Summary

7. Electromagnetic joint. Rotating magnetic field. Space-phasor theory

7.1 Electromagnetic joint

Consider an isotropic rotating machine, with one rotor winding.

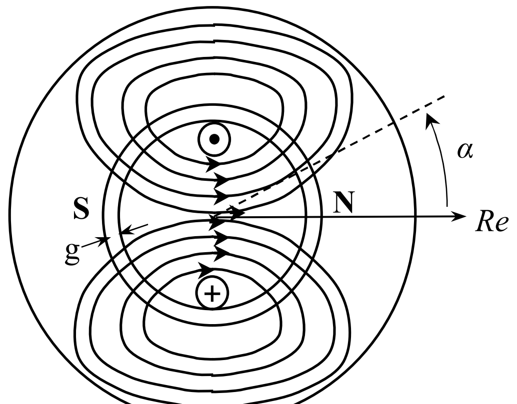


Figure 7-1 Isotropic rotor powered machine – concentrated windings

In order to calculate the magnetic field H (and the flux density B), we need to apply the Ampere's Law, that is to perform the Line integral along a closed path, i.e. one among the flux lines of Figure 7-1.

Due to the refraction law, the tangent of the angle β between the normal to the separation surface and the direction of the output flux file is equal to the ratio between the permeabilities of the two materials (ferromagnetic material and air). But $\mu_{fe}=\infty$, so the flux line is perpendicular to the separation surface, ($tg\beta=\mu_o/\mu_{fe}\approx 0$).

Furthermore, the magnetic field H inside the ferromagnetic material may be considered equal to zero ($\mu_{fe}=\infty$). Thus, the line integral of H along a flux lines has only the contribution due to the two segments of flux line inside the airgap ($N_r I_r$ is the magnetomotive force due to the rotor current):

$$\oint_C \overline{H} \cdot d\bar{l} = H2g = N_r i_r$$

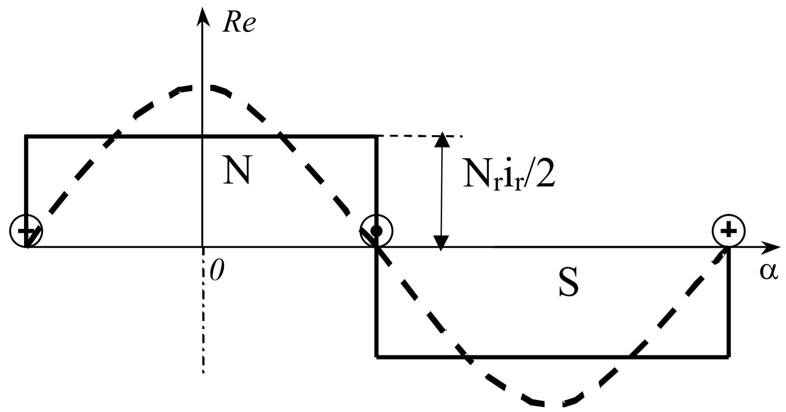


Figure 7-2 Qualitative trend of the magnetomotive force along the air gap – concentrated windings

In the air, the flux density $B=\mu_o H$. Thus, the amplitude of the flux density is constant (for half period):

$$B(\alpha) = \mu_o \frac{N_r i_r}{2g} \quad -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$$

$$B(\alpha) = -\mu_o \frac{N_r i_r}{2g} \quad \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$$

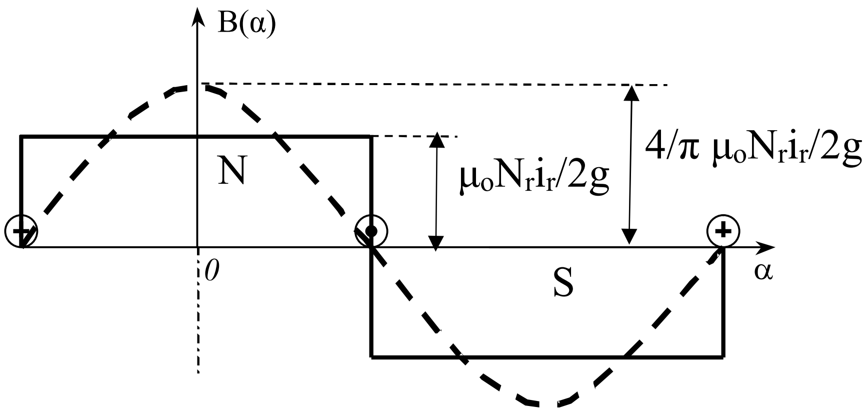


Figure 7-3 Qualitative trend of the flux density along the air gap – concentrated windings

The first harmonic of a square wave has an amplitude of 4/π of the amplitude of the square wave.

$$B_1(\alpha) = \frac{4}{\pi} \mu_o \frac{N_r i_r}{2g} \cos(\alpha)$$

Consider the Figure 7-4.

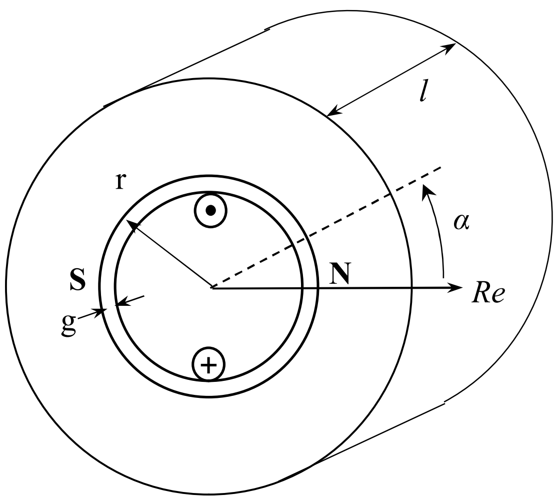


Figure 7-4 Isotropic rotor powered machine – concentrated windings

The flux linkage ψ_{rr} (flux linked with the rotor coil, due to the rotor current i_r) is

$$\psi_{rr} = N_r \varphi_{rr} = N_r \int_S \bar{b} \cdot \bar{n} dS = N_r \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_1(\alpha) rl d\alpha = N_r \frac{4}{\pi} \mu_o \frac{N_r i_r}{2g} rl 2 = N_r^{\,2} \frac{4}{\pi} \mu_o \frac{i_r}{g} rl$$

Using the definition of the inductance:

$$L_{rr} = \left.\frac{\psi_{rr}}{i_r}\right|_{i_s = 0} = N_r^{\,2} \frac{4}{\pi} \mu_o \frac{rl}{g}$$

Consider, now, the stator winding, with a generic mechanical position θ_m of the rotor respect the stator.

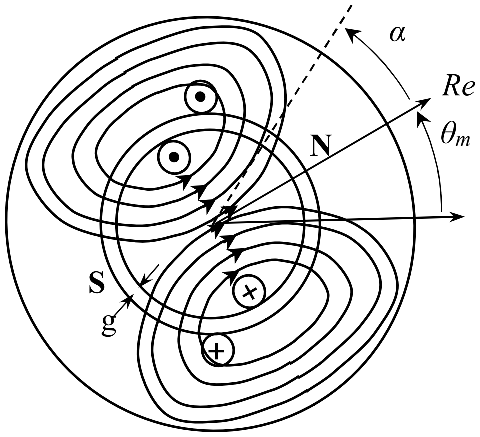


Figure 7-5 Isotropic rotor powered machine – concentrated windings

The flux linkage with the stator winding, due to the rotor current is:

$$\psi_{sr} = N_s \int_{-\frac{\pi}{2}-\theta_m}^{\frac{\pi}{2}-\theta_m} B_1(\alpha) rl d\alpha = N_s N_r \frac{4}{\pi} \mu_o \frac{i_r}{2g} rl \left[\sin\left(\frac{\pi}{2}-\theta_m\right) - \sin\left(-\frac{\pi}{2}-\theta_m\right)\right] = N_s N_r \frac{4}{\pi} \mu_o \frac{i_r}{g} rl \cos(\theta_m)$$

so

$$L_{sr} = \left.\frac{\psi_{sr}}{i_r}\right|_{i_s=0} = N_s N_r \frac{4}{\pi} \mu_o \frac{rl}{g} \cos(\theta_m) = L_{rs}$$

In a similar manner

$$L_{ss} = \left.\frac{\psi_{ss}}{i_s}\right|_{i_r=0} = N_s^2 \frac{4}{\pi} \mu_o \frac{rl}{g}$$

7.2 Number of poles

Consider the structure of Figure 7-6.

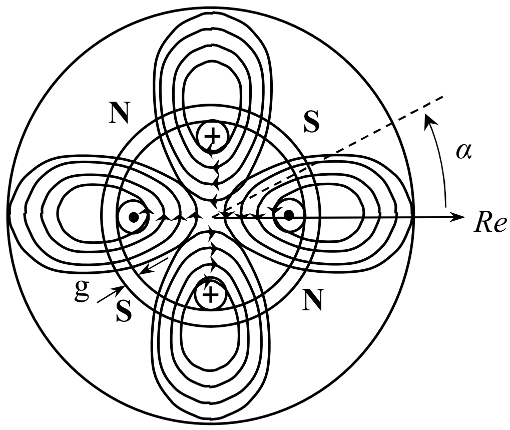


Figure 7-6 Isotropic rotor powered machine, four poles – concentrated windings

You find two North's and two South's. This machine has four poles.

If you walk along the airgap, you find a period of the magnetic quantities equal to an half of the mechanical period (2π). This means that the frequency of the electrical quantities is two times the frequency of the mechanical ones. With n_p pole pairs, the ratio is n_p. In another way, the mechanical speed seen in the electrical world is n_p times the mechanical speed seen in the mechanical world.

$$\omega_m = n_p \Omega_m$$

7.3 Distributed winding

Consider now a machine with a rotor with distributed winding instead of a concentrated one.

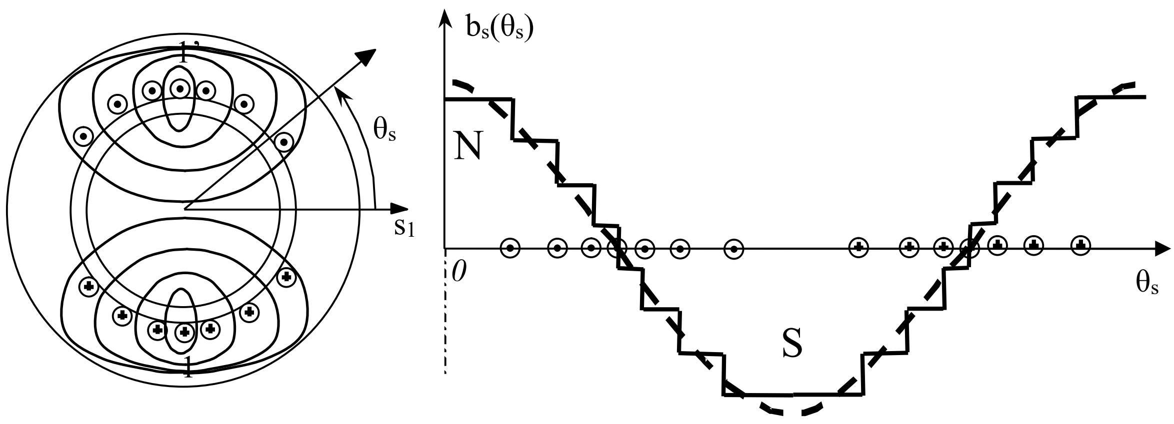


Figure 7-7: Flux density waveform along the air-gap in a distributed winding machine, due to stator current i_{s1}

The distribution of turns in the slots produces an effect (winding factor) that is easy to analyze. Each coil produces a field represented by a space phasor. The vector sum of these fields provides the resulting field (see Figure 7-8)

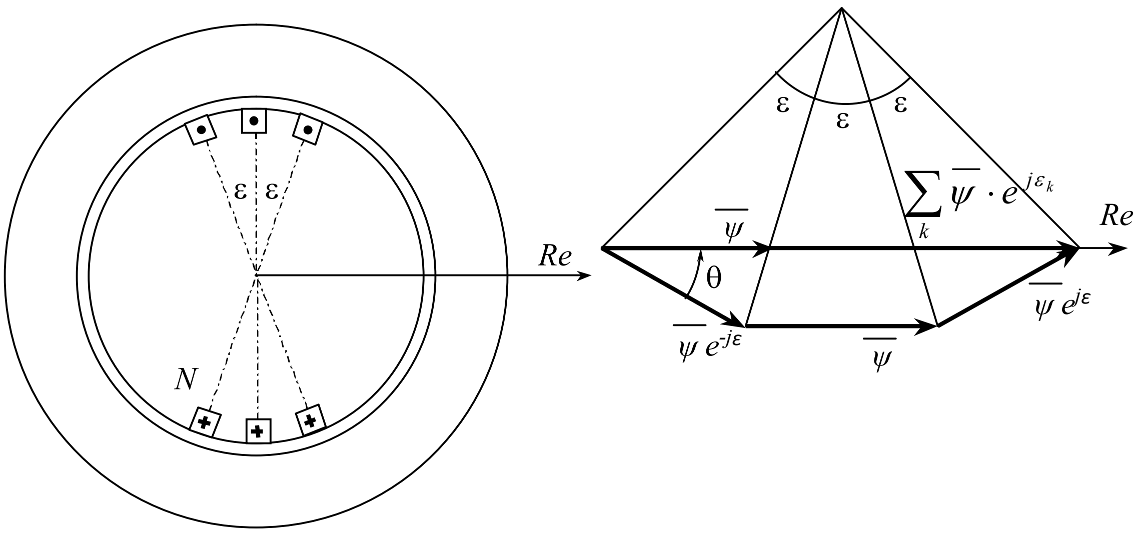


Figure 7-8: Winding factor

You need to introduce a coefficient k_w: winding factor (k_w <=1)

$$\varphi_{rr} = k_w N_r \frac{4}{\pi} \mu_o \frac{i_r}{g} rl$$

7.4 Torque expression

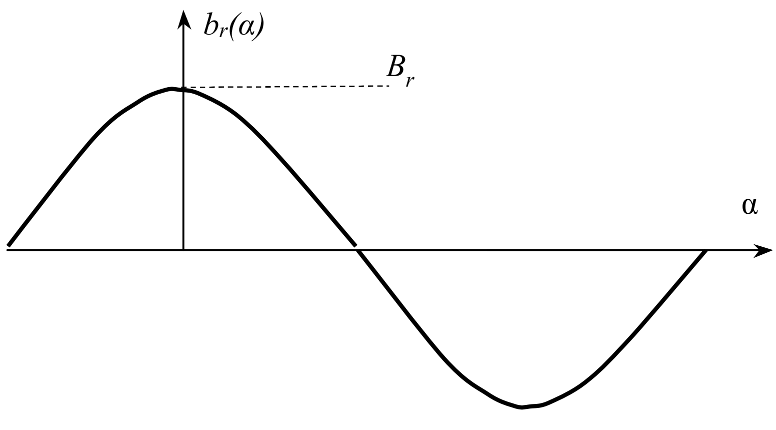
With both stator and rotor current

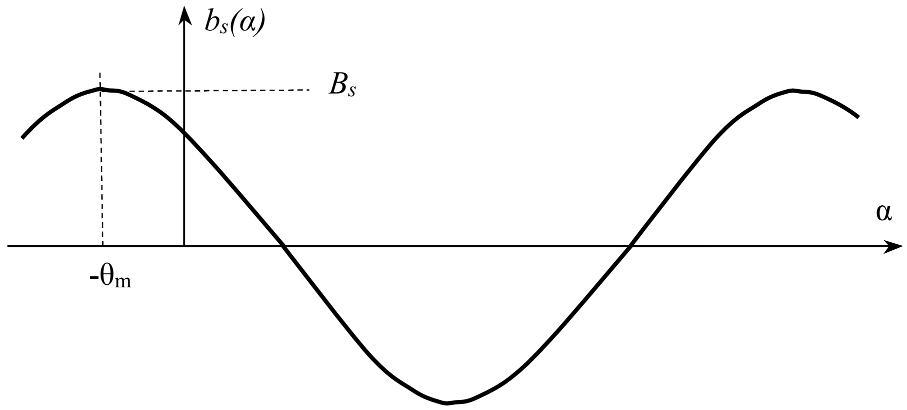
$$\psi_r = L_{rr} i_r + L_{rs}(\theta_m) i_s$$

$$\psi_s = L_{ss} i_s + L_{sr}(\theta_m) i_r$$

The torque expression is

$$T_e = \left. \frac{\partial W_m}{\partial \theta_m} \right|_{\psi_s, \psi_r = const}$$





Applying the Superposition Principle:

$$b(\alpha) = b_r(\alpha) + b_s(\alpha) = B_r \cos(\alpha) + B_s \cos(\alpha + \theta_m)$$

$$B_r = k_{wr} \frac{4}{\pi} \mu_o \frac{N_r i_r}{2g}$$

$$B_s = k_{ws} \frac{4}{\pi} \mu_o \frac{N_s i_s}{2g}$$

The energy density is 1/2 BH.

With $\mu_{fe}=\infty$, there is no energy stored in the ferromagnetic material. Therefore, the energy is stored only in the air-gap.

$$W_m = \int_V \frac{1}{2} BHdV = \int_V \frac{1}{2} \frac{B^2}{\mu_0} dV = \int_0^{2\pi} \frac{1}{2} \frac{b(\alpha)^2}{\mu_0} lrgd\alpha$$

$$T_e = \frac{\partial}{\partial \theta_m} \left\{ \frac{lrg}{2\mu_0} \left[\int_0^{2\pi} B_r^{\ 2} \cos(\alpha)^2 d\alpha + \int_0^{2\pi} B_s^{\ 2} \cos(\alpha + \theta_m)^2 d\alpha + \int_0^{2\pi} 2B_r B_s \cos(\alpha)\cos(\alpha + \theta_m) d\alpha \right] \right\}$$

The first term does not depend on the mechanical position, so its partial derivative is zero.

The second term has an average value which does not depend on the mechanical position and a $\cos[2(\alpha+\theta_m)]$ whose integral between 0 and 2π is equal to 0. The third one is:

$$\cos(\alpha)\cos(\alpha + \theta_m) = \frac{\cos(\alpha + \alpha + \theta_m) + \cos(\alpha - \alpha - \theta_m)}{2} = \frac{\cos(2\alpha + \theta_m)}{2} + \frac{\cos(-\theta_m)}{2}$$

The integral between 0 and 2π of the first part is equal to 0. Therefore the second part is the only different from zero,

$$T_e = \frac{\partial}{\partial \theta_m} \left\{ \frac{2B_r B_s lrg}{2\mu_0} \left[\int_0^{2\pi} \frac{\cos(-\theta_m)}{2} d\alpha \right] \right\} = \frac{B_r B_s lrg}{2\mu_0} 2\pi \frac{\partial}{\partial \theta_m} \cos(-\theta_m) = -\frac{B_r B_s lrg}{2\mu_0} 2\pi \sin(\theta_m)$$

Negative means an attractive torque: k $\psi_{ss}\psi_{rr} \sin(\theta_m)$ (Electromagnetic Joint)

7.5 Space phasor

If we consider only the first harmonic and neglecting the highest harmonics, the situation in the air gap may be represented by a vector whose direction is along the North polar axis (Figure 7-9).

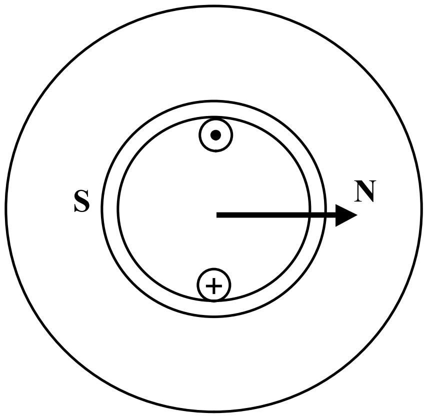


Figure 7-9: Mmf representation by means of a space phasor

The space phasor (whose amplitude is a function of time) allows the knowledge of the value of the corresponding quantity at any location within the air-gap and at each instant. In fact, in order to know the value of the amplitude at a given point of the air gap, it is sufficient to project the phasor onto the required direction. It appears that $M(\theta,t) = M(t) \cdot \cos(\theta)$ (Figure 7-10).

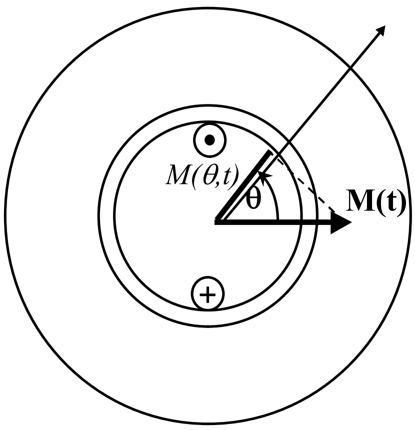


Figure 7-10: Space phasor application

All electrical quantities (voltages and currents) and magnetic quantities (mmf and fluxes) can be represented by space phasors, allowing for easy interpretation of electromagnetic phenomena.

7.6 Three-phase machine and the rotating magnetic field

Consider three windings, equal each other but with a displacement of 120°. Look at the Figure 7-11.

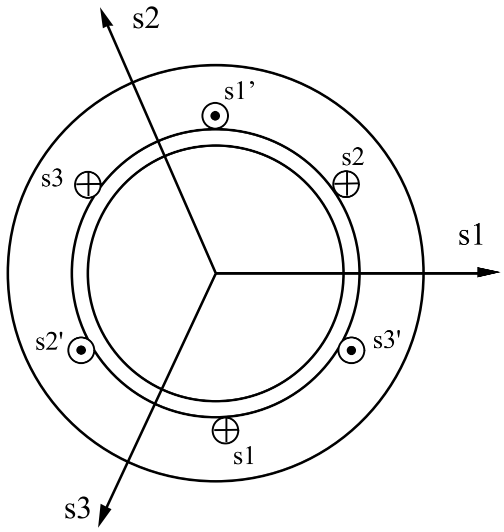


Figure 7-11: Three-phase machine

Suppose to supply the machine by means of a symmetrical three-phase power supply. At steady-state, the currents will assume the waveform of Figure 7-12: they have the same amplitude, same frequency and a displacement of 120°

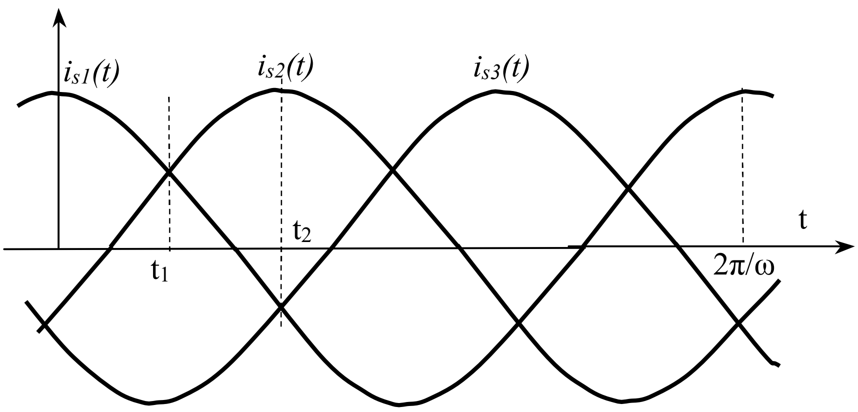


Figure 7-12: Three-phase currents

At time t=0, the current i_{s1} assumes its maximum value I_{max} while $i_{s2}=i_{s3}=I_{max}\cos(120°)=-0.5\ I_{max}$.

Applying the superposition principle, the total flux are the sum of the three fluxes due to the three currents. Thus, the total flux has the direction of s1 and an amplitude equal to 3/2 of the flux ψ_{s1} (see Figure 7-13).

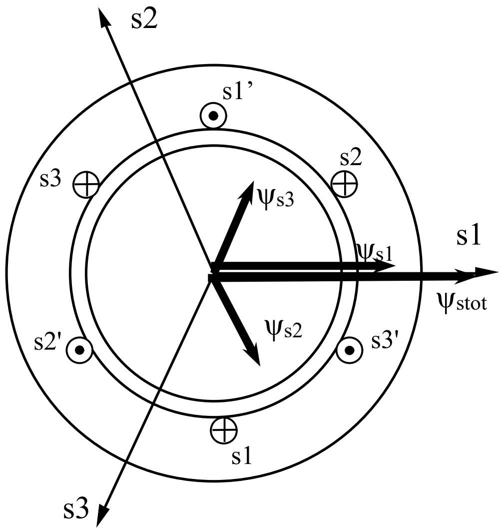


Figure 7-13: Total flux at t=0

At t=t1, the current i_{s2} assumes its maximum value I_{max} while $i_{s1}=i_{s3}=-0.5\ I_{max}$. (Figure 7-14). Thus, the total flux has the direction of s2 and an amplitude equal to the previous one.

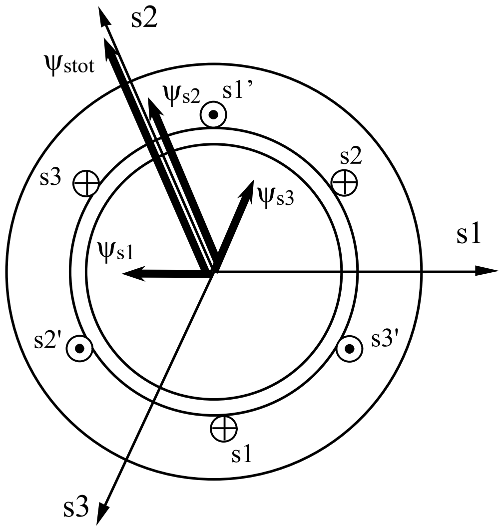


Figure 7-14: Total flux at $t=t_1$

At t=t2, the current i_{s3} assumes its maximum value I_{max} while $i_{s1}=i_{s2}=-0.5\ I_{max}$. (Figure 7-15). Thus, the total flux has the direction of s3 and an amplitude equal to the previous one.

It means that, at steady state, with a symmetrical power supply, the total flux assumes a constant value and it is moving at a constant speed: for a two poles machine, the speed of the rotating flux Ω is equal to the angular frequency ω of the electrical quantities (for n_p pole pairs, it results $\omega=n_p\ \Omega$). This effect is called "rotating field" (*campo magnetico rotante*) and is the milestone of the electromechanical conversion in ac machine.

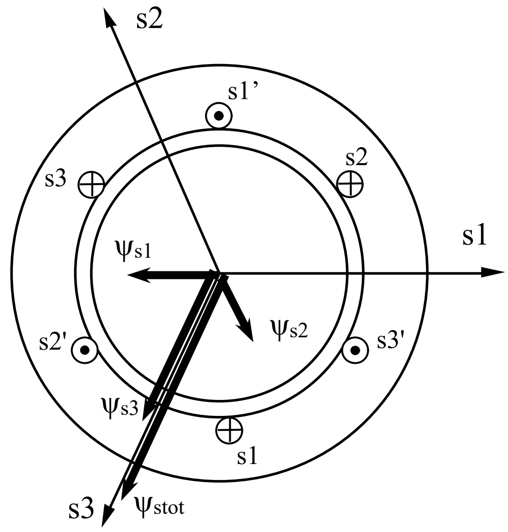


Figure 7-15: Total flux at t=t2

7.7 Space phasor algebra

Consider the reference frame α,β of Figure 7-16 (α has the same direction of the magnetic axis s1)

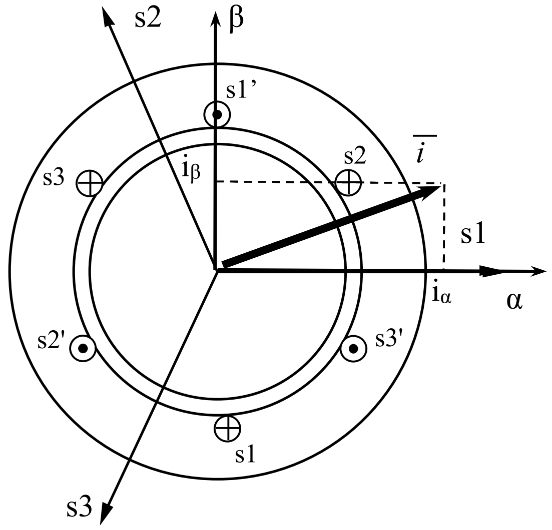


Figure 7-16: Reference frame, fixed with the stator windings

Suppose to have three currents into the windings with a total effect represented by the space phasor **I**.

The expression of the space phasor of the current as a function of the two currents i_α and i_β is the following:

$$\bar{i} = (i_\alpha + j \cdot i_\beta)$$

Conversely, knowing the space phasor, the value of winding current can be calculated simply projecting the space phasor along the direction of the corresponding magnetic axis; for example:

$$i_\alpha = R\alpha(\bar{i})$$

where $R\alpha$ is defined as the projection of the phasor onto the magnetic axis of phase "α".

The total effect achieved by using three windings, displaced by 120 electrical degrees in space, and three-phase currents, may be easily calculated by projection. The formula is as follows:

$$\bar{i} = (i_{s1} + \bar{\alpha} \cdot i_{s2} + \bar{\alpha}^2 \cdot i_{s3})$$

where $\overline{\alpha} = e^{j\frac{2}{3}\pi}$

On the other hand, it is possible to calculate, knowing the space phasor and assuming that the sum of the phase currents is zero, the value of a phase current using the formula:

$$i_{s1} = \frac{2}{3} Rs1(\bar{i})$$

where $Rs1$ is defined as the projection of the phasor onto the magnetic axis of phase "*s1*".

Traditionally (for this course), however, the space phasor is defined as follows:

$$\bar{i} = \sqrt{\frac{2}{3}}(i_{s1} + \overline{\alpha} \cdot i_{s2} + \overline{\alpha}^2 \cdot i_{s3})$$

$$i_{s1} = \sqrt{\frac{2}{3}} Rs1(\bar{i})$$

or better: $i_{s1} = \sqrt{\frac{2}{3}}\,\mathrm{Re}(\bar{i}) \quad i_{s2} = \sqrt{\frac{2}{3}}\,\mathrm{Re}(\overline{\alpha}^2 \cdot \bar{i}) \quad i_{s3} = \sqrt{\frac{2}{3}}\,\mathrm{Re}(\overline{\alpha} \cdot \bar{i})$

so as to maintain the same expressions of power and energy both in the phase quantities and in the space phasors (Re means "real part of").

The space phasor, like all vectors, is completely defined by two variables: the amplitude and the argument (or phase angle) or by its components with respect to two orthogonal axes (reference frame).

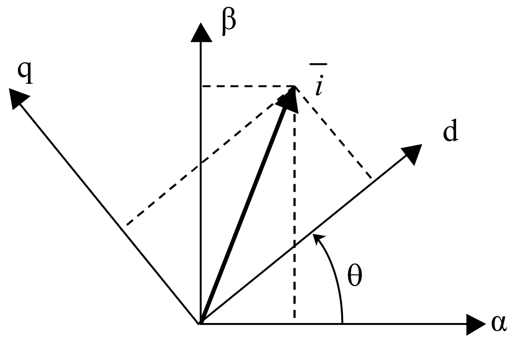


Figure 7-17: Different Reference frames

The axes of a generic reference frame are commonly marked with "d" and "q" (where d is the real axis, while q represents the imaginary axis). If the reference frame is fixed with the stator (with the real axis aligned with s1), the two axes are called α and β (as we saw). Given $i^{\alpha\beta}$ as the phasor with respect to a reference frame "αβ", the corresponding phasor in a reference frame "dq" displaced by an angle θ with respect "αβ " is:

$$\overline{i^{dq}} = \overline{i^{\alpha\beta}} \cdot e^{-j\theta}$$

The phasor is always the same, but it is "seen" from a different viewpoint.

The operation of the derivative of a phasor leads to the following relationship:

$$\frac{d\overline{i^{dq}}}{dt} = \frac{d\left(\overline{i^{\alpha\beta}} \cdot e^{-j\theta}\right)}{dt} = \frac{d\overline{i^{\alpha\beta}}}{dt} e^{-j\theta} - j\dot{\theta} \cdot e^{-j\theta} \cdot \overline{i^{\alpha\beta}} = \left(\frac{d\overline{i^{\alpha\beta}}}{dt} - j\dot{\theta} \cdot \overline{i^{\alpha\beta}}\right) \cdot e^{-j\theta}$$

7.7.1 Space phasor theory applied to an electrical machine

Suppose to supply the machine by means of a symmetrical three-phase power supply. At steady-state, the currents will assume the waveform of Figure 7-12.

Applying the Space Phasor formula [$\bar{i} = \sqrt{\frac{2}{3}}(i_{s1} + \overline{\alpha} \cdot i_{s2} + \overline{\alpha}^2 \cdot i_{s3})$] the current space phasor assumes the value $\bar{i} = \sqrt{\frac{2}{3}}\frac{3}{2}I_{\max}e^{j\omega t} = \sqrt{\frac{3}{2}}I_{\max}e^{j\omega t} = \sqrt{3}I_{rms}e^{j\omega t}$. If the same formula is applied to the voltages, the voltage space phasor has a constant amplitude, equal to the rms value of the line-line voltage and it is moving at a constant speed ω.

The active power of the three-phase circuit is traditionally calculated by: $P=3V_{ph}I_{ph}\cos(\varphi)$ or $P=\text{sqrt}(3)V_{ll}I_{ph}\cos(\varphi)$, where V_{ph} and I_{ph} represent the phase voltage and phase current, φ is the displacement angle between them and V_{ll} is the line to line voltage .

Using the space phasor representation you have: **$P=V_sI_s\cos(\varphi)$** (without any square root of 3), where V_s and I_s represent the voltage and current space phasors.