Summary

8. Synchronous machine

8.1 Principles of operation of a synchronous machine

Consider the machine shown schematically in Figure 8-1.

It consists of a cylindrical stator with a symmetrical three-phase winding so as to generate a distribution of sinusoidal magnetomotive force in the air-gap, and a rotor flux created by a winding, supplied by a constant current. We also admits that the stator and the rotor are made of laminated material with infinite permeability.

There are two main solutions for the rotor: isotropic structure (typical of a high speed application) as in Figure 8-1 and with Salient Poles (anisotropic structure) like Figure 8-2.

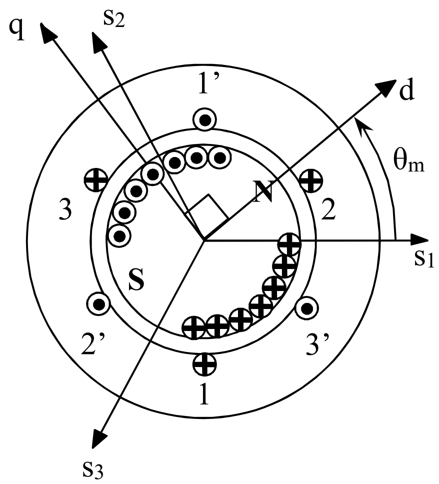


Figure 8-1: isotropic machine

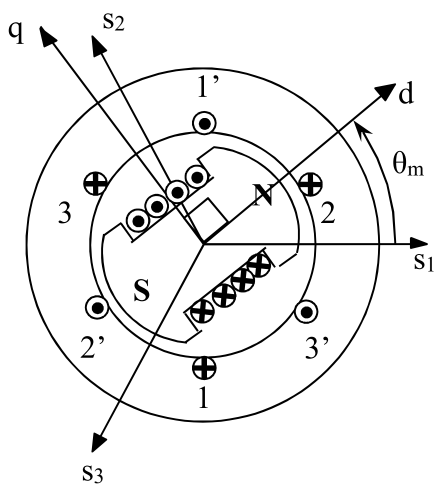


Figure 8-2: machine with salient poles (anisotropic machine)

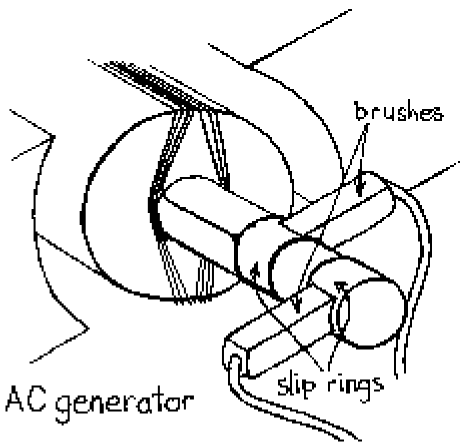


Figure 8-3: slip rings (*http://www.lselectric.com/*)

Rotor winding is supplied by means of two rings (slip rings) and two brushes (Figure 8-3) and it is called Field winding or excitation winding.

In order to understand the behaviour of the synchronous machine, we can start looking at an electromagnetic joint. The torque expression is:

$$T_e = k_e \cdot \psi_{ss} \cdot \psi_{rr} \cdot \sin(\delta)$$

where δ is the angle between the stator flux and the rotor one.

The torque expression may be written in the following way:

$$T_e = k_e \cdot \psi_{ss} \cdot \psi_{rr} \cdot \sin(\delta) = k_e \cdot \mathrm{Im}\left(\overline{\psi_{ss}}\,\underline{\psi_{rr}}\right)$$

The ψ_{rr} (flux linked with the rotor windings due to the rotor field or excitation current) is proportional to the magnetomotive force M_f due to the field (excitation) current i_f. The total flux linked with stator (ψ_s) is given by the sum of the effect of the flux due to the rotor current (ψ_{sr}) and the one due to the stator current (ψ_{ss}) and is proportional to the resultant air-gap flux per pole. Therefore $\overline{\psi}_s = \overline{\psi}_{ss} + \overline{\psi}_{sr}$ and ψ_{sr} (flux linked with the stator windings, due to the excitation current i_f) has the same direction of M_f and of ψ_{rr} ($\mathrm{Im}\left[\overline{\psi_{sr}}\,\underline{\psi_{rr}}\right] = 0$). So the torque expression may be:

$$T_e = k_e\,\mathrm{Im}\left[\left(\overline{\psi_s} - \overline{\psi_{sr}}\right)\underline{\psi_{rr}}\right] = k_e\,\mathrm{Im}\left[\left(\overline{\psi_s} - \overline{\psi_{sr}}\right)\underline{\psi_{rr}}\right] = k_e\,\mathrm{Im}\left[\overline{\psi_s}\,\underline{\psi_{rr}}\right] = k_e\,\mathrm{Im}\left[\overline{\psi_s} k_2 \underline{M_f}\right] = k \cdot \psi_s \cdot M_f \cdot \sin\left(\delta_{rf}\right)$$

The angle δ_{rf} is called "Torque angle", while δ is called "Power angle".

If M_f is leading the total ψ_s, the machine is operating like a generator and the torque angle δ_{rf} is positive (see Figure 8-4).

The torque is constant at steady state if the torque angle is constant, that is the speed of the total flux ψ_s (related to the stator voltages) is equal to the speed of M_f (fixed to the rotor), so as to the mechanical speed (this is why this machine is called synchronous).

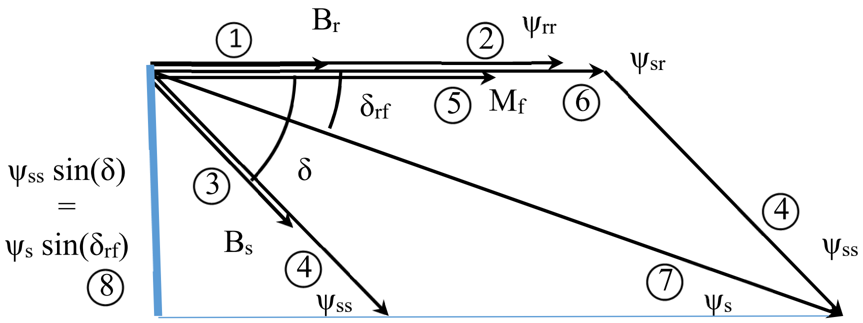


Figure 8-4. Phasor diagram of a synchronous machine

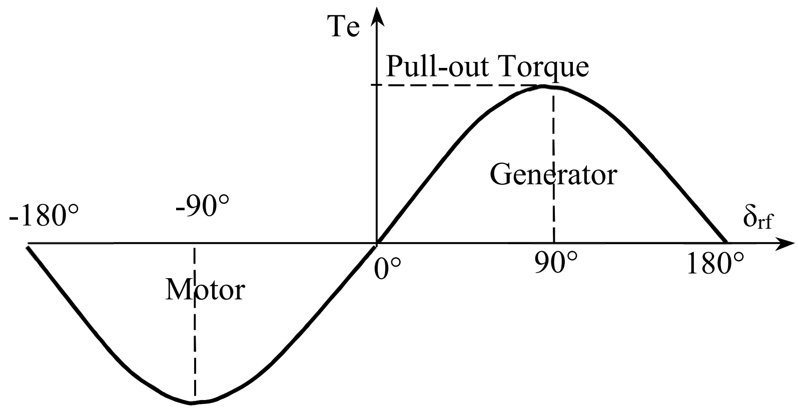


Figure 8-5: Electromagnetic Torque

8.1.1 Loss of synchronism or Pulling out of step (*perdita di passo*)

If the resistive torque assumes a value higher than the Pull-out Torque, the power is no more balanced. So the machine accelerates, and the electromagnetic torque (and the output power) assumes alternate values, with an average value equal to zero. This situation is called "Loss of synchronism or Pulling out of step". In order to make the system safe, the input mechanical power has to be suddenly set to zero.

8.2 Steady-state equivalent circuit

Consider an isotropic machine. For each winding may be written the classical relationship between the voltage, the current and the flux. The flux linkage depends on all the stator currents and the rotor one.

$$v_{s1} = R_s i_{s1} + p\psi_{s1}$$
$$v_{s2} = R_s i_{s2} + p\psi_{s2}$$
$$v_{s3} = R_s i_{s3} + p\psi_{s3}$$
$$v_f = R_f i_f + p\psi_f$$
$$\psi_{s1} = L_{ss} i_{s1} + M_{ss} i_{s2} + M_{ss} i_{s3} + L_{sf}(\theta_m) i_f$$
$$\psi_{s2} = L_{ss} i_{s2} + M_{ss} i_{s1} + M_{ss} i_{s3} + L_{sf}(\theta_m - \frac{2}{3}\pi) i_f$$
$$\psi_{s3} = L_{ss} i_{s3} + M_{ss} i_{s1} + M_{ss} i_{s2} + L_{sf}(\theta_m - \frac{4}{3}\pi) i_f$$

where $L_{sf}(\theta_m)$ depends on the mechanical angle and may be considered as a sinusoidal function of it:

$$L_{sf}(\theta_m) = L_{sf}\cos(\theta_m)$$

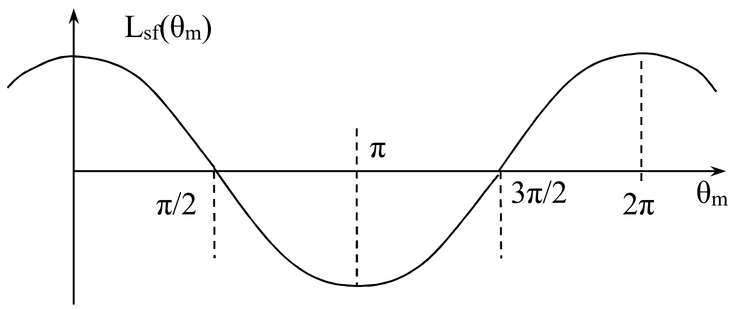


Figure 8-6: Waveform of the mutual inductance between the excitation and stator windings

If the sum of the three-phase currents is zero, for example due to the connection (in isolated star or delta connection), we have that ($i_{s1} + i_{s2} + i_{s3} = 0$):

$$\psi_{s1} = L_s i_{s1} + L_{sf}(\theta_m) i_f$$

$$\psi_{s2} = L_s i_{s2} + L_{sf}(\theta_m - \frac{2}{3}\pi) i_f$$

$$\psi_{s3} = L_s i_{s3} + L_{sf}(\theta_m - \frac{4}{3}\pi) i_f$$

where $L_s = L_{ss} - M_{ss}$ is the synchronous inductance.

Applying now the space phasor formula in a stationary reference frame (α axis has the same direction of the magnetic axis of the first winding $s1$), we obtain the following relationships:

$$\bar{v}_s^{\alpha\beta} = R_s \bar{i}_s^{\alpha\beta} + p\bar{\psi}_s^{\alpha\beta}$$

$$\bar{\psi}_s^{\alpha\beta} = L_s \bar{i}_s^{\alpha\beta} + \sqrt{\frac{3}{2}} L_{sf} e^{j\theta m} i_f$$

Defining the mutual inductance M as

$$M = \sqrt{\frac{3}{2}} L_{sf}$$

and passing to a reference frame fixed with the excitation flux (axis " d " fixed with the North of the excitation flux), we obtain the following relationships ($\overline{F}^{\alpha\beta} = \overline{F}^{dq} e^{j\theta m}$)

$$\bar{v}_s = R_s \bar{i}_s + p\bar{\psi}_s + j\dot{\theta}_m \bar{\psi}_s$$

$$\bar{\psi}_s = L_s \bar{i}_s + M i_f$$

At steady state, the angular speed ω of the electrical quantities is equal to the mechanical speed ω_m and the electrical quantities are constant if seen from the reference frame fixed with the rotor (the derivatives are zero). Therefore, it results

$$\bar{v}_s = R_s\bar{i}_s + j\omega L_s\bar{i}_s + j\omega M i_f$$

$$\bar{v}_s = R_s\bar{i}_s + jX_s\bar{i}_s + \bar{E}_{sf}(i_f)$$

$$v_f = R_f i_f$$

X_s is called Synchronous Reactance while E_{sf} is the no-load electromotive force.

The equivalent circuit of an isotropic machine is:

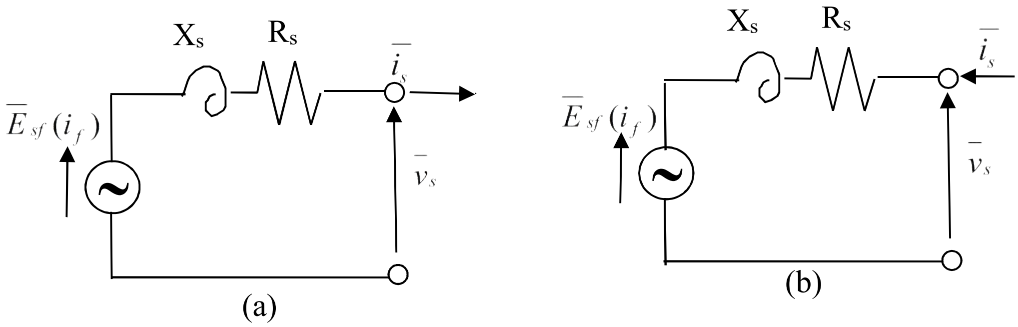


Figure 8-7: Equivalent circuit of an isotropic machine ((a) generator, (b) motor)

8.3 Open circuit (no-load)

In no-load condition (the machine is disconnected and the stator current are zero), the stator voltage is equal to the no-load electromotive force E_{sf}.

$$\bar{v}_s = \bar{E}_{sf}(i_f)$$

Due to the non-linear behaviour of the ferromagnetic material, the no-load electromotive force E_{sf} is a non-linear function of the excitation current i_f. This means that the mutual inductance M is a function of the excitation current. On the knee of the characteristic, the rated values of E_{sf} and i_f are defined.

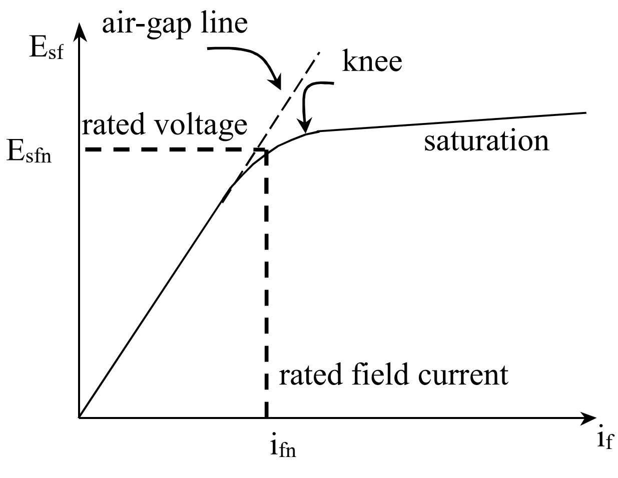


Figure 8-8: Open-circuit (no-load) characteristic of a synchronous machine

An increase in the excitation current over the rated value does not correspond to an increase of the electromotive force, due to the saturation condition of the ferromagnetic material. The operation is limited to an excitation current between 0 and the rated current i_{fn}. The no-load voltage is proportional to the mechanical speed (at steady state, $\omega_m = \omega$).

8.4 Short-circuit characteristic

The stator voltage is set to zero (v_s=0). Consider the operation like a generator.

$$0 = -R_s \bar{i}_s - jX_s \bar{i}_s + \overline{E}_{sf}(i_f)$$

$$\bar{i}_s = \frac{\overline{E}_{sf}(i_f)}{R_s + jX_s}$$

Starting with the current i_s, we can draw the phasor diagram:

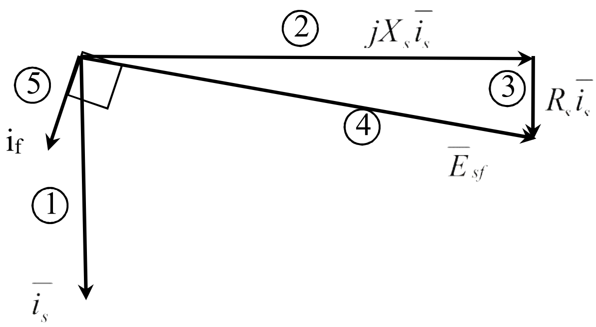


Figure 8-9. Phasor diagram of a synchronous machine in short-circuit condition

Due to the low value of the voltage drop on the impedance, the value of E_{sf} (and i_f) is low (in the linear segment of the characteristic of Figure 8-8).

So the short-circuit characteristic (i_s as a function of i_f) of a synchronous machine is linear.

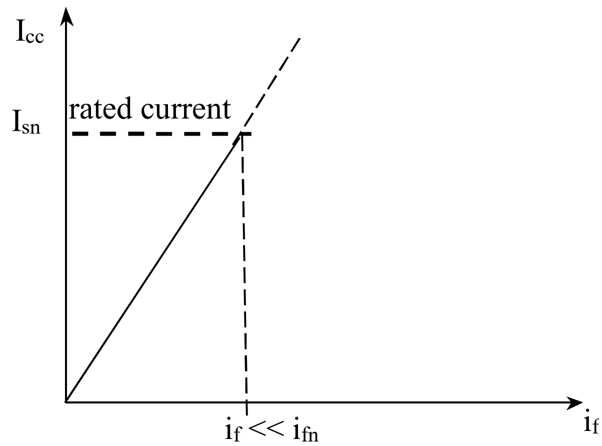


Figure 8-10. Short-circuit characteristic

8.5 Connection to the grid (Behn-Eshemburg diagram)

Consider a circuit with two voltage generators E1 and E2, a reactance X and a resistance R.

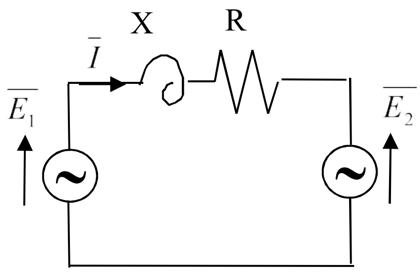


Figure 8-11. Simple equivalent circuit

It results:

$$\overline{E_1} = \overline{E_2} + R\bar{I} + jX\bar{I}$$

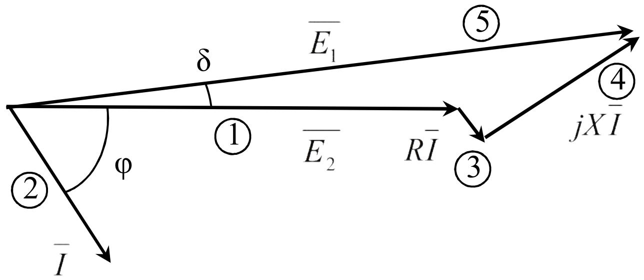


Figure 8-12. Phasor diagram of the simple equivalent circuit (the current is lagging the voltage E_2)

The active power on the right side of the circuit is:

$$P_2 = E_2 I \cos(\varphi) = \mathrm{Re}(\overline{E_2 \underline{I}})$$

Neglect the voltage drop on the resistance.

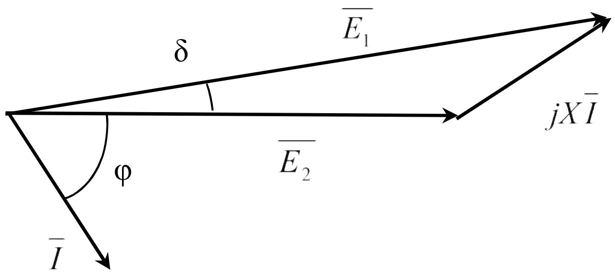


Figure 8-13. Phasor diagram of the simple equivalent circuit (no voltage drop on the resistance)

So the transmitted active power P is equal to P_2 (no losses).

The expression of the current *I* is:

$$\bar{I} = \frac{\overline{E_1} - \overline{E_2}}{jX}$$

The phasor $\overline{E_1}$, if $\overline{E_2}$ is put on the real axis, may be described by: $E_1 e^{j\delta}$.

$$\bar{I} = \frac{E_1 e^{j\delta} - E_2}{jX}$$

The transmitted power P results:

$$P = \mathrm{Re}(\overline{E_2 \underline{I}}) = E_2\,\mathrm{Re}(\underline{I}) = E_2\,\mathrm{Re}\left(\frac{E_1 e^{-j\delta} - E_2}{-jX}\right) = E_2\,\mathrm{Re}\left(\frac{E_1 e^{-j\delta}}{-jX}\right) - E_2\,\mathrm{Re}\left(\frac{E_2}{-jX}\right) = E_2\,\mathrm{Re}\left(\frac{E_1 e^{-j\delta}}{-jX}\right)$$

so

$$P = E_2\,\mathrm{Im}\left(\frac{E_1 e^{-j\delta}}{-X}\right) = \frac{E_2 E_1 \sin(\delta)}{X}$$

The angle δ is the Power Angle

Coming back to a synchronous generator connected to the grid.

The resistance of the generator and the one of the grid are in series, so they are equivalent to a single resistance R_{eq}. The same for the reactances.

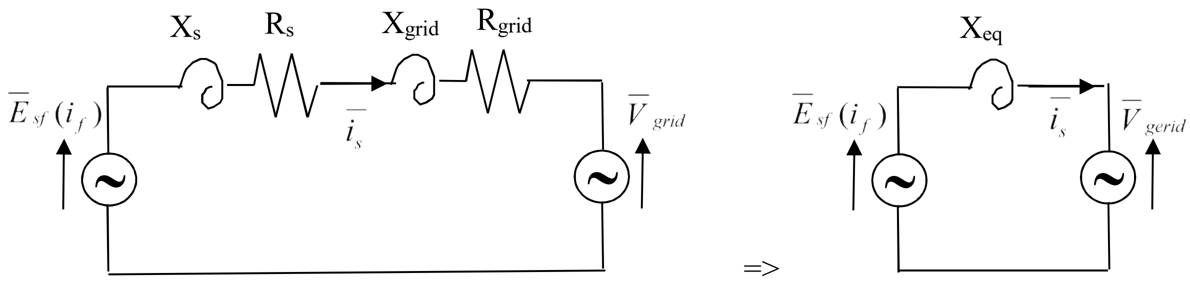


Figure 8-14. Equivalent circuit of a synchronous machine connected to the grid

In order to connect a synchronous generator to the grid, you have to follow the following steps:

1. by means of the prime mover you can control the frequency of the electromotive force in order to make it equal to the frequency of the grid voltage; you need also that the angular position of E_{sf} has to be equal to the angular position of V_{grid};
2. by means of the excitation current you can control the amplitude of the electromotive force in order to make it equal to the amplitude of the grid voltage (the excitation current i_f assumes a value (i_{f0}) a little bit lower than its rated value i_{fn})
3. you have to pay attention to the sequence of the three-phase voltages
4. in these conditions, you can close the switch in order to connect the generator to the grid.
5. the current, now, is 0 (no power flow and δ=0) (see Figure 8-15)

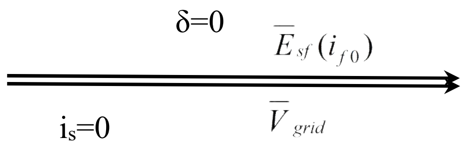


Figure 8-15. Phasor diagram at the connection of the synchronous machine to the grid

Now you may operate on the system changing the excitation current and/or the prime mover mechanical power.

Suppose you increase the excitation current, starting from the condition with no current.

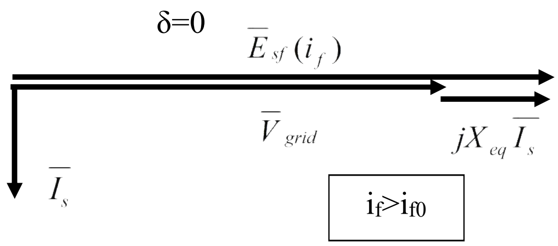


Figure 8-16. Phasor diagram in over-excitation condition

The power angle δ still assumes null value. The voltage drop on the equivalent reactance is parallel to V_{grid}. So the current phasor I_s is lagging the grid voltage by 90°.

The terminals of the grid appears to the generator like an inductive load. From the grid point of view, the synchronous machine is seen like a capacitance.

Suppose you decrease the excitation current, starting from the condition with no current.

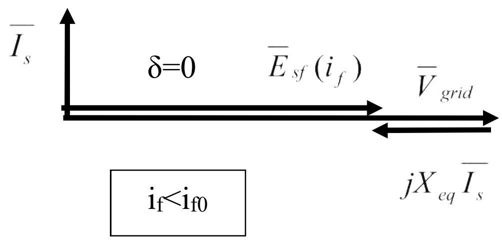


Figure 8-17. Phasor diagram in under-excitation condition

The power angle δ still assumes null value. The voltage drop on the equivalent reactance is parallel to V_{grid}, but with different direction. So the current phasor I_s is leading the grid voltage by 90°.

The terminals of the grid appears to the generator like the terminals of a capacitive load. From the grid point of view, the synchronous machine is seen like an inductance.

Suppose you increase the mechanical power of the prime mover, starting from the condition with no stator current. The rotor speed increases and the electromotive phasor is moving away from the grid voltage phasor. The power angle δ increases, so the active power P. There is a stable operating point where the input mechanical power would be equal to the output electrical power (neglecting the losses).

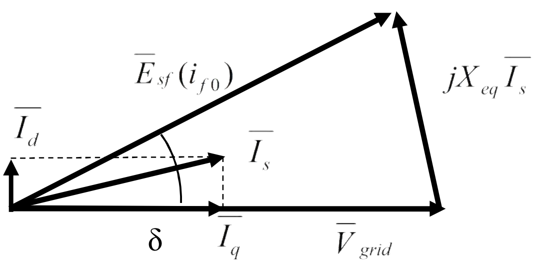


Figure 8-18. Phasor diagram with positive input mechanical power

In this condition, there is a component of I_s (I_q) in phase to the grid voltage (active power) and another one (I_d) orthogonal to the grid voltage. This means that the reactive power is not zero. In order to have only active power, an increase of the value of the excitation current is required.

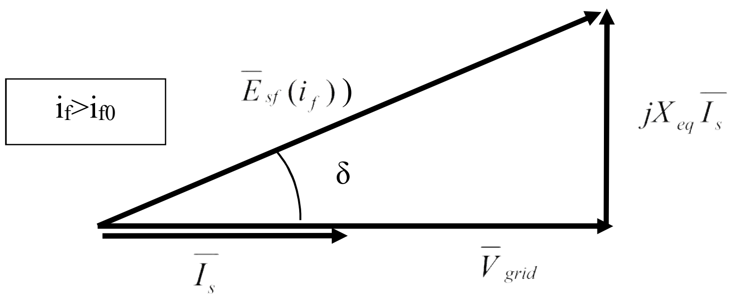


Figure 8-19. Phasor diagram with positive input mechanical power and no reactive power

8.6 Capability curves of a synchronous machine

The capability curves give an idea about the maximum active and reactive power that a synchronous generator, connected to the grid, may produce, in steady state condition. As a generator, the active power P is only positive.

The first constraint is given by the maximum stator current. The value is limited by thermal problem (that is: power losses, cooling system and service duty).

Given the grid voltage V_{grid} and the excitation current (it means a constant E_{sf}), in a Gaussian plane P-Q, the limit of the stator current may be represented by a circle with a centre in the origin (the product of V_{grid} and I_s is constant, so the apparent power is constant).

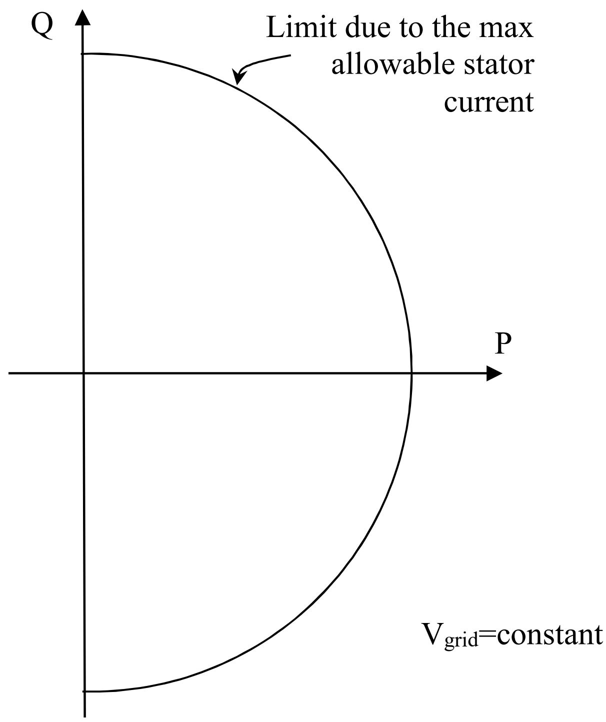


Figure 8-20. Limits due to the maximum stator current

But there is another constraint, due to the maximum value of the excitation current (that is the maximum $E_{sf\max}=E_{sf}(i_{f\max})$), which limits the maximum transmittable active power, given by δ=90°.

$$P_{t\max} = \frac{E_{sf\max} V_{grid} \sin(90°)}{X_{eq}} = \frac{E_{sf\max} V_{grid}}{X_{eq}}$$

Look at the expression of the stator current:

$$\overline{I_s} = \frac{\overline{E_{sf}(i_f)} - \overline{V_{grid}}}{jX_{eq}}$$

The complex power $\overline{A}$ is given by:

$$\overline{A} = \overline{V_{grid}}\,\underline{I_s} = P + jQ$$

so

$$P + jQ = \overline{V_{grid}}\,\frac{\underline{E_{sf}} - \underline{V_{grid}}}{-jX_{eq}} = \overline{V_{grid}}\,\frac{\underline{E_{sf}}}{-jX_{eq}} - \overline{V_{grid}}\,\frac{\underline{V_{grid}}}{-jX_{eq}} = \overline{V_{grid}}\,\frac{\underline{E_{sf}}}{-jX_{eq}} - j\frac{V_{grid}^2}{X_{eq}}$$

then

$$P + j\left(Q + \frac{V_{grid}^2}{X_{eq}}\right) = \overline{V_{grid}}\,\frac{\overline{E_{sf}}}{-jX_{eq}}$$

The term on the right may be simplified, when the transmitted power is at its maximum: $E_{sf}=E_{sf\max}$ and $E_{sf\max}$ is orthogonal to V_{grid}. This means that, considering $\overline{V_{grid}} = V_{grid} e^{j\varphi v}$:

$$\overline{E_{sf\max}} = E_{sf\max} e^{j\varphi e}$$

$$\underline{E_{sf\max}} = E_{sf\max} e^{-j\varphi e}$$

$$\overline{V_{grid}}\,\frac{\underline{E_{sf\max}}}{-jX_{eq}} = \frac{V_{grid} e^{j\varphi v} E_{sf\max} e^{-j\varphi e}}{-jX_{eq}} = \frac{V_{grid} E_{sf\max} e^{j(\varphi v - \varphi e)}}{-jX_{eq}} = \frac{V_{grid} E_{sf\max} e^{j(\delta + \pi/2)}}{X_{eq}}$$

which represents a complex number with constant amplitude $\frac{V_{grid} E_{sf\max}}{X_{eq}}$

Applying the Pythagoras' theorem

$$P^2 + \left(Q + \frac{V_{grid}^2}{X_{eq}}\right)^2 = \left(\frac{V_{grid} E_{sf\max}}{X_{eq}}\right)^2 = P_{t\max}^2$$

This equation represents a circle with a centre in

$$P = 0$$

$$Q = -\frac{V_{grid}^2}{X_{eq}}$$

with a radius equal to $P_{t\max}$

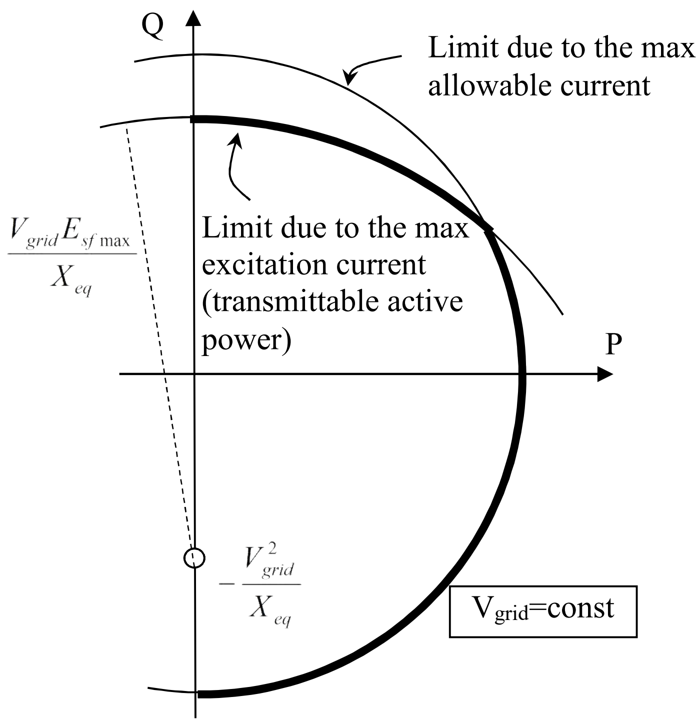


Figure 8-21. Limits due the maximum currents

8.7 "V" curves of a synchronous machine

These curves are calculated with V_{grid} and P constant.

jump

8.8 Effects of the Salient Poles

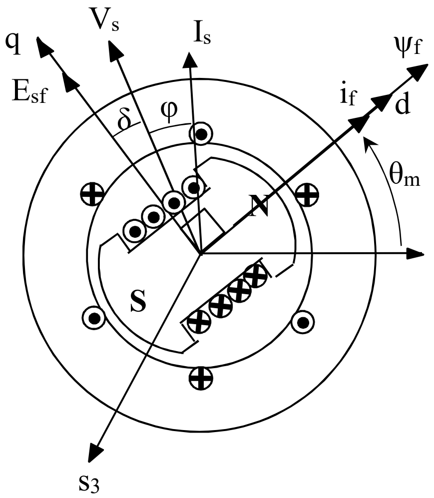


Figure 8-22: machine with salient poles (anisotropic machine)

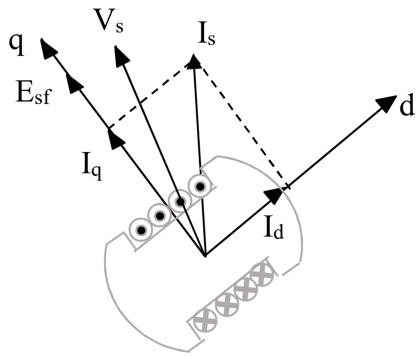


Figure 8-23: synchronous machine with salient poles (anisotropic machine), rotor only

The stator current may be represented by its components along the rotor flux ψ_f (d-axis) and orthogonal to it (q-axis), aligned with the no-load voltage E_{sf}.

The flux due to the current I_q has to pass through a large airgap. Otherwise, the flux due to the current I_d has to pass through a lower airgap. So $X_d > X_q$.

It results:

$$\overline{E}_{sf} = R_s \overline{I_s} + jX_d \overline{I_d} + jX_q \overline{I_q} + \overline{V_s}$$

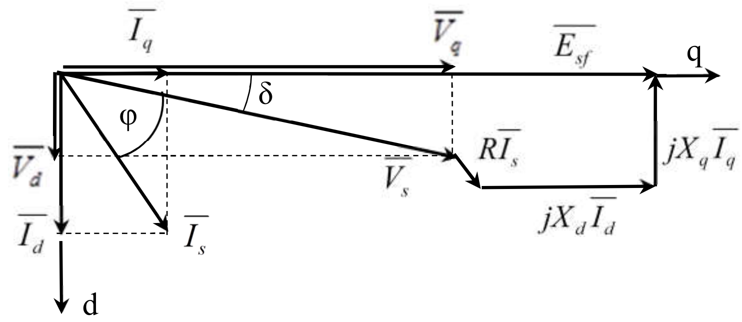


Figure 8-24. Phasor diagram for a salient poles machine (the axes are rotated by 90°)

The active power generated by the machine is:

$$P = \mathrm{Re}(\overline{V_s}\underline{I_s})$$

But $\overline{V_s} = V_d + jV_q$ and $\overline{I_s} = I_d + jI_q$.

So

$$P = \mathrm{Re}(\overline{V_s}\underline{I_s}) = V_d I_d + V_q I_q$$

But $V_d = V_s \sin(\delta)$ and $V_q = V_s \cos(\delta)$

Suppose to neglect the voltage drop on the resistance R_s.

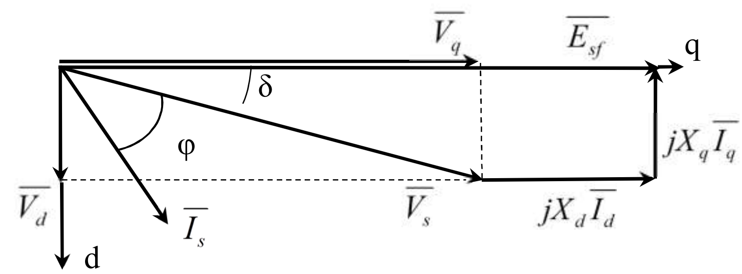


Figure 8-25. Phasor diagram for a salient poles machine (neglecting Rs effect)

On the real axis ("d"), $0 = \overline{V_d} + jX_q\overline{I_q} = V_d + jX_q jI_q = V_d - X_q I_q$ => $I_q = \dfrac{V_d}{X_q} = \dfrac{V_s \sin(\delta)}{X_q}$.

On the imaginary axis ("q"), $\overline{E_{sf}} = \overline{V_q} + jX_d\overline{I_d}$ => $jE_{sf} = jV_q + jX_d I_d$ => $E_{sf} = V_q + X_d I_d$ => $I_d = \dfrac{E_{sf} - V_q}{X_d} = \dfrac{E_{sf} - V_s \cos(\delta)}{X_d}$.

So

$$P = V_s \sin(\delta)\left(\frac{E_{sf} - V_s \cos(\delta)}{X_d}\right) + V_s \cos(\delta)\frac{V_s \sin(\delta)}{X_q}$$

$$P = \frac{E_{sf} V_s \sin(\delta)}{X_d} + \left(\frac{1}{X_q} - \frac{1}{X_d}\right) V_s^2 \frac{1}{2}\sin(2\delta)$$

with $X_d > X_q$.

The first term is equal to the one of the isotropic machine (where $X_s = X_d = X_q$). The second term is typical of an anisotropic machine (salient poles) and increases the power P for power angle δ lower than 90°.

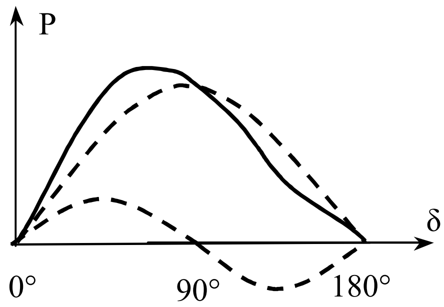


Figure 8-26: Produced Power

8.9 Permanent Magnet

draft

Consider a rotor with permanent magnet instead of an excitation winding. The model of this machine may be easily found considering that the rotor flux given by the permanent magnets would be constant.

$\psi_{PM} = Mi_f$ constant

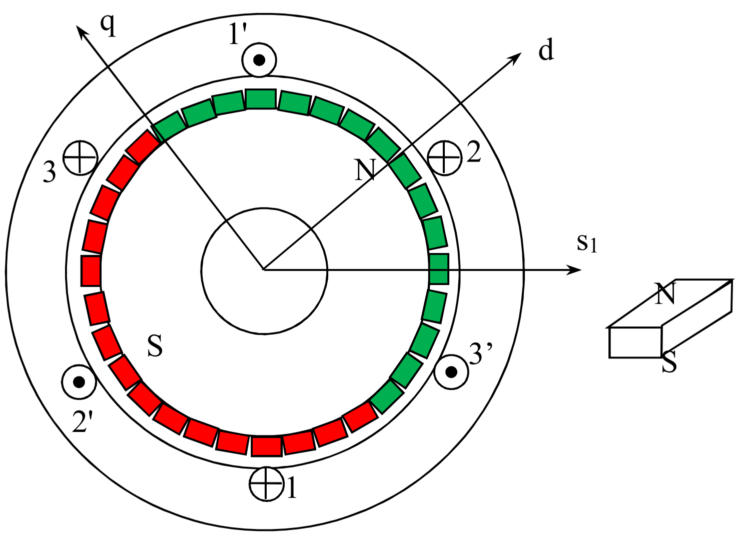


Figure 8-27: Example of a SMPM (Surface Mounted Permanent Magnet) synchronous machine, 2 poles (green magnets have North outward while the South towards the rotor; red ones vice versa)

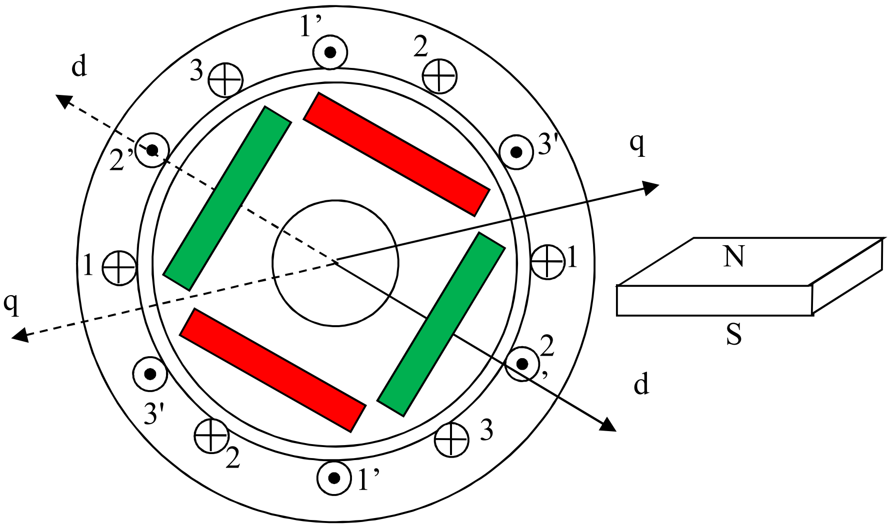
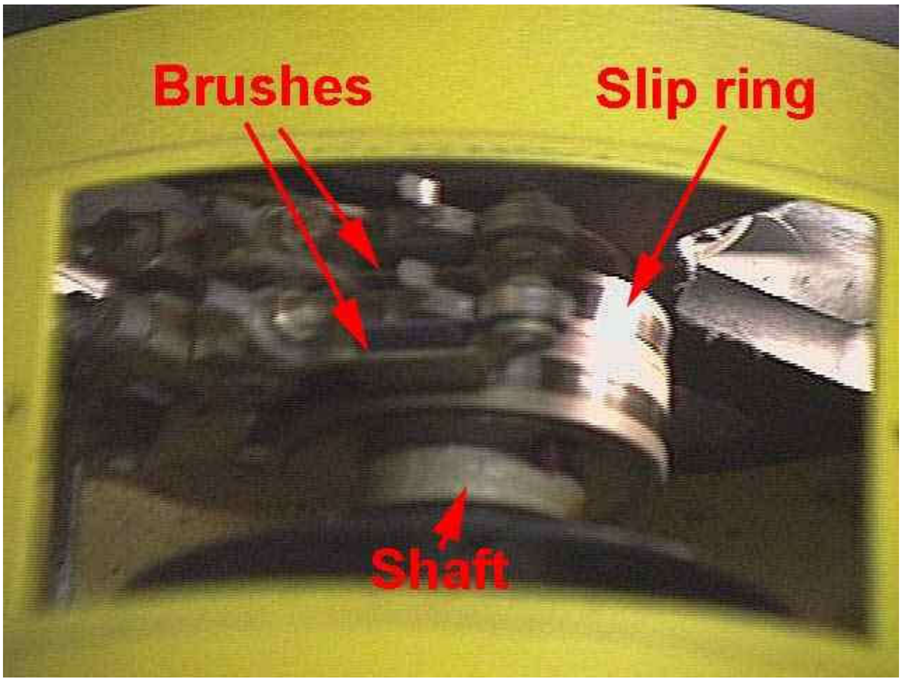


Figure 8-28: Example of IPM (Interior Permanent Magnet) synchronous machine, 4 poles (green magnets have North outward while the South towards the rotor; red ones vice versa)

The model of the machine is the same as above, but the excitation current is constant.

Appendix



(*http://emadrlc.blogspot.it/*)

Exciter: brushless excitation system