Summary

4. Transformer

4.1 Ideal transformer

Consider the structure of Figure 4-1, in which the permeability of the ferromagnetic material is infinite. The horizontal segment of the ferromagnetic structure are called "yoke" (*giogo*) while the vertical segment are called "limb" (or leg) (*colonna*)

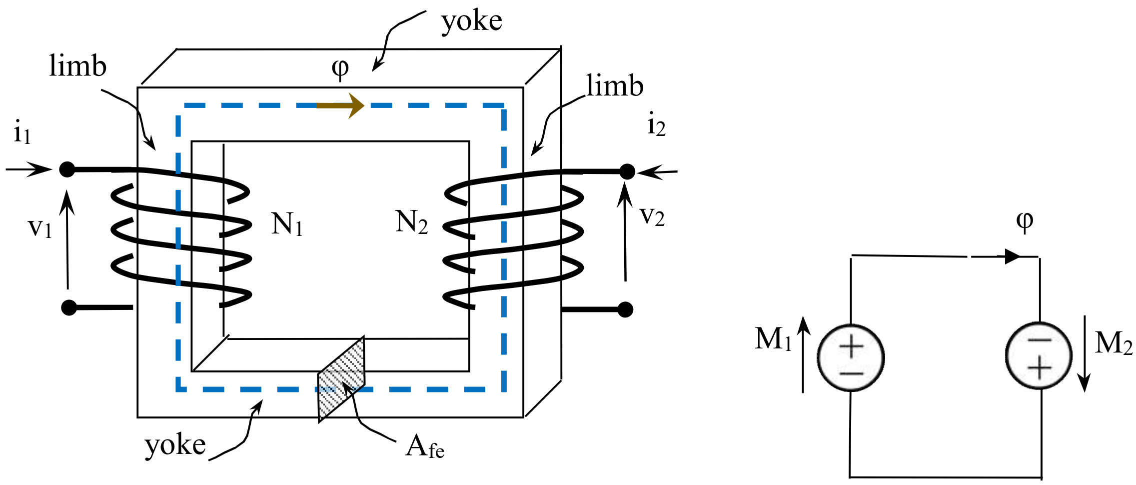

Figure 4-1: Transformer structure

In this condition, the reluctance θ_{fe} is zero. Applying the Kirchhoff Voltage Law, it results: $M_1+M_2=0$. It means that $N_1 i_1=-N_2 i_2$ and

$$i_2 = -\frac{N_1}{N_2} i_1$$

where

$$k = \frac{N_1}{N_2}$$

the ratio between the numbers of turns of each coil is called ratio of transformation.

On the other side, given the magnetic flux φ, the voltages at the terminals of each coil are:

$$\begin{aligned} v_1 &= \frac{d\psi_1}{dt} = \frac{dN_1\varphi}{dt} = N_1\frac{d\varphi}{dt} \\ v_2 &= \frac{d\psi_2}{dt} = \frac{dN_2\varphi}{dt} = N_2\frac{d\varphi}{dt} \end{aligned} \quad \longrightarrow \quad \frac{v_1}{v_2} = \frac{N_1}{N_2} = k$$

k is also called voltage ratio.

The device of Figure 4-1 is defined as Ideal Transformer. The energy balance says:

$$p_1 = v_1 i_1 = \frac{N_1}{N_2} v_2 \left(-\frac{N_2}{N_1} i_2 \right) = -v_2 i_2 = -p_2 \Rightarrow p_1 = -p_2$$

In an ideal transformer, there are no losses.

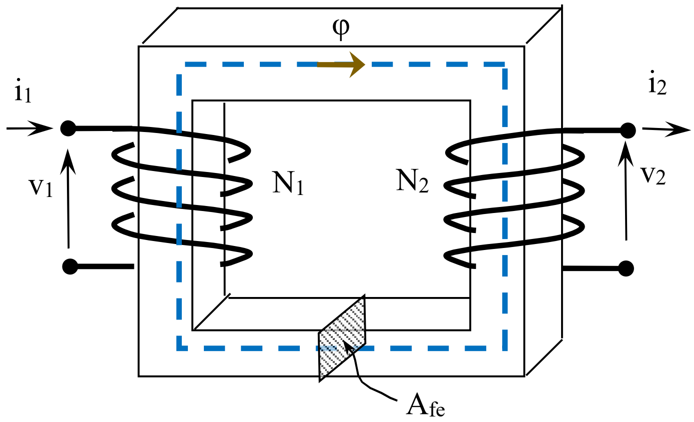

Figure 4-2: Ideal Transformer

If you change the direction of the current i_2 you have:

$$v_1 = \frac{N_1}{N_2} v_2$$

$$i_1 = \frac{N_2}{N_1} i_2$$

$$p_1 = p_2$$

The power is moving from left to right. The winding on the left (incoming power) is called "primary winding" while the winding on the right "secondary winding" (outcoming power). The structure of the ferromagnetic material is called "core".

If you apply a voltage to the primary winding, the flux linkage (and magnetic flux φ) is directly related to the voltage, by means of an integration.

In a sinusoidal steady state the integration of a phasor is 1/jω times the phasor itself. Therefore, with a sinusoidal voltage source, the flux linkage (and the magnetic flux) is proportional to the voltage; with a constant voltage, the magnetic flux φ is constant.

The symbols of the ideal transformer are:

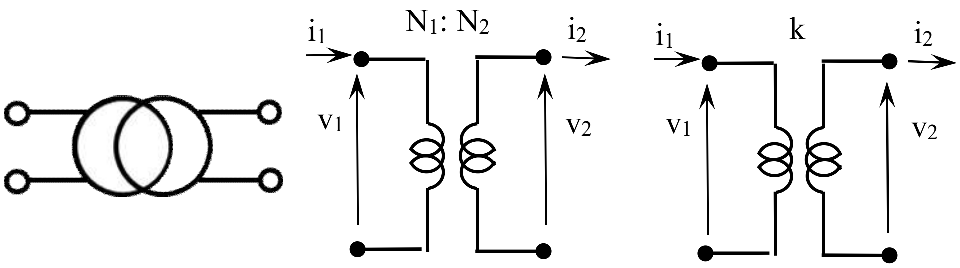

Figure 4-3: Ideal Transformer symbols

If the N_1 is higher than N_2, the device is called step-down transformer, otherwise step-up transformer.

4.2 Properties of an ideal transformer

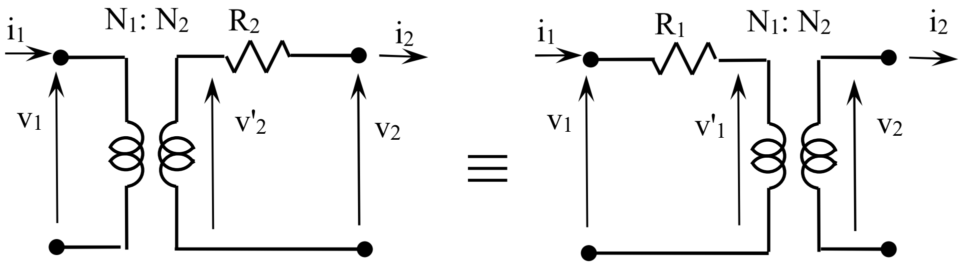

Figure 4-4: Ideal Transformer property

The two circuits of Figure 4-4 are equivalent each other (from the outer terminals point of view) if the powers are equal. In particular, the losses are the same.

$$R_2 i_2^{\ 2} = R_1 i_1^{\ 2}$$

$$i_1 = \frac{N_2}{N_1} i_2$$

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = k^2 R_2$$

So, for an ideal transformer, it is possible to "move" a resistance (but also an inductance or a capacitance or an impedance) from secondary to primary (and vice versa) on condition that it is multiplied by the transformation ratio squared.

This is true also for

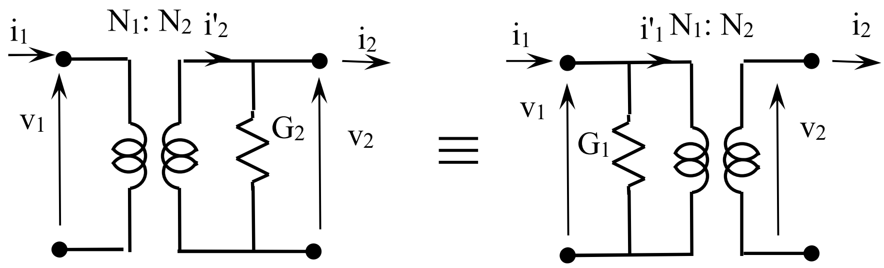

Figure 4-5: Ideal Transformer property 2

$$G_2 v_2^{\ 2} = G_1 v_1^{\ 2}$$

$$v_1 = \frac{N_1}{N_2} v_2$$

$$G_1 = \left(\frac{N_2}{N_1}\right)^2 G_2$$

$$R_1 = \frac{1}{G_1} = \left(\frac{N_1}{N_2}\right)^2 \frac{1}{G_2} = \left(\frac{N_1}{N_2}\right)^2 R_2 = k^2 R_2$$

4.3 Real transformer

The ferromagnetic material has a finite permeability. The reluctance of the ferromagnetic path is different from zero.

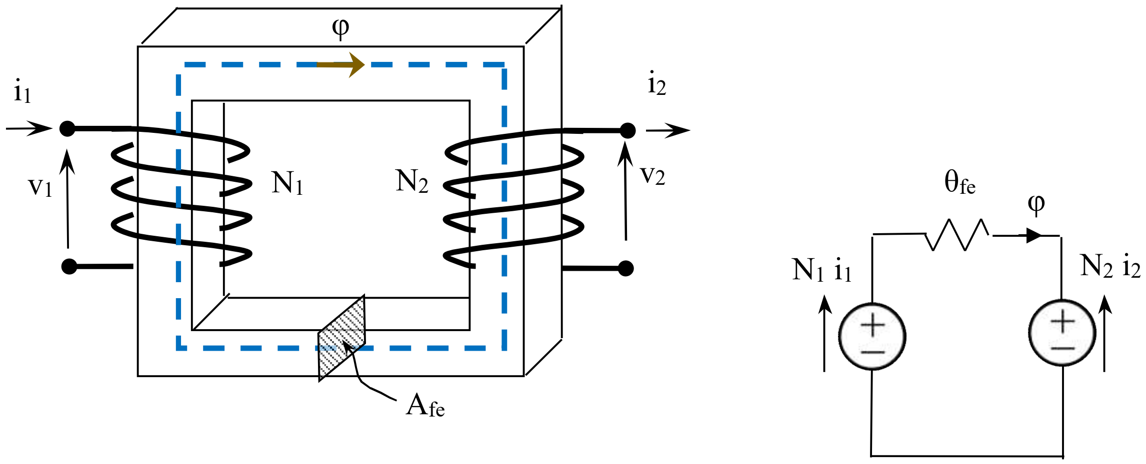

Figure 4-6: Real Transformer - 1st step: $\mu_{fe} \neq \infty$

With this condition, if the magnetic flux φ is different from zero, $N_1 i_1 \neq -N_2 i_2$ but

$$\varphi = \frac{N_1 i_1 - N_2 i_2}{\theta_{eq}} = \frac{M_1 - M_2}{\theta_{eq}} = \frac{M_0}{\theta_{eq}}$$

This is true also when the output current $i_2=0$.

$$\varphi = \frac{N_1 i_{10}}{\theta_{eq}} = \frac{M_0}{\theta_{eq}}$$

where i_{10} is called "no-load current" (*corrente a vuoto*)

On the other side, with $i_1=0$

$$\varphi = \frac{-N_2 i_{20}}{\theta_{eq}} = \frac{M_0}{\theta_{eq}}$$

There is the need of a magnetizing current to keep a magnetic flux inside the structure different from zero. This idea is represented in Figure 4-7 [(a) the magnetizing current is supplied by the primary winding; (b) by the secondary one].

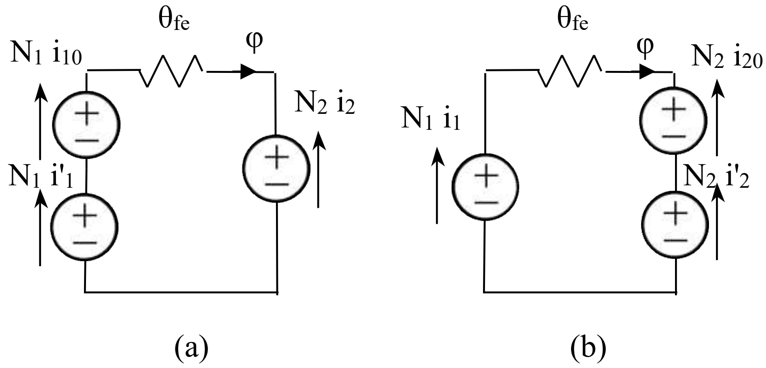

Figure 4-7: Real Transformer - magnetizing current

In Figure 4-7 (a) it results:

$$N_1 i'_1 - N_2 i_2 = 0$$

$$i_2 = \frac{N_1}{N_2} i'_1$$

In Figure 4-7 (b) it results:

$$N_1 i_1 - N_2 i'_2 = 0$$

$$i'_2 = \frac{N_1}{N_2} i_1$$

Both relationships are well described by an ideal transformer.

The magnetic flux is common to the windings, so it results again:

$$v_1 = \frac{N_1}{N_2} v_2$$

The equivalent electric circuits of the transformer may be those shown in Figure 4-8.

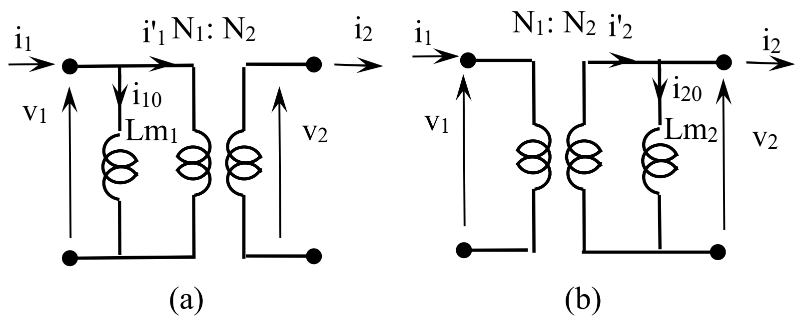

Figure 4-8: Equivalent circuits of a real transformer - 1st step

where

$$L_{m1} = \frac{N_1^{\ 2}}{\theta_{fe}}$$

$$L_{m2} = \frac{N_2^{\ 2}}{\theta_{fe}}$$

$$L_{m1} = \frac{N_1^{\ 2}}{N_2^{\ 2}} L_{m2} = k^2 L_{m2}$$

If the condition of infinite permeability is removed, there is another effect: the magnetic flux is no more bounded into the ferromagnetic structure, but some flux lines (represented in Figure 4-9 as φ_l) pass through the air. Due to the permeability of the air (μ_0), much lower than the ferromagnetic one, the reluctance of this path is very high respect to the reluctance of the ferromagnetic path (this is why the flux in the air is neglected, sometimes).

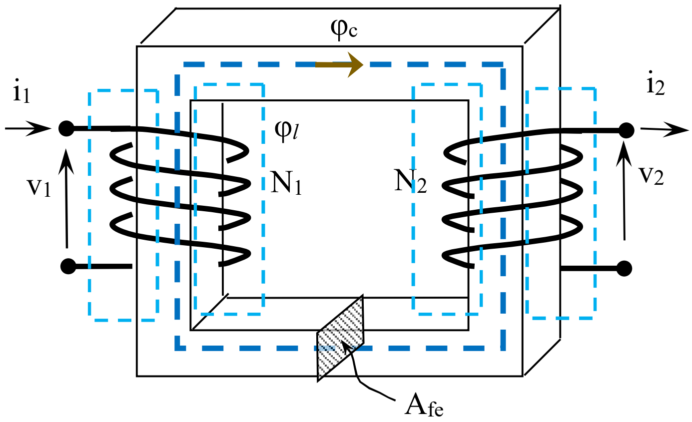

Figure 4-9: Real Transformer - 2nd step: leakage flux

This flux is called leakage flux (*flusso disperso*), while φ_c is the "common flux". Due to the constant value of μ_0, the reluctance of the path in the air may be considered constant.

The magnetic circuit, which represents this situation, is shown in Figure 4-10 (as the current i_2 has been changed the sign, in order to have v_2 equal to the derivate of the flux linkage ψ_2, the flux φ_2 has to be chosen positive going downward)

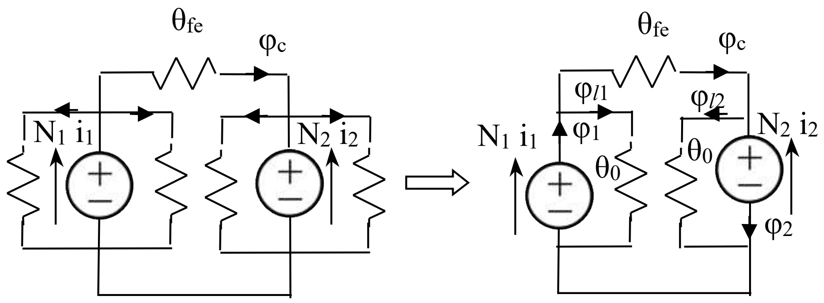

Figure 4-10: Real Transformer - magnetic circuit

So

$$\varphi_1 = \varphi_{l1} + \varphi_c$$
$$\varphi_2 = -\varphi_{l2} + \varphi_c$$

and the flux linkage

$$\psi_1 = N_1\varphi_1 = N_1\varphi_{l1} + N_1\varphi_c$$
$$\psi_2 = N_2\varphi_2 = -N_2\varphi_{l2} + N_2\varphi_c$$

and the voltages

$$v_1 = \frac{d\psi_1}{dt} = \frac{dN_1\varphi_{l1}}{dt} + \frac{N_1 d\varphi_c}{dt} = \frac{d\psi_{l1}}{dt} + \frac{N_1 d\varphi_c}{dt}$$
$$v_2 = \frac{d\psi_2}{dt} = -\frac{dN_2\varphi_{l2}}{dt} + \frac{N_2 d\varphi_c}{dt} = -\frac{d\psi_{l2}}{dt} + \frac{N_2 d\varphi_c}{dt}$$

The leakage flux φ_{l1} (and ψ_{l1}) is a function of the only current i_1, while φ_{l2} (and ψ_{l2}) of the only current i_2.

The leakage flux linkage ψ_{l1} is proportional to the current i_1, due to the linear behavior of the air path. The parameter is an inductance: L_{l1}. Similar considerations may be done for the secondary winding: L_{l2}.

Call the term $\frac{N_1 d\varphi_c}{dt} = e_1$ and $\frac{N_2 d\varphi_c}{dt} = e_2$. It results

$$e_1 = \frac{N_1}{N_2} e_2$$

It is typical of an ideal transformer.

$$v_1 = \frac{d\psi_1}{dt} = \frac{d\psi_{l1}}{dt} + \frac{N_1 d\varphi_c}{dt} = \frac{L_{l1} di_1}{dt} + e_1$$
$$v_2 = \frac{d\psi_2}{dt} = \frac{d\psi_{l2}}{dt} + \frac{N_2 d\varphi_c}{dt} = -\frac{L_{l2} di_2}{dt} + e_2$$

The equivalent circuit of the transformer of Figure 4-9 is shown in Figure 4-11 (in which the magnetizing current is supplied by the primary winding).

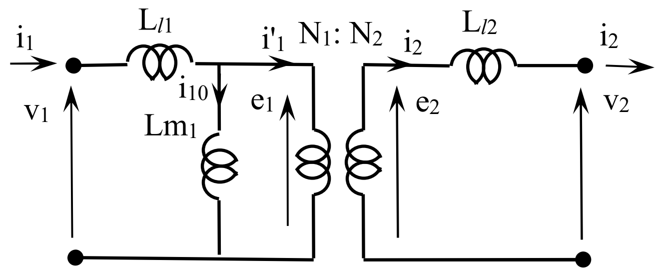

Figure 4-11: Equivalent circuit of a real transformer - 2^{nd} step

From the losses point of view, a transformer is made by ferromagnetic material, so eddy-current and hysteresis losses are present. They are a function of the flux density, therefore of the magnetic flux, that is of the flux linkage and, for a sinusoidal steady state, of the voltage. The losses in the ferromagnetic material are called "core losses" (*perdite nel ferro*) and they are represented by a resistance R_c put in parallel to the magnetizing inductance, as in Figure 4-12

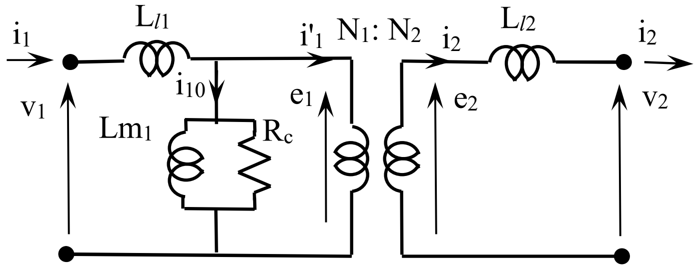

Figure 4-12: Equivalent circuit of a real transformer - 3rd step: core losses

Furthermore, the coils are made by conductive material, with a resistivity different from zero. Joule losses in the coils are present, as the function of the current squared.

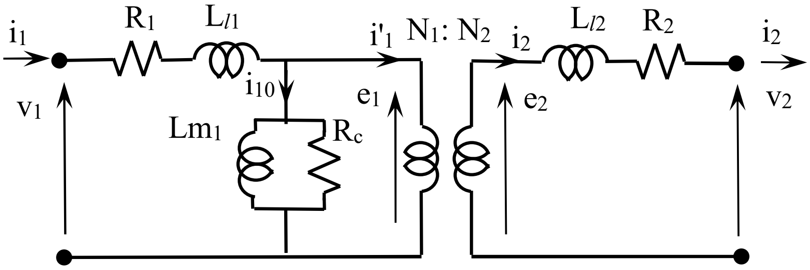

Figure 4-13: Equivalent circuit of a real transformer - 4th step: Joule losses

The efficiency (*rendimento*) of the transformer is defined as the ratio between the output power P_2 and the input one P_1 (P_c and P_J are the core and Joule losses respectively).

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + P_c + P_J}$$

The nameplate data (*dati di targa*) are:

- V_{1n}/V_{2n} the transformation ratio (rated values of the voltages)
- I_{1n}/I_{2n} : rated values of the currents
- A_n: rated apparent power (it is equal to V_{1n} I_{1n} and V_{2n} I_{2n})
- f_n: rated frequency

4.4 Sinusoidal steady state

Suppose to supply the primary winding of the transformer by means of a constant sinusoidal voltage source: $v_1(t)=V_{1\ max}\cos(\omega t)$. Applying the phasor approach, the voltage source is represented by a phasor $\overline{V_1} = V_{1rms}$ where $V_{1\ rms}$ is the rms value of voltage. The inductances may be represented by the corresponding reactance ($X=\omega L$). The equivalent circuit is shown in Figure 4-14.

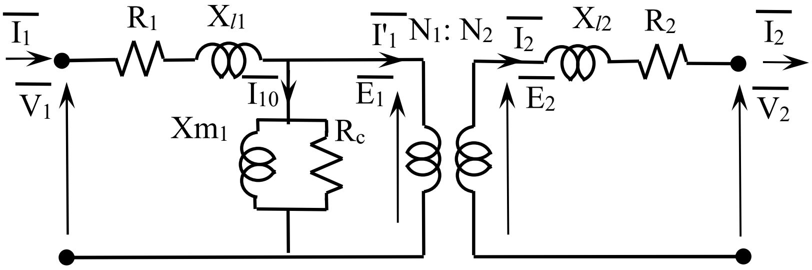

Figure 4-14: Equivalent circuit of a real transformer in sinusoidal steady state

Suppose that the losses are negligible respect to the power flowing through the transformer.

The dynamic equations

$$v_1 = \frac{d\psi_1}{dt} = \frac{d\psi_{l1}}{dt} + \frac{N_1 d\varphi_c}{dt} = \frac{L_{l1} di_1}{dt} + e_1$$
$$v_2 = \frac{d\psi_2}{dt} = \frac{d\psi_{l2}}{dt} + \frac{N_2 d\varphi_c}{dt} = -\frac{L_{l2} di_2}{dt} + e_2$$

become (the derivative of a phasor is jω times the phasor itself):

$$\overline{V_1} = j\omega\overline{\Psi_1} = j\omega L_{l1}\overline{I_1} + \overline{E_1} = jX_{l1}\overline{I_1} + \overline{E_1}$$
$$\overline{V_2} = j\omega\overline{\Psi_2} = -j\omega L_{l2}\overline{I_2} + \overline{E_2} = -jX_{l2}\overline{I_2} + \overline{E_2}$$

Usually a transformer is supplied by a voltage source (see Figure 4-15).

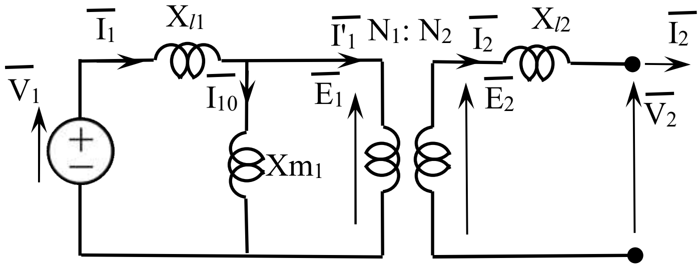

Figure 4-15: Equivalent circuit of a real transformer supplied by a sinusoidal voltage source

If the amplitude of the voltage source is constant, the flux linkage is constant.

$$\left|\overline{\Psi_1}\right| = \frac{\left|\overline{V_1}\right|}{\omega}$$
$$\left|\overline{\Psi_2}\right| = \frac{\left|\overline{V_2}\right|}{\omega}$$

4.5 Reduced equivalent circuits

The equivalent circuit of Figure 4-14 is complex to be treated. Furthermore, the type tests are not able to measure all the parameters of the circuit. So, some reduced equivalent circuits of a real transformer are introduced. Consider the Figure 4-16.

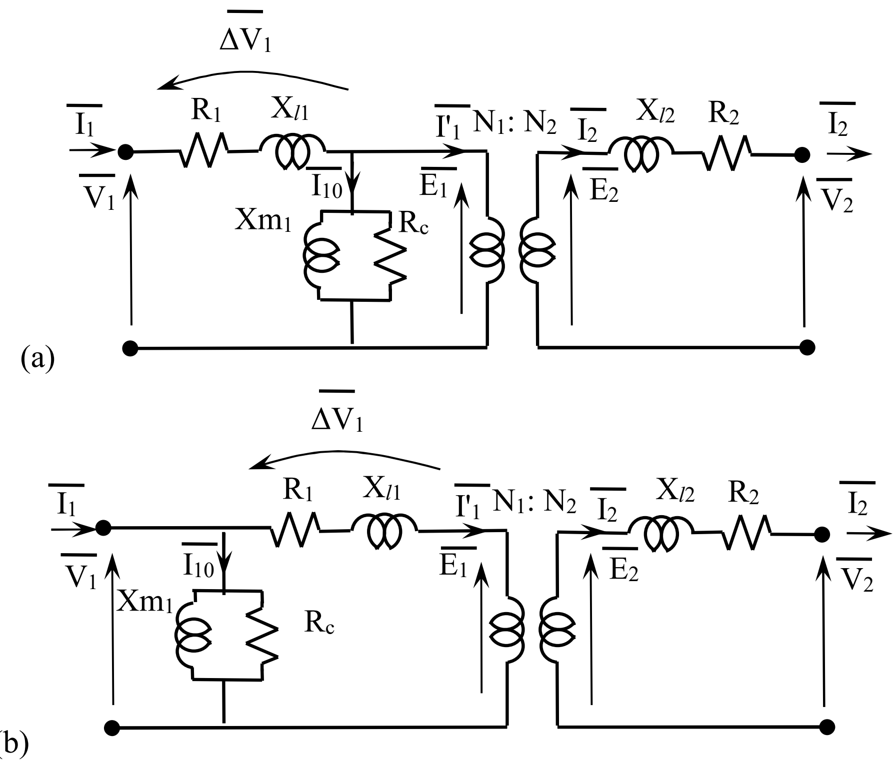

Figure 4-16: First simplification of the equivalent circuit of a real transformer in sinusoidal steady state

The voltage drop on the resistance R_1 and on the reactance X_{l1} (impedance Z_1) is usually negligible respect to V_1. This is true also for the no-load current I_{10} respect to typical value of I_1.

So, it is possible to move the impedance Z_1 after the magnetizing reactance and the core losses resistance (Figure 4-16 (b)). But now Z1 may be moved on the other side of the ideal transformer, dividing it by the transformation ratio squared.

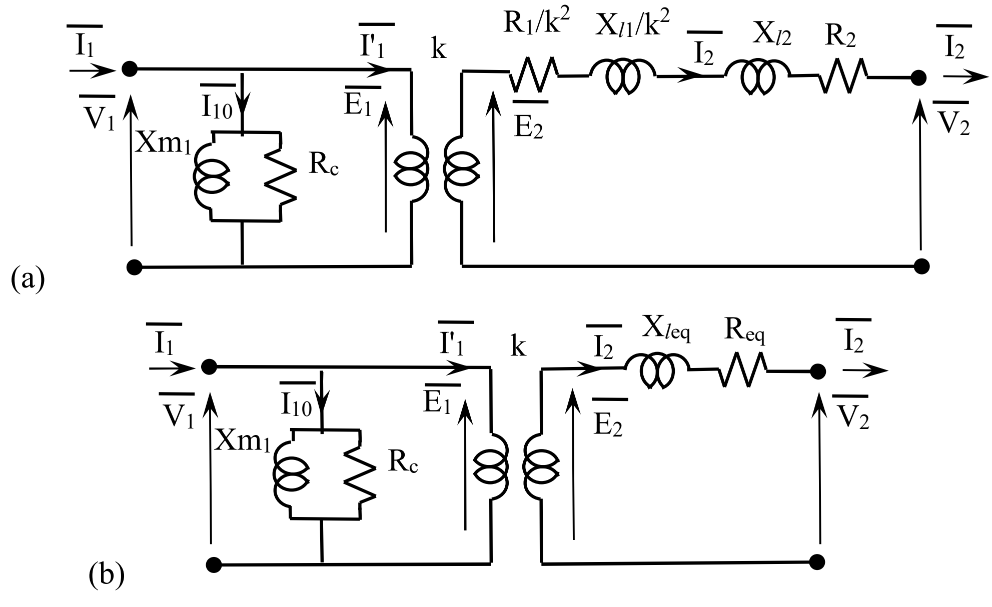

Figure 4-17: Four parameters equivalent circuit of a real transformer in sinusoidal steady state

Figure 4-17 (b) represents the so-called "Four parameters equivalent circuit" of a real transformer in sinusoidal steady state. This equivalent circuit is usually used for type test analysis.

With a similar approach (moving the impedance Z_2 to the left of the ideal transformer), you may obtain the four parameters equivalent circuit of

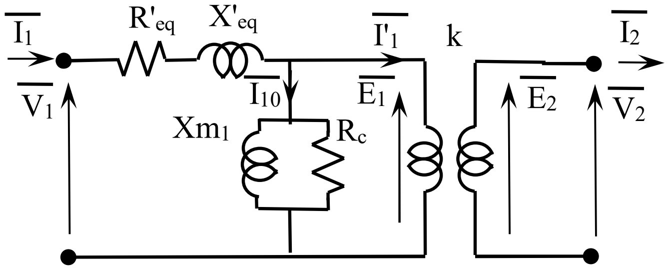

Figure 4-18: Four parameters equivalent circuit of a real transformer (left side)

Another approximation may be made neglecting the no-load current I_{10} (see Figure 4-19).

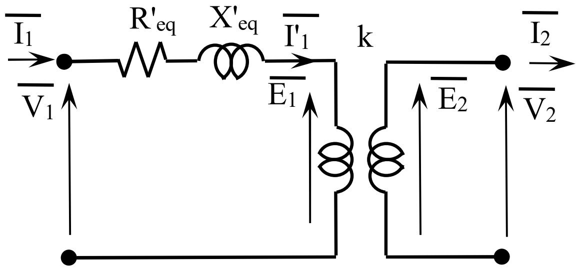

Figure 4-19: Equivalent circuit of a real transformer with only a series impedance.

A further approximation is shown in Figure 4-20, in which the equivalent series resistance is neglected.

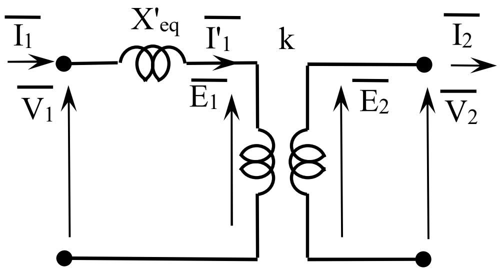

Figure 4-20: Equivalent circuit of a real transformer with only a series reactance.

4.6 Type tests of a transformer

In order to find the four parameters of the equivalent circuit of Figure 4-17 (b), two type tests are performed.

Open-circuit test

This test is performed supplying the primary with the rated voltage, at the rated frequency. The secondary is disconnected (no secondary current). In this condition, the equivalent circuit is that of Figure 4-21. If $I_2=0 \rightarrow I'_1=0$ and $V_2=E_2=E_1/k=V_1/k$ and $I_1=I_{10}$.

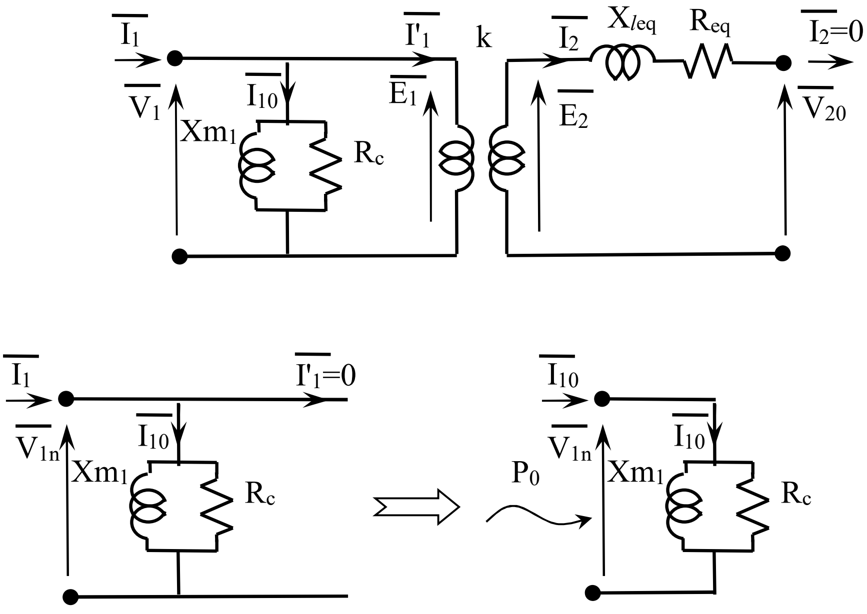

Figure 4-21: Equivalent circuit during an open-circuit test

The measurement performed during the open-circuit test are about the no-load current I_{10} and the active power P_0.

The results of this test are the no-load current referred to the rated primary current and the active power referred to the rated apparent power of the transformer.

$$i_0\% = \frac{I_0}{I_{1n}}100$$

$$p_0\% = \frac{P_0}{A_n}100$$

Starting from the values of I_0 and P_0, the parameters of R_c and X_{m1} may be easily found.

$$P_0 = \frac{V_{1n}^{\ 2}}{R_c} \qquad R_c = \frac{V_{1n}^{\ 2}}{P_0}$$

$$A_0 = V_{1n} I_{10} \qquad \cos\varphi_0 = \frac{P_0}{A_0}$$

$$Q_0 = \sqrt{A_0^{\ 2} - P_0^{\ 2}} \qquad X_{m1} = \frac{V_{1n}^{\ 2}}{Q_0}$$

From this test, the transformation ratio k may be found, starting from the measure of the secondary (no-load) voltage V_{20}.

$$k = \frac{V_{1n}}{V_{20}}$$

The no-load active power P_0 represents the core losses P_c.

Short-circuit test

The other test is realized by means of a short-circuit of the secondary. The transformer is supplied by a low voltage such to have a primary current equal to the rated one (safety condition). The equivalent circuit is shown in Figure 4-22.

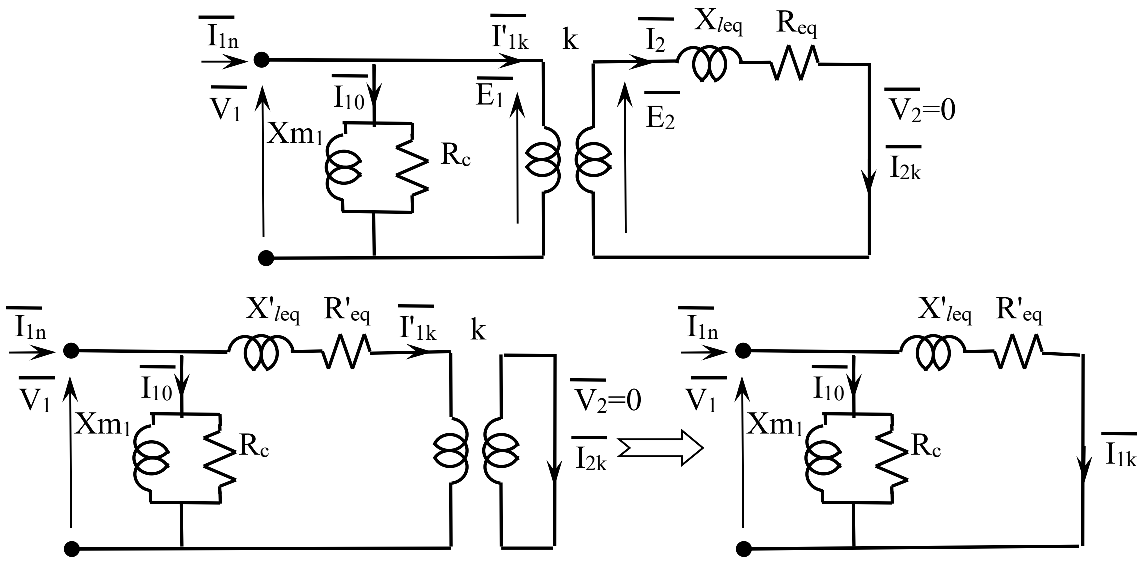

Figure 4-22: Equivalent circuits during a short-circuit test

Due to the low values of the series impedance respect to the shunt impedance (Rc and Xm1), the shunt impedance may be neglected. The reduced equivalent circuit is shown in Figure 4-23

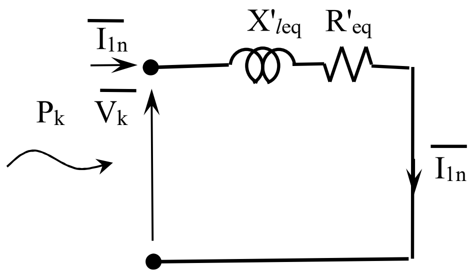

Figure 4-23: Equivalent circuit during a short-circuit test

The measurement performed during the short-circuit test are about the short-circuit voltage V_k and the active power P_k.

The results of this test are the short-circuit voltage referred to the rated primary voltage and the active power referred to the rated apparent power of the transformer.

$$v_k\% = \frac{V_k}{V_{1n}}100$$

$$p_k\% = \frac{P_k}{A_n}100$$

Starting from the values of V_k and P_k, the parameters of R'_{eq} and X'_{leq} may be easily found.

$$P_k = R'_{eq} I_{1n}^2 \quad R'_{eq} = \frac{P_k}{I_{1n}^2}$$

$$A_k = V_k I_{1n} \quad \cos\varphi_k = \frac{P_k}{A_k}$$

$$Q_k = \sqrt{A_k^2 - P_k^2} \quad X'_{leq} = \frac{Q_k}{I_{1n}^2}$$

The short-circuit active power P_k represents the copper losses P_{cu}.

4.7 Voltage variation

Consider the equivalent circuit of Figure 4-24.

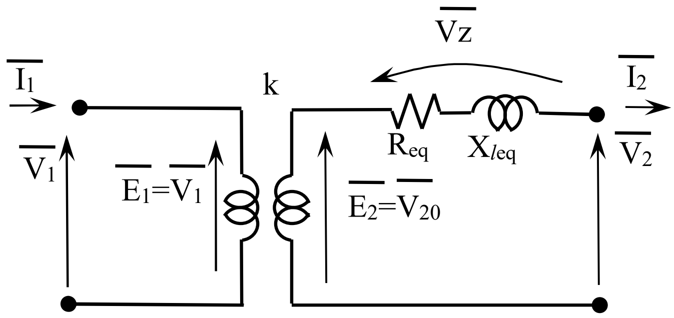

Figure 4-24: Equivalent circuit of a real transformer with only a series impedance (secondary side).

The secondary voltage V_2 with no-load (I_2=0) is equal to V_{20}, which is equal to V_1/k.

With a load (I_2≠0), it results:

$$\overline{V_{20}} = \overline{V_2} + \overline{Vz}$$
$$\overline{Vz} = \left(R_{eq} + jX_{leq}\right)\overline{I_2}$$

It may be represented by the diagram of Figure 4-25, in the case of a resistive-inductive load (the current lags the voltage).

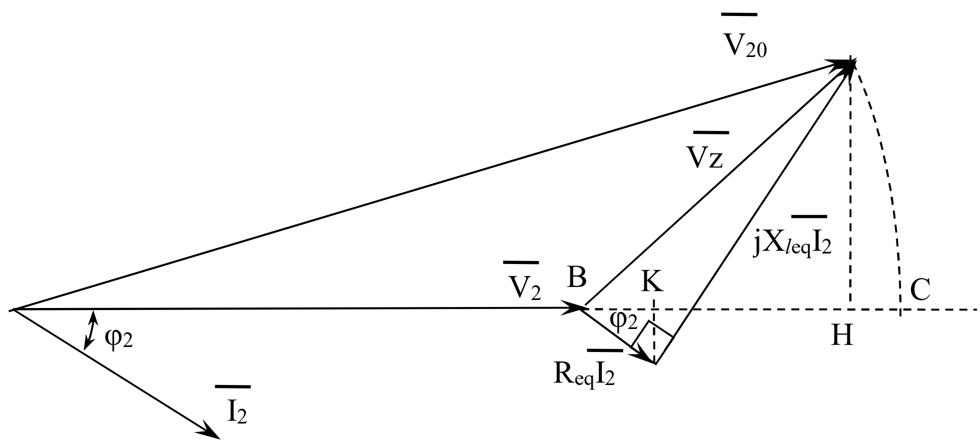

Figure 4-25: Voltage variation diagram for a resistive-inductive load

The voltage variation ΔV is the difference between the amplitudes of the voltage V_2 and the no-load voltage V_{20}, without taking care of the complex number (segment BC in Figure 4-25):

$$\Delta V = \left|\overline{V_{20}}\right| - \left|\overline{V_2}\right|$$

Usually the voltage variation assumes low values, such as you may simplify the expression considering the segment BH instead of BC. But BH=BK+KH and BK=$R_{eq}I_2 \cdot \cos(\varphi_2)$ and KH=$X_{leq}I_2 \cdot \sin(\varphi_2)$. So

$$\Delta V = R_{eq} I_2 \cos(\varphi_2) + X_{leq} I_2 \sin(\varphi_2)$$

Consider now a resistive-capacitive load (the current leads the voltage).

The diagram of Figure 4-26 shows this condition.

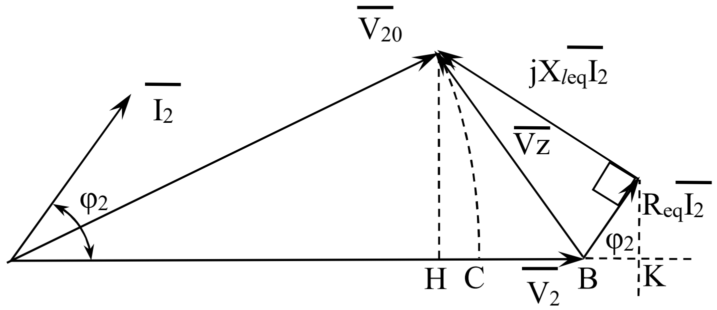

Figure 4-26: Voltage variation diagram for a resistive-capacitive load

In this case, the output voltage V_2 could be higher than the no-load voltage V_{20}. Instead, for a resistive-inductive load, the output voltage V_2 is always lower than the no-load voltage V_{20}.

4.8 Transformers in parallel

In the case of more than one transformer connected to the same voltage, supplying the same load, you have transformers in parallel.

Consider, for each transformer, the equivalent circuit of Figure 4-24.

Figure 4-27 shows the parallel connection of two transformers.

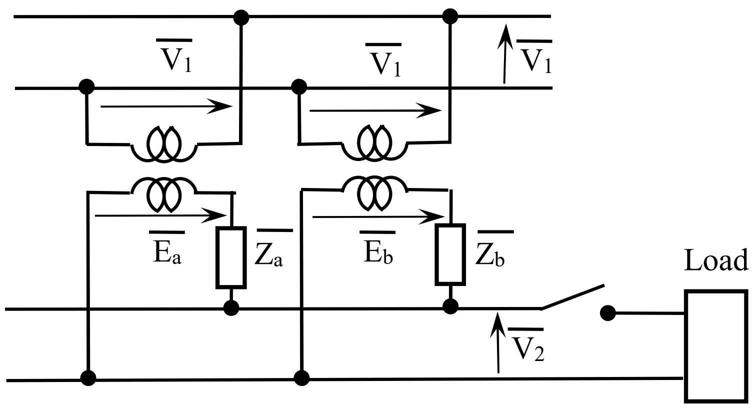

Figure 4-27: Two transformers in parallel

In order to have an equal distribution of the load power, between the two transformers (proportional to the respective rated apparent power), it is necessary the following condition:

1. The transformers must have the same transformation ratio V_{1n}/V_{2n}.
2. They must have the same short-circuit voltage v_k%

These sentences may be demonstrated looking at the no-load and load condition.

No-load condition

In this condition, the load current is zero.

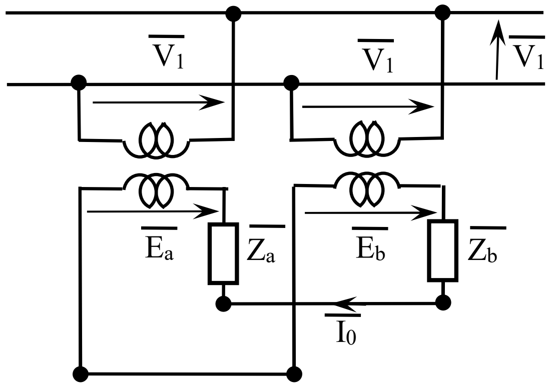

Figure 4-28: Two transformers in parallel with no load

In order to reduce to zero the circulating current $I_0=(E_b-E_a)/(Z_b+Z_a)$, E_b has to be equal to E_a. It means that the two transformers must have the same transformation ratio ($V_1/E_a=V_1/E_b$). Otherwise, a small difference between the two secondary voltages could supply a very high value, due to the very low value of the impedance Z_a and Z_b.

Load operation

Suppose that the first condition is verified. It means that $E_a=E_b=E_2$.

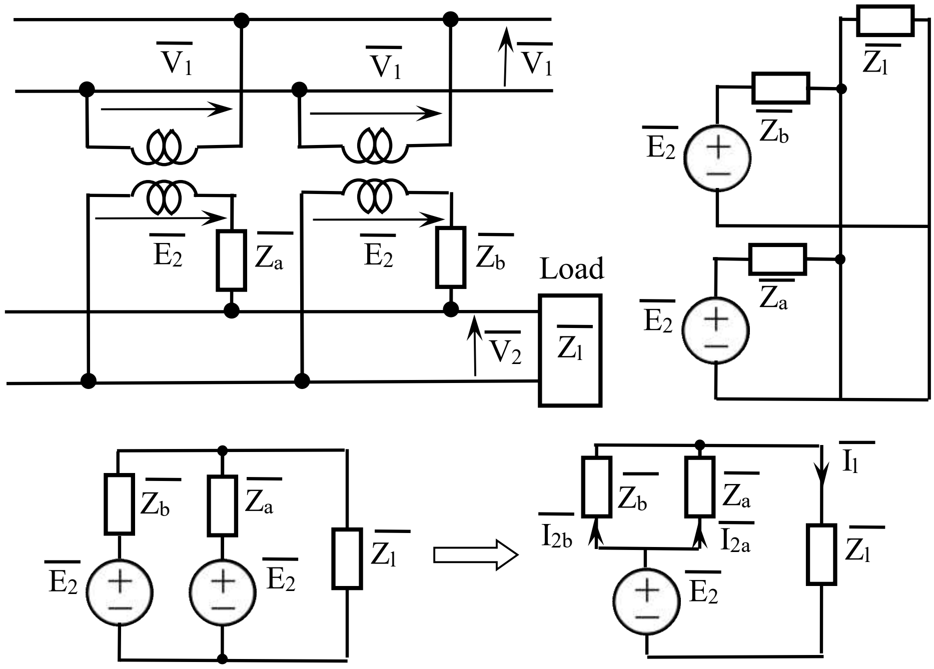

Figure 4-29: Two transformers in parallel - load operation

It is:

$$\overline{I_l} = \overline{I_{2a}} + \overline{I_{2b}}$$

and (Z_b and Z_a are in parallel):

$$\overline{Z_a} \cdot \overline{I_{2a}} = \overline{Z_b} \cdot \overline{I_{2b}}$$

thus (current divider):

$$\overline{I_{2a}} = \frac{\overline{Z_b}}{\overline{Z_a} + \overline{Z_b}} \overline{I_l} \quad \overline{I_{2b}} = \frac{\overline{Z_a}}{\overline{Z_a} + \overline{Z_b}} \overline{I_l}$$

Then

$$\frac{\overline{I_{2a}}}{\overline{I_{2b}}} = \frac{\overline{Z_b}}{\overline{Z_a}} \Rightarrow \frac{I_{2a}}{I_{2b}} = \frac{Z_b}{Z_a}$$

Multiplying by the ratio between the rated currents, it results:

$$\frac{I_{2a}}{I_{2b}} \frac{I_{2bn}}{I_{2an}} = \frac{Z_b}{Z_a} \frac{I_{2bn}}{I_{2an}} = \frac{V_{bk}}{V_{ak}} = \frac{V_{bk}}{V_{ak}} \frac{V_{1n}}{V_{1n}} \frac{100}{100} = \frac{v_{bk}\%}{v_{ak}\%} = \frac{I_{2a}}{I_{2an}} \frac{I_{2bn}}{I_{2b}}$$

In order to have a good distribution of the current in the two transformers (proportional to the respective rated apparent power) it needs that:

$$\frac{A_{2a}}{A_{2an}} = \frac{A_{2b}}{A_{2bn}} \Rightarrow \frac{V_{2n} I_{2a}}{V_{2n} I_{2an}} = \frac{V_{2n} I_{2b}}{V_{2n} I_{2bn}} \Rightarrow \frac{I_{2a}}{I_{2an}} = \frac{I_{2b}}{I_{2bn}} \Rightarrow \frac{I_{2a}}{I_{2an}} \frac{I_{2bn}}{I_{2b}} = 1 = \frac{v_{bk}\%}{v_{ak}\%} \Rightarrow v_{ak}\% = v_{bk}\%$$

4.9 Autotransformer (Variac)

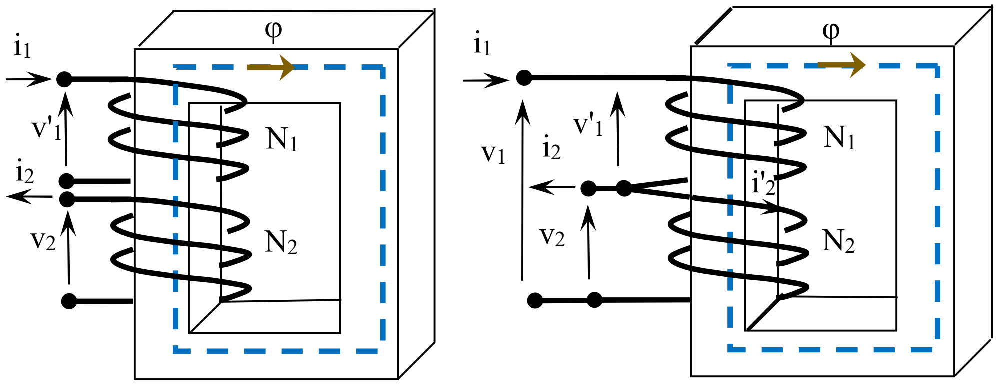

Figure 4-30: Autotransformer

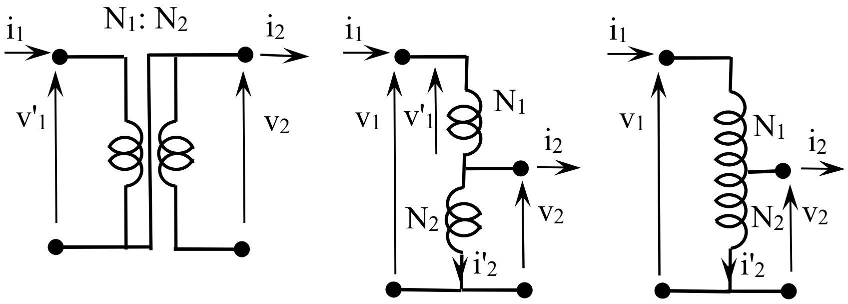

Figure 4-31: Autotransformer representation

Starting from a classical transformer, connect two terminals of the two windings, as in Figure 4-30.

Consider now the primary voltage as the total voltage v_1. Given the magnetic flux φ, neglecting any leakage flux, it results:

$$v'_1 = \frac{d\psi_1}{dt} = \frac{dN_1\varphi}{dt} = N_1\frac{d\varphi}{dt}$$
$$v_2 = \frac{d\psi_2}{dt} = \frac{dN_2\varphi}{dt} = N_2\frac{d\varphi}{dt}$$
$$v_1 = (N_1 + N_2)\frac{d\varphi}{dt} = v'_1 + v_2$$
$$\frac{v_1}{v_2} = \frac{N_1 + N_2}{N_2}$$

From the current point of view, with $\mu_{fe}=\infty$, $N_1i_1+N_2i'_2=0$, while $i'_2=i_1- i_2$. Therefore,

$N_1i_1+N_2(i_1-i_2)=0 \rightarrow (N_1+N_2)i_1-N_2-i_2=0$:

$$\frac{i_1}{i_2} = \frac{N_2}{N_1 + N_2}$$

The autotransformer is a transformer with a transformation ratio equal to $(N_1+N_2)/N_2$. Respect to a classical transformer $N_1:N_2$, the rated power of the winding N_1 is V'_1I_1 and the power for the winding N_2 is $V_2I'_2$ (ideally equal to V'_1I_1). Now, the rated power of the primary is V_1I_1 while for the secondary V_2I_2 (ideally equal to V_1I_1). But $V'_1=V_1*N_1/(N_1+N_2)$, so the rated power of the autotransformer is $(N_1+N_2)/N_1$ times the classical transformer $N_1:N_2$, with the same ferromagnetic material and the same copper quantities. This means that a autotransformer has a les size and weight of a classical transformer of the same power and transformation ratio. The drawback is mainly the no insulation between the primary and secondary windings, typical of a classical transformer: in an autotransformer the primary is electrically connected to the secondary.

An autotransformer may be considered like an inductive voltage divider.

If you use the secondary like a primary (and viceversa) the autotransformer may be a step-up transformer.

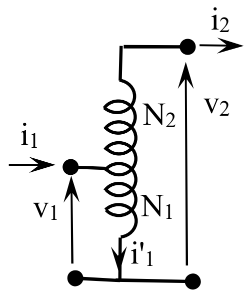

Figure 4-32: Step-up Autotransformer

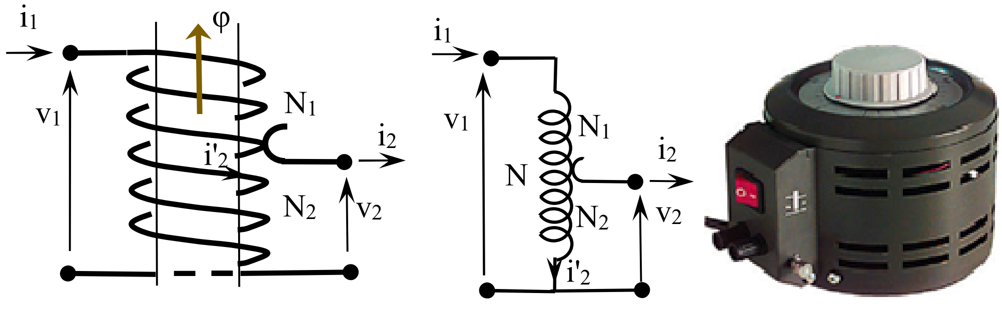

Figure 4-33: Variac

Based on the same principle, the Variac device is able to change the transformation ratio by means of a mechanical sliding contact. The output voltage V_2 may vary from V_1 to 0.

Figure 4-34 shows a Variac with an output voltage V_2 that may be higher (and lower) than V_1.

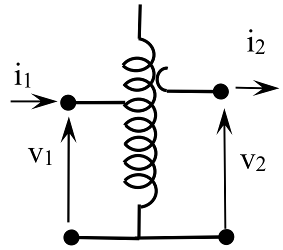

Figure 4-34: Step-up/down Variac

4.10 Multiwinding transformer

Given the same magnetic structure of a two-winding transformer. Consider more than two windings on this structure (for example 4 windings as in Figure 4-35). The first winding is the primary. The transformer of the figure has three secondary windings.

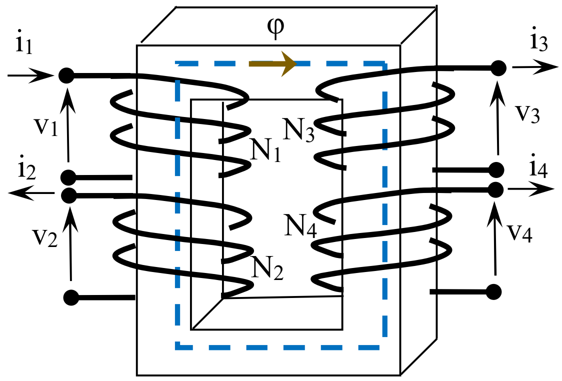

Figure 4-35: Multiwinding transformer

Neglecting any leakage flux, it results:

$$v_1 = \frac{d\psi_1}{dt} = \frac{dN_1\varphi}{dt} = N_1\frac{d\varphi}{dt}$$

$$v_2 = \frac{d\psi_2}{dt} = \frac{dN_2\varphi}{dt} = N_2\frac{d\varphi}{dt} = \frac{N_2}{N_1}v_1$$

$$v_3 = \frac{d\psi_3}{dt} = \frac{dN_3\varphi}{dt} = N_3\frac{d\varphi}{dt} = \frac{N_3}{N_1}v_1$$

$$v_4 = \frac{d\psi_4}{dt} = \frac{dN_4\varphi}{dt} = N_4\frac{d\varphi}{dt} = \frac{N_4}{N_1}v_1$$

The voltage of each secondary winding is proportional to the voltage of the primary: the coefficient is the ratio between the turns of the secondary and of the primary.

From the current point of view, with $\mu_{fe}=\infty$, $N_1i_1-N_2i_2-N_3i_3-N_4i_4=0 \rightarrow i_1=(N_2i_2+N_3i_3+N_4i_4)/N_1$. Therefore, the current of the primary is a combination of the current of the secondaries.

A multiwinding transformer has more than two windings and its behavior is well described by the above considerations.

4.11 Three-phase transformer

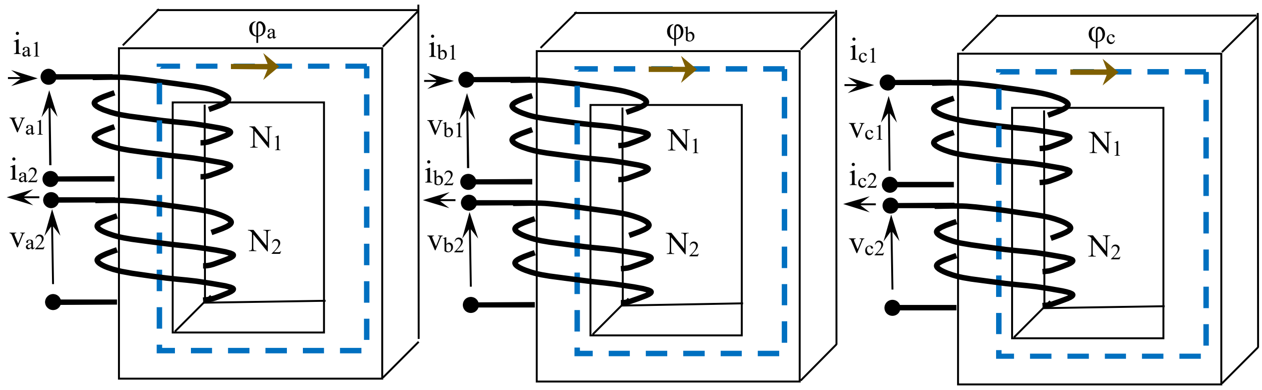

Figure 4-36: Three single-phase transformers

Suppose the primaries would be supplied by a symmetric three-phase power supply system. The three magnetic fluxes are sinusoidal, with the same amplitude and frequency, and with a displacement of 120° each other. Thus, their sum is zero.

The structure of Figure 4-37 is magnetically equivalent to the three transformers of Figure 4-36

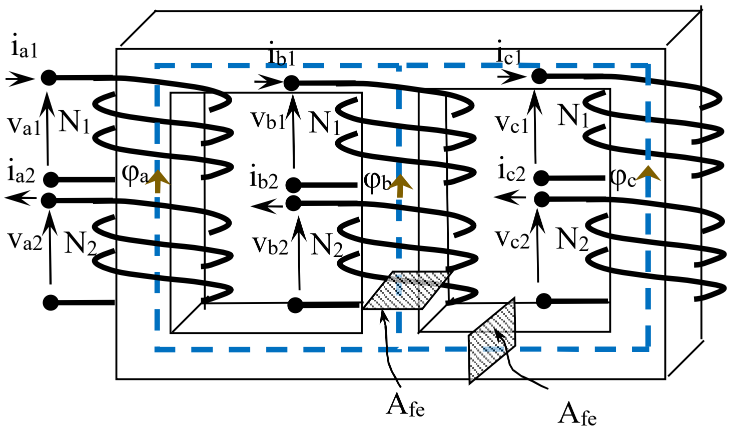

Figure 4-37: Three-phase transformer (three limbs/legs)

If the supply system is not symmetrical, the sum of the three fluxes is not zero. A different type of transformer (five limbs) is usually used (see Figure 4-38)

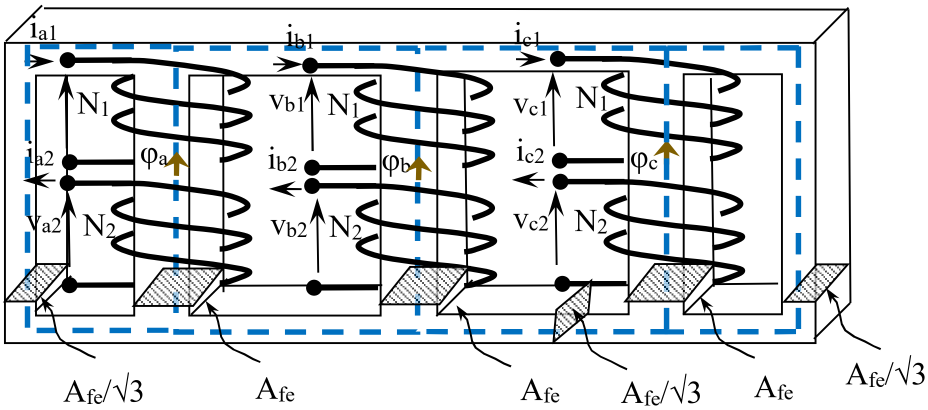

Figure 4-38: Three-phase transformer (five limbs/legs)

In a three-phase power supply system, the terminals are three, while the primaries of the three-phase transformer are six (as for the secondary). It is possible to connect to the supply the three primaries of the transformer with a wye (star) (Y) (*stella*) or delta (Δ or D) (*triangolo*) connection.

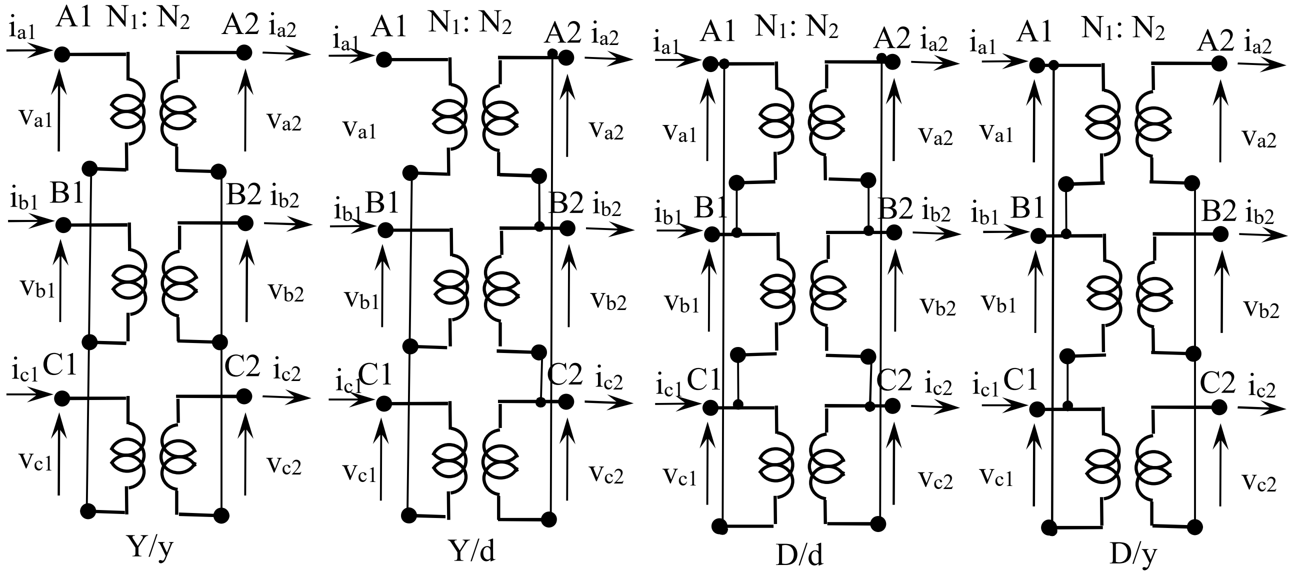

Figure 4-39: Different types of connection

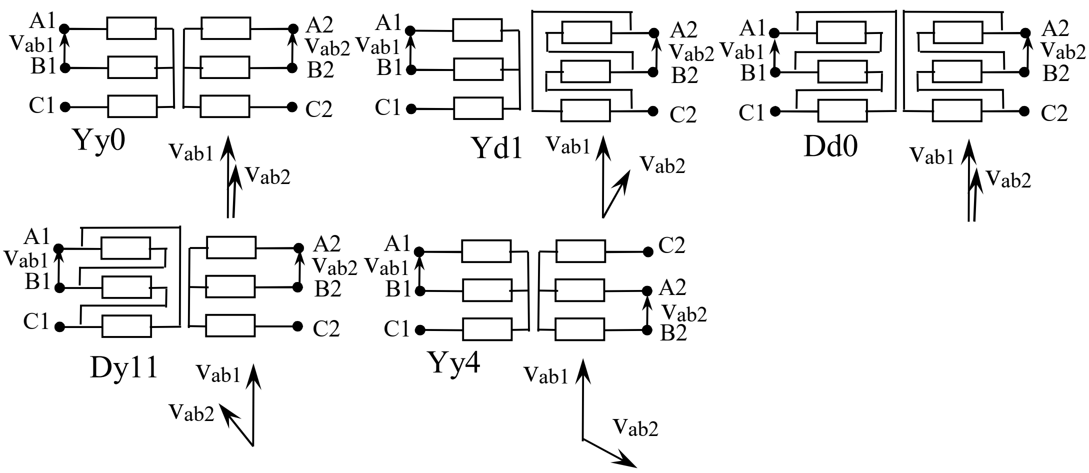

Figure 4-40: Some connection groups

Parallel of three-phase transformers

In order to have an equal distribution of the load power, between the two transformers (proportional to the respective rated apparent power), it is necessary the following condition:

1. The transformers must have the same transformation ratio V_{1n}/V_{2n}.
2. They must have the same short-circuit voltage $v_k\%$
3. They must have the same connection group