i_{sd} + \frac{R_r}{M}\cdot\psi_r + \dot{\theta}_s\cdot L_{ks}\cdot i_{sq}\right]$$

$$pi_{sq} = \frac{1}{L_{ks}}\left[v_{sq} - R_{ks}\cdot i_{sq} - \dot{\theta}_m\cdot\psi_r - \dot{\theta}_s\cdot L_{ks}\cdot i_{sd}\right]$$

$$p\psi_r = R_r\cdot i_{sd} - \frac{R_r}{M}\cdot\psi_r \qquad (5\text{-}9)$$

$$0 = R_r\cdot i_{sq} - \dot{\theta}_r\cdot\psi_r$$

$$p\omega_m = \frac{n_p}{J}\left(T_e - T_r\right)$$

$$T_e = n_p\cdot\psi_r\cdot i_{sq}$$

Looking at the third and the last equation we see that the two components of the stator current act separately on the rotor flux and torque, because the flux depends only on the component i_{sd} while i_{sq} component acts only on the torque.

The behavior of the induction machine controlled by field orientation is thus similar to that of a DC machine: in this analogy, the direct component of stator current assumes the role of the excitation current and the quadrature component of the armature current. Of course, while in DC machine the two currents flow in two distinct windings, in this case the i_{sd} and i_{sq} are the components along the axes d and q of a unique single-phase system of currents: the transition from one to the other system is obtained through the formula of the space phasor.

The decoupling, obtained between the effects of the two components of the stator current, can simplify the control of the mechanical variables of the drive. In fact, if the rotor flux is kept constant with the aim to exploit well the iron, the torque is directly proportional to the quadrature component of stator current i_{sq} whose reference value can be directly derived from the desired value of the torque. A change of the torque value, however, may be obtained by a variation of flux acting on the i_{sd}, but given the presence of high time constant with which i_{sd} influences the flux (M/R_r), which is higher than the time constant between stator current and stator voltage (L_{ks}/R_{ks}), this mode is not suitable for a rapid adjustment of the torque T_e.

The outline of the induction machine in a rotor flux reference frame is represented in Figure 5-12.

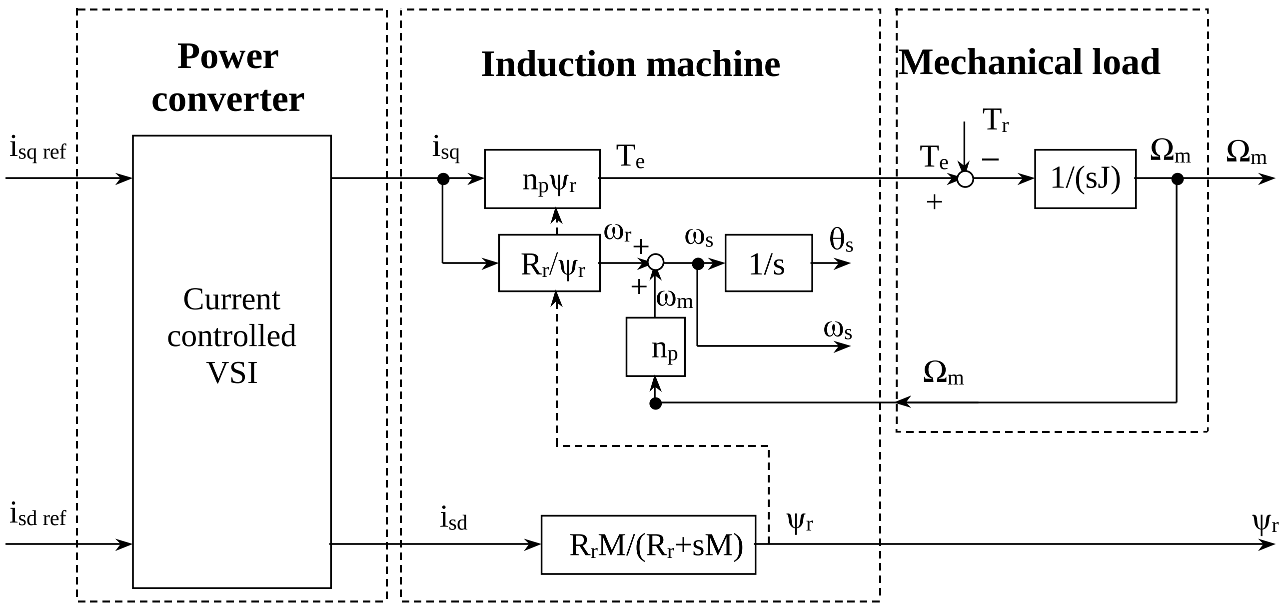

Figure 5-12 Diagram of the induction machine in a rotor flux reference frame, considering the stator currents as inputs

The schematic diagram of the drive control system is then:

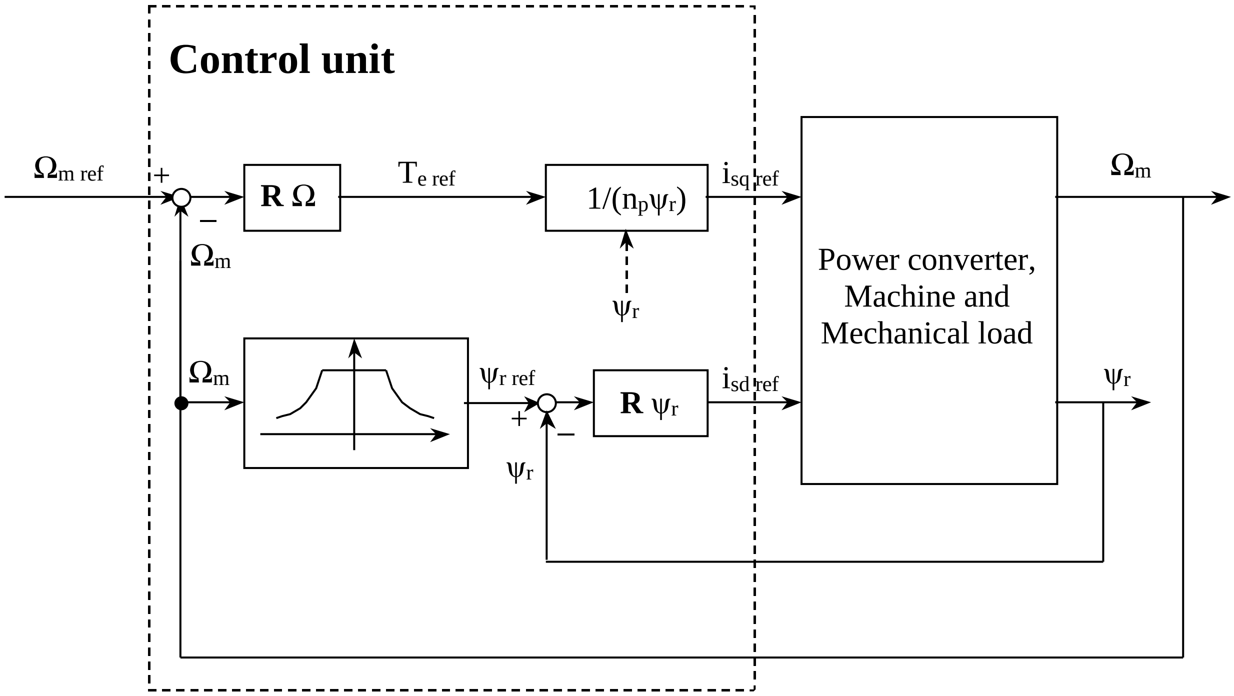

Figure 5-13 Diagram of the control

In the Figure 5-13 the block "**R** Ω" controls the mechanical speed and determines the desired value of torque; the block "**R** ψ_r" acts on the component i_{sd} so that the rotor flux is equal to its reference value $\psi_{r\ ref}$. At low speed, where the drive must provide high torque regardless of speed itself, the reference value of the flux is kept constant and equal to the maximum allowed by the magnetic circuit of the machine. At high speeds, in which the drive has to work at maximum power, the reference value of flux follows the trend dictated by the operating regions.
The implementation of field-oriented control can be performed according to different approaches (direct or indirect control, voltage or current controlled), but in any case, the obtained decoupling enables the synthesis of the control system of mechanical variables in a way completely independent.
At this point there are clear advantages of this type of control:

- direct access to the flux and the torque in an independent manner allowing field weakening action and torque and current control;
- decoupling is active both in the transient and in steady state;
- at steady state the control system works with constant quantities which makes the control less sensitive to unavoidable delays or phase displacement on the signals.

Given these advantages, the field-oriented control has some serious hurdles to overcome, mainly the acquisition of a flux signal, frequency independent, which gives module and position of the space phasor of the rotor flux. Secondly, there is a certain computational complexity mainly due to the necessary transformations of the variables.

5.6 Power supply

In the general theory of field oriented control it has been assumed that the stator currents were easily adjusted, because the power supplier has been considered as an ideal controllable current source, resulting in a considerable simplification of the model, since this hypothesis has led to neglect the equations representing the behavior of the stator circuits.
According to this view, therefore, the field oriented control scheme of a machine can be represented as in Figure 5-14 where the block "Flux estimator " is an observer designed according to the considerations reported later .

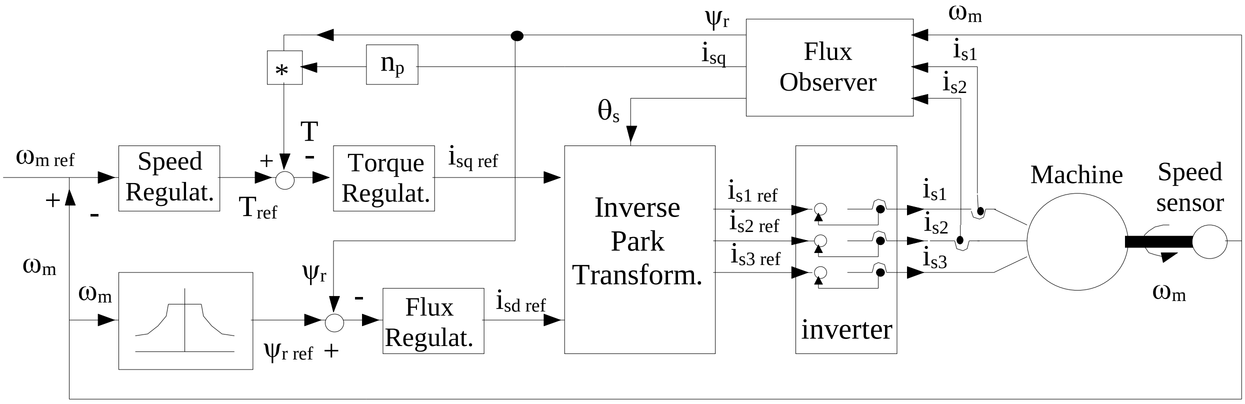

Figure 5-14 Complete diagram of field oriented control with current amplifier

In this schematic diagram, the inverter is represented by a device equipped with a fast current control so that the currents follow closely their reference values, and the inverter can be represented by a block of unity gain or a pure delay, ie type $e^{-s\tau}$, or, at most, a first order transfer function $1/(1+s\tau)$.
This solution is valid with the assumption that the power converter is composed by very fast semiconductor elements, necessary to quickly obtain a good tracking of the currents (switching frequencies above 15 kHz), which is applicable to power up to 15 to 20 kW. This structure, as seen, has the advantage of using a current setting very simple.
If you want to be released from the limitations of the switching frequency and power introduced above, you have to use a more complex control philosophy, which allows the use of slower semiconductor devices (up to 800 Hz), and to realize MW applications.
The current control is characterized by a decoupling action and is based on the PWM modulation,which involves the use of a voltage source inverter.
This situation considers the dynamics of the induction machine stator, so it will be necessary to revise the model of the machine presented in Figure 5-12.
The control scheme is as follows:

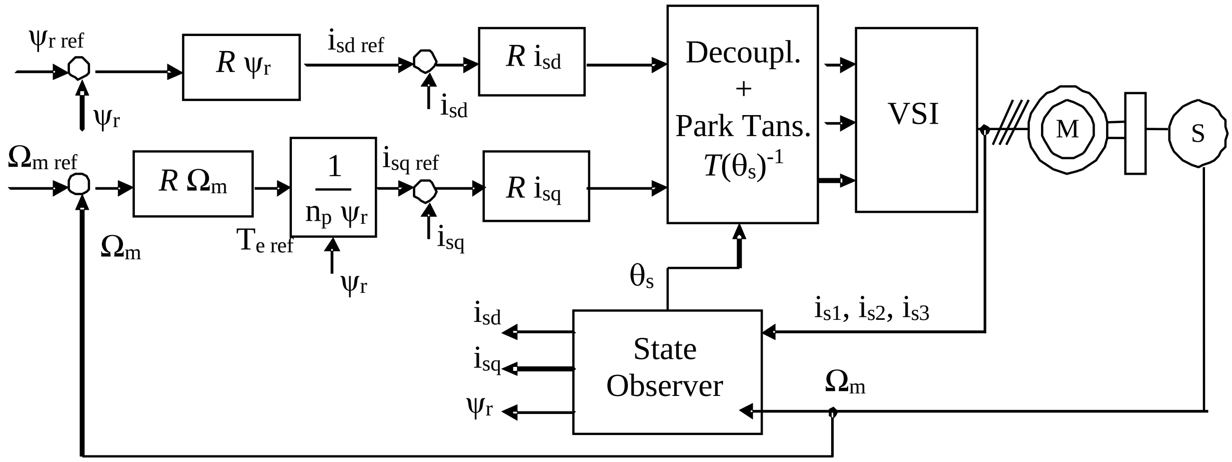

Figure 5-15 Diagram of field oriented control with decoupled current regulator

As you can see, the current control using this methodology is made by two regulators: one on the "*d*" axis and the other on "*q*" axis.

5.7 Control with a decoupled current controller

In the eq. (5-8) the real voltages able to increase or decrease the currents i_{sd} and i_{sq} are:

$$u_{sd} = R_{ks} \cdot i_{sd} + L_{ks} p i_{sd}$$
$$u_{sq} = R_{ks} \cdot i_{sq} + L_{ks} p i_{sq}$$

The term "$-\frac{R_r}{M} \cdot \psi_r$" seems to be a disturbance while "$+\dot{\theta}_m \cdot \psi_r$" is very similar to a back emf (like in a DC machine).
The terms "$-\dot{\theta}_s \cdot L_{ks} \cdot i_{sq}$" and "$+\dot{\theta}_s \cdot L_{ks} \cdot i_{sd}$" are coupling terms between the i_{sd} and i_{sq} control loops.
The decoupled current controller, with a compensation of the disturbances and the ability to perform a "start on fly", has the structure of Figure 5-16.

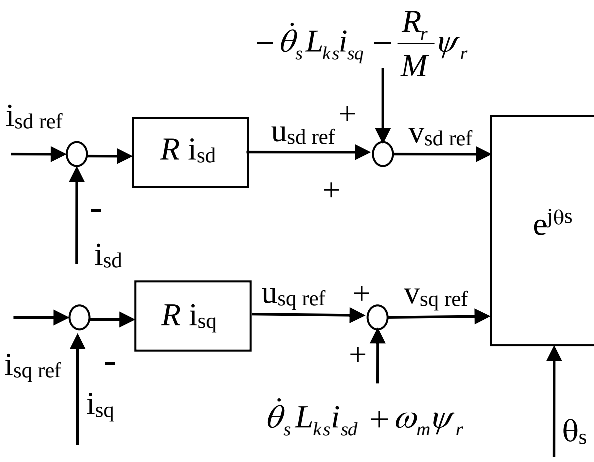

Figure 5-16 decoupling and disturbance compensation

5.7.1 Current control loops

Suppose to have an estimator free from errors, so as to be able to calculate the decoupling terms and feed-forward (disturbances compensation) accurately, the current regulator scheme becomes:

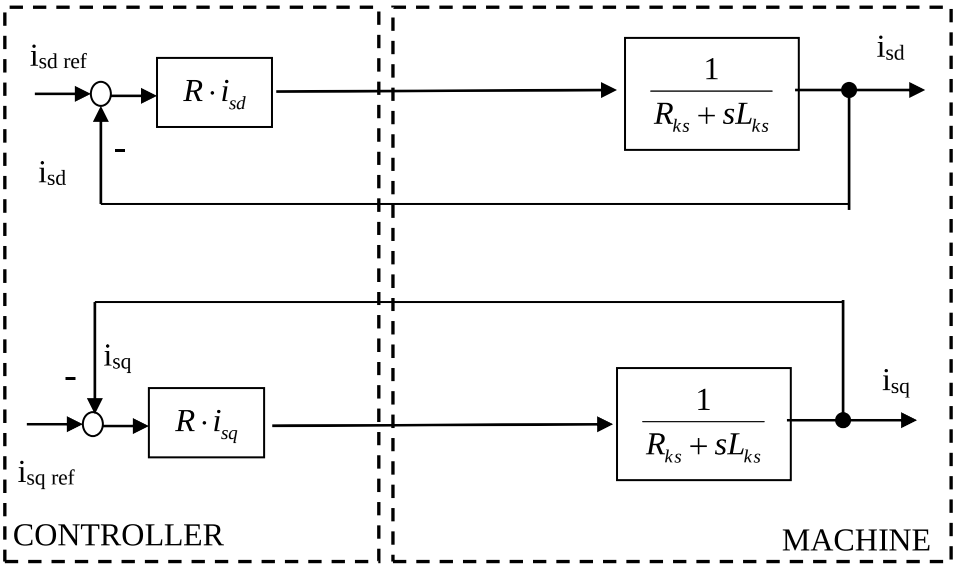

Figure 5-17 Equivalent block diagram of the current control with an ideal decoupling

The simplification is possible because the coupling terms and the disturbances in the machine are compensated by terms calculated by the controller itself.

In these conditions there are no difference between the "*d*" and "*q*" current components regulators. This suggests that the controller synthesis is carried out by calculating the gains for only one loop.
The system transfer function under control is the following:

$$BI(s) = \frac{1}{R_{ks} + s \cdot L_{ks}}$$

The closed loop transfer function, useful for the synthesis of the speed and flux control loop, is:

$$LI(s) = \frac{(KpI + \frac{KiI}{s}) \cdot BI(s)}{1 + (KpI + \frac{KiI}{s}) \cdot BI(s)}$$

where for the current regulator a PI controller is used and *KpI* and *KiI* are the gains of the PI.

5.7.2 Flux control loop

Now consider the flux control loop that acts on the current reference along the "*d*" axis:

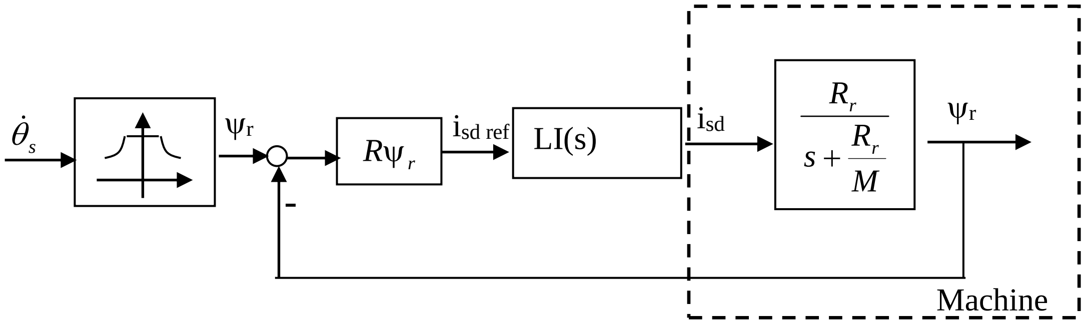

Figure 5-18 Flux control loop

In this case, the transfer function of the controlled process depends on the relationship between the flux ψ_r and the current i_{sd} and on the closed loop current control function:

$$B\psi(s) = LI(s)\frac{R_r}{s + \frac{R_r}{M}}$$

the closed loop transfer function is:

$$L\psi(s) = \frac{(Kp\psi + \frac{Ki\psi}{s}) \cdot B\psi(s)}{1 + (Kp\psi + \frac{Ki\psi}{s}) \cdot B\psi(s)}$$

where for the flux regulator a PI controller is used and $Kp\psi$ and $Ki\psi$ are the gains of the PI.

5.7.3 Speed control loop

It now addresses the synthesis of the speed control loop.

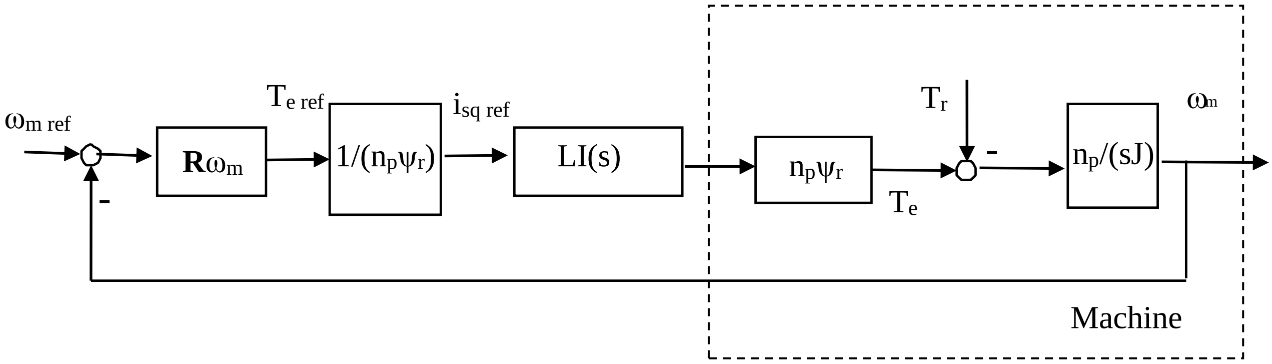

Figure 5-19 Speed control loop

The transfer function of the controlled process depends on the mechanical load and on the "*q*" axis current, which has the same transfer function of the d-axis

$$B_{\omega m}(s) = \frac{LI(s)\cdot n_p\psi_r}{n_p\psi_r}\cdot\frac{n_p}{J\cdot s}$$

The torque T_r is neglected in the expression of the mechanical load because it is considered as an external disturbance.

5.8 State estimators

For both types of control seen above, the estimation of the state variables, in the coordinates of the rotor flux, is required. Basically you need to know the rotor flux in module and position.

5.8.1 I-Ω estimator

Figure 5-20 shows the block diagram of a rotor flux estimator, called I-Ω. For the realization of this estimator, the phase currents measurements and the mechanical speed are necessary.

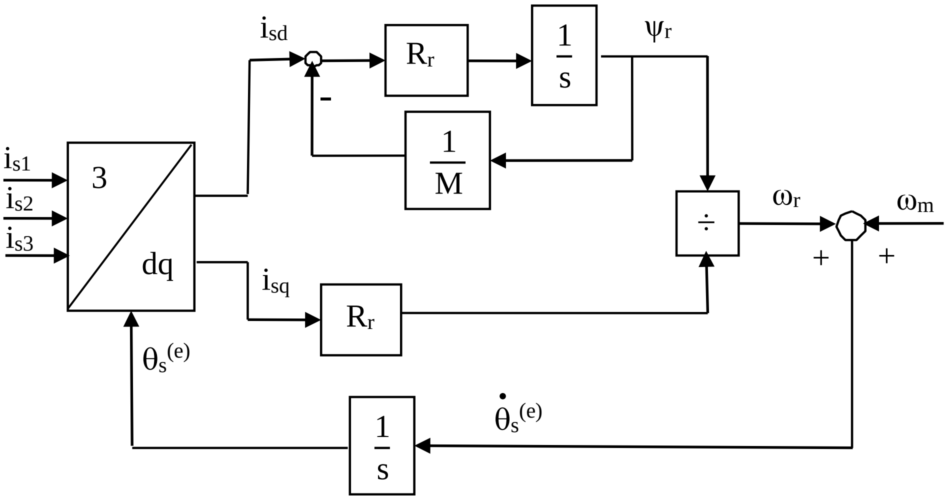

Figure 5-20 "I-Ω" rotor flux estimator

The used relations are derived from the equations of the rotor (5-9):

$$p\psi_r = R_r\cdot i_{sd} - \frac{R_r}{M}\cdot\psi_r$$

$$0 = R_r\cdot i_{sq} - \dot{\theta}_r\cdot\psi_r = R_r\cdot i_{sq} - \left(\dot{\theta}_s - \omega_m\right)\cdot\psi_r$$

and

$$p\psi_r = R_r \cdot i_{sd} - \frac{R_r}{M} \cdot \psi_r$$

(5-10)
$$\dot{\theta}_s = \omega_m + \frac{R_r \cdot i_{sq}}{\psi_r}$$

$$\theta_s = \int \dot{\theta}_s dt$$

From the knowledge of the position of the rotor flux, it is possible, through the reference frame rotation of this angle, to calculate the two current components, from which we get the position angle and the value of the rotor flux.
Some problems may arise in the use of a constant value of the parameter M (magnetizing inductance). In fact, the parameter M is not constant because it depends on the level of iron saturation. To make the correct calculations you need to replace the block 1/M with a non-linear relationship that takes into account the effects of nonlinearity.

5.8.2 V-I estimator

Another estimator is called V-I.
The equations are referred to a stationary reference frame. The inputs are the stator voltages and currents. There is no need of the mechanical speed. The stator flux is obtained from the integration of the voltage after the stator resistance, while the rotor flux is calculated from the stator flux and the stator current. The inputs of the estimator are, therefore, the stator currents and voltages on axes fixed with stator.
The equations become:

$$\psi_{s\alpha} = \int \left(v_{s\alpha} - R_s i_{s\alpha}\right) dt$$
$$\psi_{s\beta} = \int \left(v_{s\beta} - R_s i_{s\beta}\right) dt$$
$$\psi_{r\alpha} = (\psi_{s\alpha} - L_{ks} i_{s\alpha})$$
$$\psi_{r\beta} = (\psi_{s\beta} - L_{ks} i_{s\beta})$$

$$\theta_s = \arctan\left(\frac{\psi_{r\beta}}{\psi_{r\alpha}}\right)$$
$$\dot{\theta}_s = \frac{d\theta_s}{dt}$$

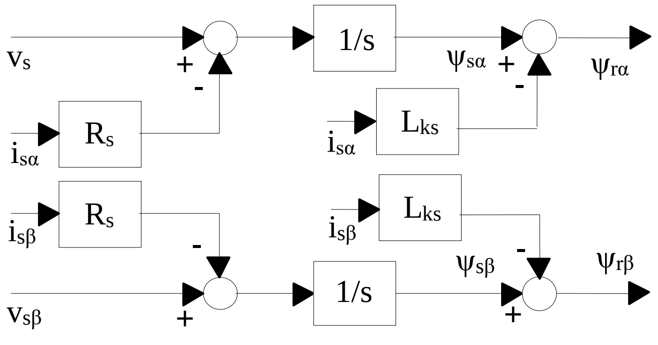

Figure 5-21 Pure integral

This estimator doesn't work fine at low speed, because the errors on the stator voltage (voltage drop on the switches or dead time), on the resistance and current may be neglected only at high speed (when the voltage is high). This method is also affected by an offset in the currents, introduced by the sensors, because the integral is pure. To avoid this problem, a low pass filter is used instead of the pure integral, but the behavior is different at low speed.

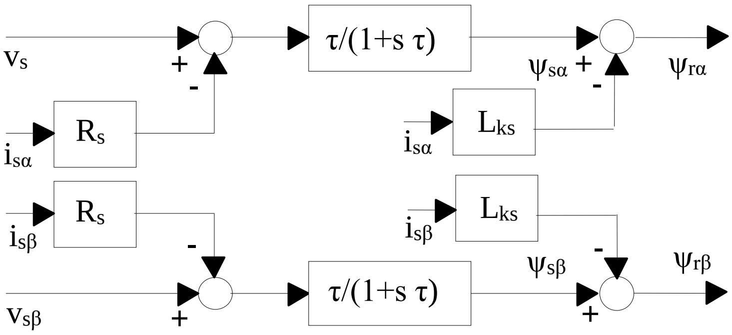

Figure 5-22 Low-pass filter instead of pure integral

5.9 Complete FOC scheme

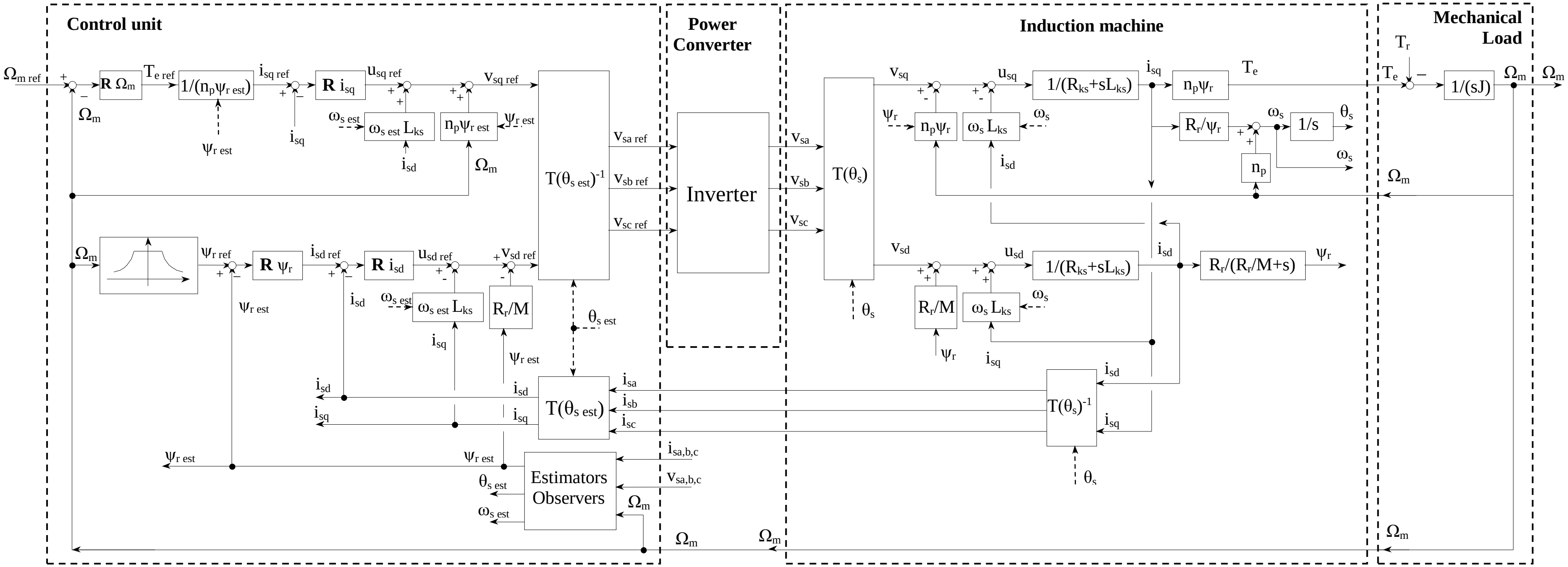

Figure 5-23 Complete FOC scheme

5.10 Operating regions

The operating regions of the machine are very similar to that presented for the dc machine. In fact, the considerations made on the maximum flux stator and rotor, the rated emf, the maximum amplitude of the stator current (as a function of the ventilation system and the type of service), the base speed and the maximum mechanical speed still apply for the induction machine. Figure 5-24 presents some mechanical characteristics as the voltage and the frequency change (maintaining constant the ratio between the amplitude of the stator voltage phasor and frequency).

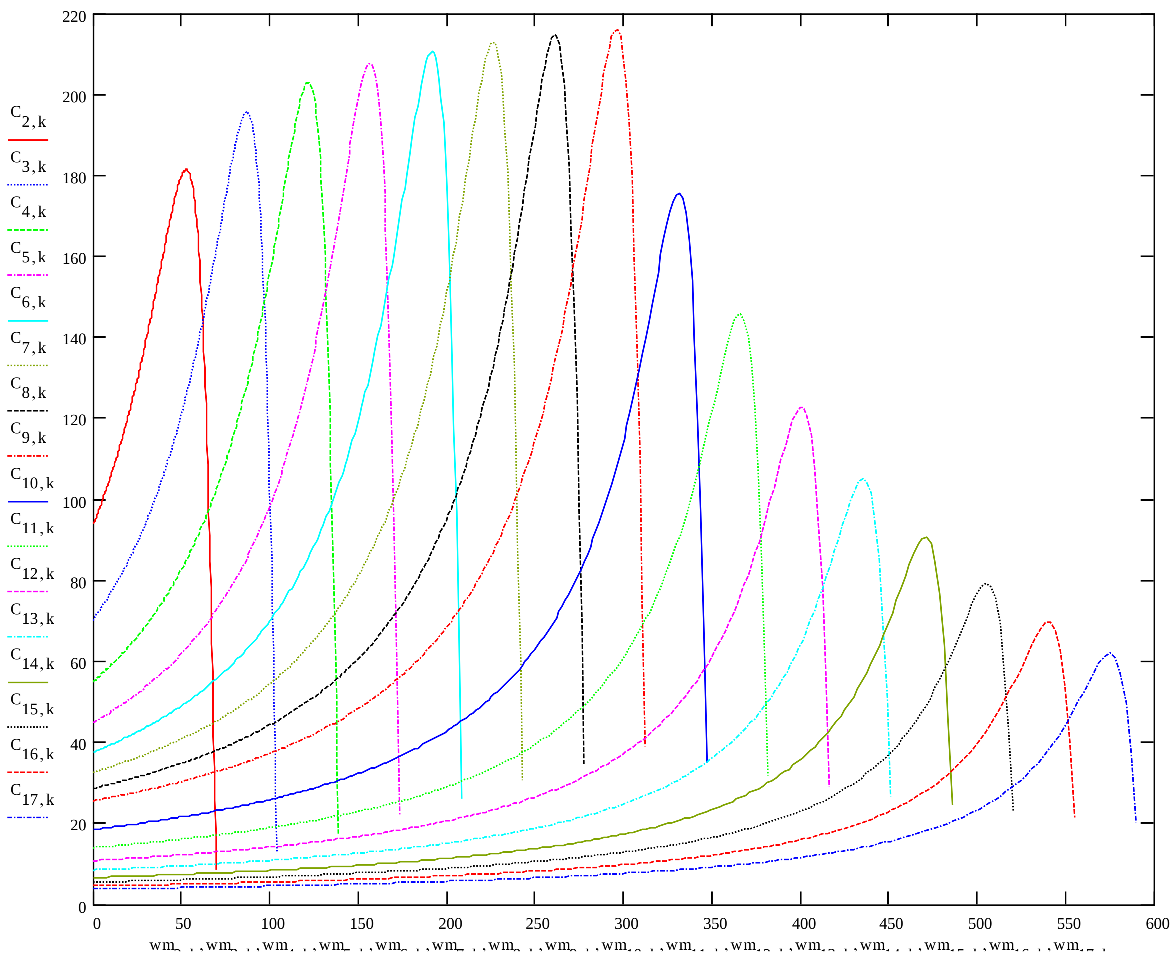

Figure 5-24 Torque/speed characteristic with voltage and frequency variation

This suggests that the voltage, current and flux trends are still valid, as a function of the mechanical speed, as shown in Figure 5-25.

It starts, first, by proper exploitation of iron (saturation limits), so by the rotor flux, which will be held constant (rated value) as far as possible (base speed Ω_b). The direct axis current component i_{sd} is equal, at steady state, to ψ_r/M, and therefore it will have the same trend of the flux.

Since the amplitude of the current must be limited due to thermal problems (losses, cooling system, service cycle duties) the q axis current will be also limited (Pythagoras theorem).

It follows that the electromagnetic torque Te has the same trend as current i_{sq}.

The stator voltage is slightly greater than the emf $\omega_m \psi_r$ (the difference is due to the voltage drop on resistances and reactances). The limits imposed by the power supply (inverter) relate to the maximum stator voltage, for which the attainment of this value (holding a margin for adjustment) identifies the base speed.

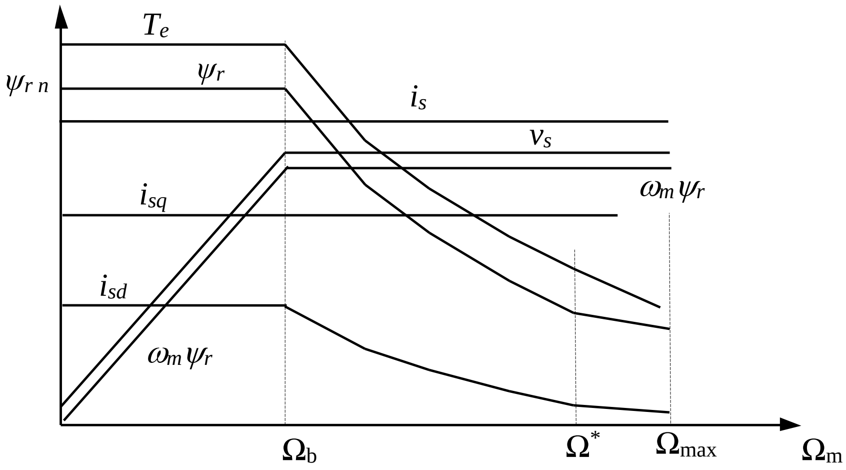

Figure 5-25 i, v e ψ as a function of the mechanical speed

For higher speeds, it is necessary to decrease the rotor flux (and therefore also the current i_{sd}) as $1/\Omega$. Maintaining the same limit on the amplitude of the stator current, one could increase the value of the current i_{sq}. Usually this operation is not implemented, and its value is kept constant.
Change, however, the torque characteristic such as, for higher speeds than the base speed, the rated torque decreases like $1/\Omega$ while the maximum torque as $1/\Omega^2$. There will be, therefore, a speed Ω^* beyond which the limit is no longer required by the rated torque but by the peak torque.

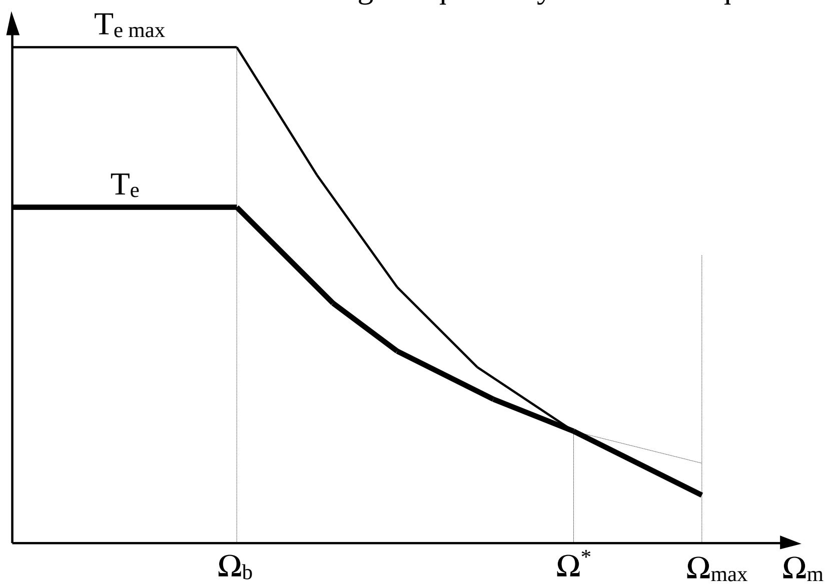

Figure 5-26. Torque trends as a function of the mechanical speed

5.11 Volt/Hz

Simple solutions can be obtained as the evolution of the classic control schemes: the scalar control or V/f. Figure 5-27 shows the typical V/f control.

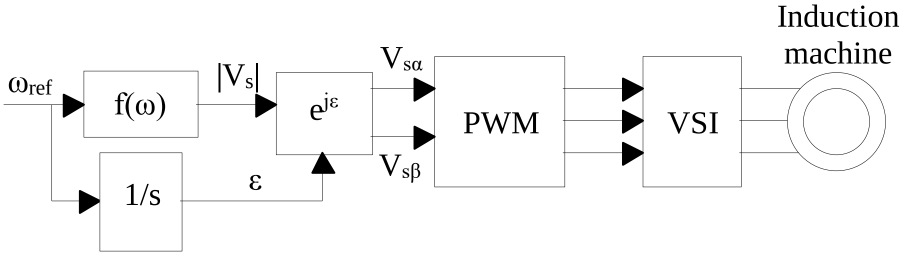

Figure 5-27 Diagram of V/f control without a speed sensor

With this control, the speed of the voltage phasor is equal to the reference. If the slip is limited (this is true when the control works well) the mechanical speed will be near the reference. The error between the speed and the reference depends on the slip, that is on the load.
In steady state, the stator equation becomes:

$$\overline{v_s} = R_s \cdot \overline{i_s} + p\overline{\psi_s}$$

$$\overline{v_s} = R_s \cdot \overline{i_s} + j\omega\overline{\psi_s}$$

At medium and high speed, the voltage drop on the resistance may be neglected:

$$\overline{v_s} = j\omega\overline{\psi_s}$$

$$v_s = \omega\,\psi_s$$

so, with a constant value of the ratio between voltage and frequency, the module of the stator flux remains almost constant.
Usually the slope of the speed reference is limited in order to limit the slip, during the transient.

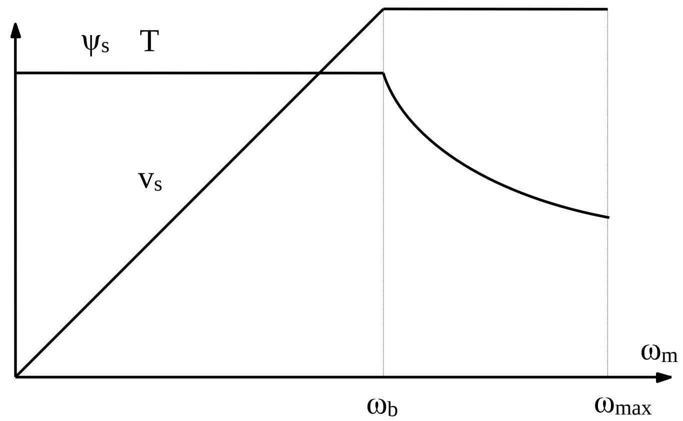

Figure 5-28 Operating region

Over the base speed, you have a constant value of the voltage (due to the limit introduced by the power converter) but an increase of the speed; so the flux decreases. This is usually called "field weakening region".

Figure 5-29 shows a closed loop scheme, in which the speed is controlled acting on the slip.

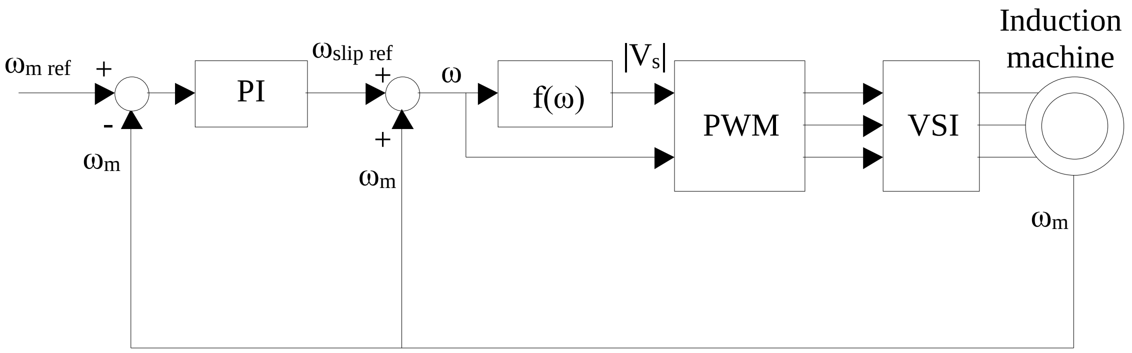

Figure 5-29 Diagram of V/f control with a speed sensor

In fact if the slip is limited, the relationship between the torque and the slip frequency is quite linear.

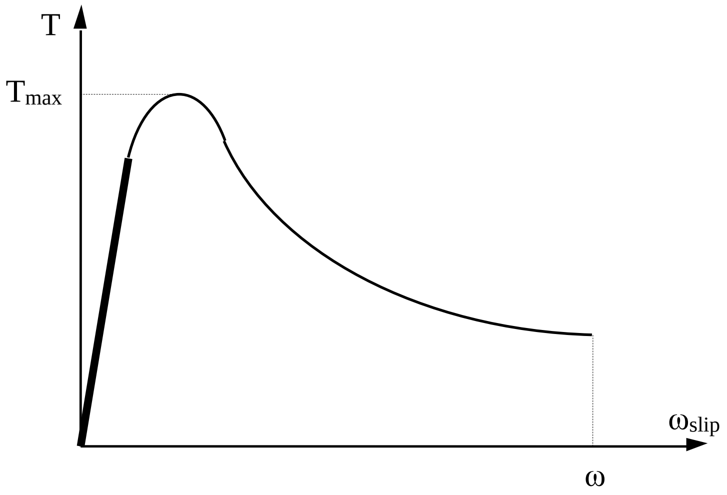

Figure 5-30: Torque/slip curve

Therefore the output of the speed controller, instead of being the reference torque, can be directly the slip frequency ω_{slip}.. Added to the current mechanical speed (in electrical radians per second) it gives the value of the stator voltage frequency.
The output of the speed controller can be saturated to values that limit the slip and never exceed the maximum torque. In this case, the system works well also with a speed reference step.