Summary

5. The electromechanical energy conversion

5.1 The primitive machine

Before addressing the study of electrical machines, it is convenient to introduce the study of the basic principles of electromechanical energy conversion.
The starting point for understanding this theory is the primitive machine:

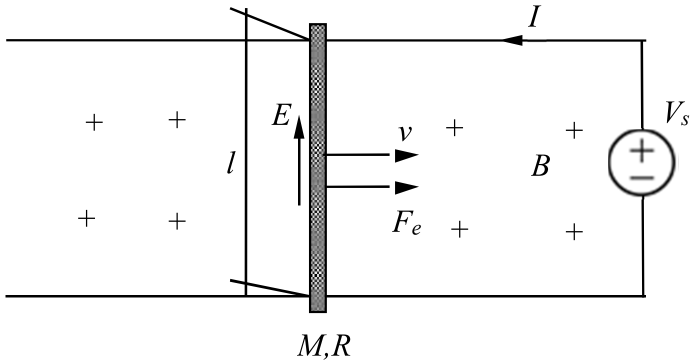

Figure 5-1: the primitive machine

It is a device consisting of:

- a straight conductor of length "*l*", mass "*M*" and resistance "*R*" free to move in horizontal direction
- two rails made by ideal conductor (R_{rail}=0); there is no friction between conductor and rails
- a uniform magnetic field B perpendicular to the conductor rails plane (in the Figure 5-1 it is directed into the page).

It follows that, if the conductor moves at a linear speed "*v*", an electromotive force is induced across an element "*dl*" of the conductor.

$$dE = \overline{v} \times \overline{B} \cdot \overline{dl}$$

which, integrated over the length "*l*", in the case of Figure 5-1 becomes:

$$E = Blv$$

If it is carrying a current "*I*", the element "*dl*" is subjected to a force:

$$\overline{dF_e} = \overline{dl} \times \overline{B} \cdot I$$

which, integrated over the length "*l*", in the case of Figure 5-1 becomes:

$$F_e = BlI$$

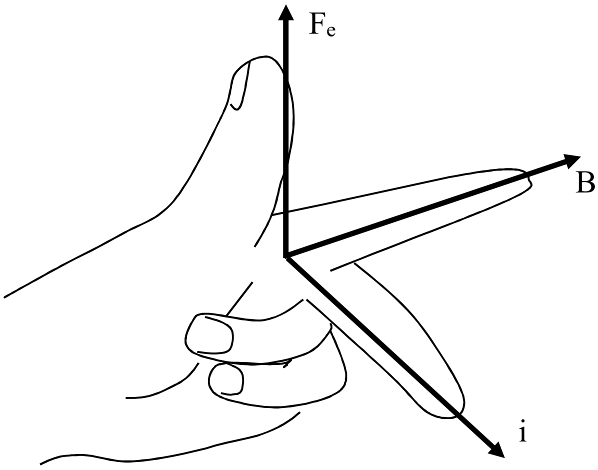

Figure 5-2: Left-hand law (*regola della mano sinsitra*)

If the conductor moves and, at the same time, a current is flowing, the electric power transmitted matches the mechanical power:

$$P_e = E \cdot I = B \cdot l \cdot v \cdot \frac{F_e}{B \cdot l} = F_e \cdot v = P_m$$

Consider, then, the evolution of the following phenomenon (the conductor is free to move):

- initial status: conductor stands still and no current is flowing
- a voltage V_s is applied, so an initial (starting) (*corrente di avviamento*) current begins to flow: $I_{st} = V_s / R$
- it generates an electromagnetic force (starting force) (*forza di avviamento*) on the conductor: $F_e = F_{st} = B \cdot l \cdot I_{st}$
- the conductor accelerates, and this generates an induced electromotive force: $E = Blv$
- in every instants you have, then, that the current becomes: $I = \dfrac{V_s - E}{R}$
- equilibrium is reached, without any friction and resistive force, when the current vanishes (I=0) and therefore when $V_s = E$
- the final speed is called no-load speed (v_o) (*velocità a vuoto*) and it results: $V_s = E = Blv_o$

If braking forces (resistive forces) (*forza resistente*) go into action, then the balance is reached when $F_e = F_r$ (the total resistive force) and therefore the presence of a current is required. In particular:

$$F_r = F_e = BlI = Bl\frac{V_s - E}{R} = Bl\frac{Blv_o - Blv}{R} = \frac{B^2l^2}{R} \cdot (v_o - v)$$

$$v = v_o - \frac{F_r}{\frac{B^2l^2}{R}}$$

where "v" is always less than the no load speed v_o with positive resistive force (motor behavior) and higher than v_o with negative resistive force (generator).

The relationship between force and speed is linear. There are two typical points: the starting force and the no-load speed.

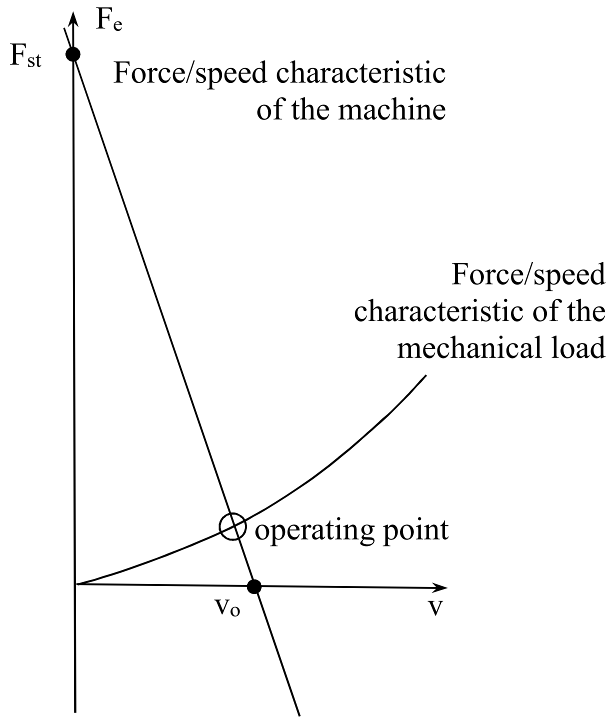

Figure 5-3: Force/speed characteristics (*caratteristica elettromeccanica*)

What we said for the primitive linear machine remains valid for a rotating machine, making the appropriate substitutions:

- instead of the *Bl* product, the flux linkage Ψ has to be considered
- the electromagnetic force F_e becomes electromagnetic torque T_e
- the linear speed v becomes angular speed Ω
- the inertia force becomes inertia torque
- the braking force becomes braking torque (resistive torque).

The fundamental relationships then become:

$$E = \Omega\Psi \quad T_e = \Psi I \quad EI = T_e\Omega$$

We can thus obtain, in a similar way, the values of the operating speed:

- at no load (without braking torque)

$$\Omega_o = \frac{V_s}{\Psi}$$

- at a load (with braking torque)

$$\Omega = \Omega_o - \frac{T_r}{\Psi^2/R}$$

- with locked rotor (zero speed), the starting torque (*coppia di avviamento*) is

$$T_e = T_{st} = \Psi \cdot I_{st} = \Psi \cdot V_s / R$$

5.1.1 The torque/speed curve

The locus of the operation points (at steady state) of the electric machine is called torque/speed curve. The remarkable points are:

- the intersection with the torque axis (zero speed = standstill), which represents the starting torque.
- the intersection with the speed axis (zero torque = no load), which provides no load speed.

The intersection of this curve with the torque/speed curve of the mechanical load identifies the operating point.
Torque-speed characteristics of typical mechanical load:

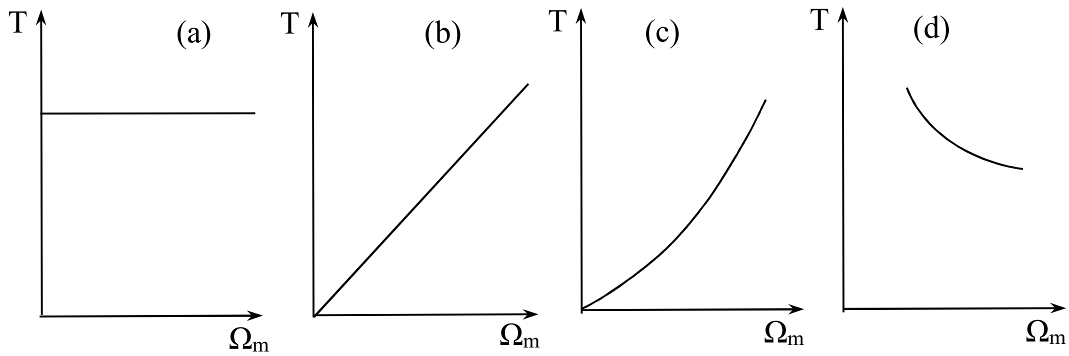

Figure 5-4: Torque-speed characteristics of typical mechanical load: (a) Constant torque, (b) linearly proportional, (c) proportional to the speed squared, (d) inverse proportional to the speed

Type of machine	**Torque law depending on speed**
Conveyor (*nastro trasportatore*)	Constant
Rotary press (*rotativa*)	Constant
Centrifugal pump (*pompa centrifuga*)	Torque increasing with the speed squared
Fans and blowers (*ventilatori e soffiatori*)	Torque increasing with the speed squared
Screw compressor (*compressore a vite*)	Constant
Piston compressor (*compressore volumetrico*)	Constant
Extruding machine (*estrusore*)	Constant or decreasing linearly with speed
Mechanical press (*pressa*)	Constant
Winders, unwinders (*avvolgitore, svolgitore*)	Constant or decreasing linearly with speed
Mixer (*miscelatore*)	Torque increasing linearly with speed
Kneader (*impastatrice*)	Constant or decreasing linearly with speed
Centrifuge (*centriguga*)	Torque increasing with the speed squared
Hoist, lift (*montacarichi, ascensore*)	Constant

This operating point may be stable or unstable.
It will be stable if an increase of the speed is a deficiency of torque so the machine slows down; conversely, a decrease of speed must be matched by a surplus of torque so the machine accelerates.
Knowledge of the mechanical behavior of the load is a fundamental starting point for the electromechanical device design and for the choice of the machine to be used.
By studying the behavior of the mechanical load, you can in fact identify the needs in terms of torque, starting from the time profile of the required speed. You may also obtain the points of maximum acceleration and deceleration, that are the points of maximum torque for the electric machine.

5.1.2 Reluctance torque and excitation torque

In this section, we want to highlight the different contributions to the origin of electro-mechanical action.
Basically, there are two different cases:
- torque resulting from the magnetic structure of the circuit, that is caused by anisotropy
- resulting from the interaction between two magnetic coils.

Consider the first case, referring to Figure 5-5, where you can highlight a magnetic structure made by a fixed part on which is mounted a winding and a rotating part.

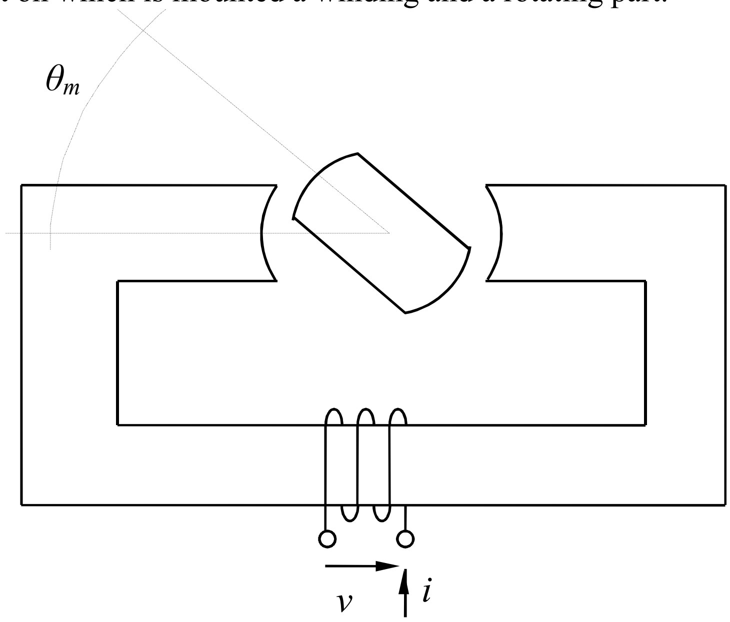

Figure 5-5: reluctance torque in a primitive machine

From the electrical point of view, the system may be represented by a one-port, as a series of a variable inductance (it depends on the mechanical position θ_m, which may be a function of time) with a resistance. Thus, we have:

$$v = Ri + \frac{d}{dt}\left[L(\theta_m)i\right] = Ri + L(\theta_m)\frac{di}{dt} + i\frac{dL(\theta_m)}{dt}$$

Multiplying both sides by the current "i", we obtain the equation, expression of the energy balance:

$$vi = Ri^2 + iL(\theta_m)\frac{di}{dt} + i^2\frac{dL(\theta_m)}{dt}$$

Recalling that the instantaneous power absorbed by the magnetic field can be obtained by differentiating the energy expression, it results:

$$p_\mu = \frac{dU}{dt} = \frac{d}{dt}\left[\frac{1}{2}L(\theta_m)i^2\right] = L(\theta_m)i\frac{di}{dt} + \frac{1}{2}i^2\frac{dL(\theta_m)}{dt}$$

Comparing this expression with the energy balance of the circuit, we find that the total electric power inlet (on the left of the equation) is divided into three contributions:

- power dissipated in the resistance (Joule losses)

$$p_J = Ri^2$$

- power related to the variation of the energy stored in the magnetic field

$$p_\mu = L(\theta_m)i\frac{di}{dt} + \frac{1}{2}i^2\frac{dL(\theta_m)}{dt}$$

- mechanical power

$$p_m = \frac{1}{2}i^2\frac{dL(\theta_m)}{dt}$$

If we consider that the inductance in the reference frame varies with periodic sinusoidal pattern, you can highlight an expression for the anisotropy torque:

$$p_m = \frac{1}{2}i^2\frac{dL(\theta_m)}{dt} = \frac{1}{2}i^2\frac{dL(\theta_m)}{d\theta_m}\frac{d\theta_m}{dt} = \frac{1}{2}i^2\frac{dL(\theta_m)}{d\theta_m}\Omega_m$$

$$T_m = \frac{p_m}{\Omega_m} = \frac{1}{2} i^2 \frac{dL(\theta_m)}{d\theta_m}$$

Now, suppose to change the old structure so to include a new winding on the rotating part as in Figure 5-6.

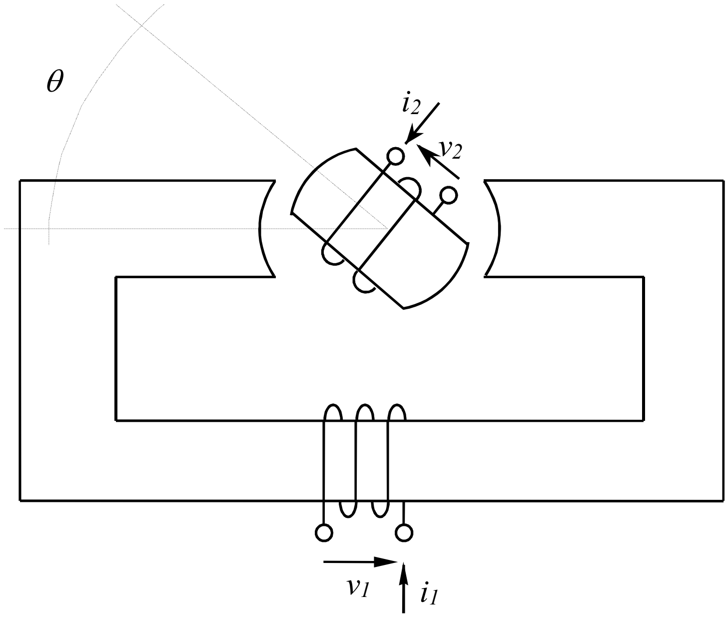

Figure 5-6: reluctance and excitation torque in a simple machine

The equations describing this structure are that of a mutual inductor with variable parameters. You can then write:

$$v_1 = R_1 i_1 + \frac{d}{dt}\left[L_1(\theta_m) i_1\right] + \frac{d}{dt}\left[L_m(\theta_m) i_2\right]$$

$$v_2 = R_2 i_2 + \frac{d}{dt}\left[L_2(\theta_m) i_2\right] + \frac{d}{dt}\left[L_m(\theta_m) i_1\right]$$

and developing the derivatives:

$$v_1 = R_1 i_1 + L_1(\theta_m)\frac{di_1}{dt} + i_1 \frac{dL_1(\theta_m)}{dt} + L_m(\theta_m)\frac{di_2}{dt} + i_2 \frac{dL_m(\theta_m)}{dt}$$

$$v_2 = R_2 i_2 + L_2(\theta_m)\frac{di_2}{dt} + i_2 \frac{dL_2(\theta_m)}{dt} + L_m(\theta_m)\frac{di_1}{dt} + i_1 \frac{dL_m(\theta_m)}{dt}$$

If, therefore, similar to what we saw before, we extract the contributions related to the mechanical power, it is obtained:

for the primary windings $\quad \frac{1}{2} i_1^2 \frac{dL_1(\theta_m)}{dt} + \frac{1}{2} i_1 i_2 \frac{dL_m(\theta_m)}{dt}$

for the secondary windings $\quad \frac{1}{2} i_2^2 \frac{dL_2(\theta_m)}{dt} + \frac{1}{2} i_1 i_2 \frac{dL_m(\theta_m)}{dt}$

In a similar way as before, we can highlight the following contributions to the torque:

$$T_m = \frac{1}{2} i_1^2 \frac{dL_1(\theta_m)}{d\theta_m} + \frac{1}{2} i_2^2 \frac{dL_2(\theta_m)}{d\theta_m} + i_1 i_2 \frac{dL_m(\theta_m)}{d\theta_m}$$

where the first two terms are similar to the previous terms of anisotropy and the last torque is called excitation torque:

$$T_m = i_1 i_2 \frac{dL_m(\theta_m)}{d\theta_m}$$

5.1.3 Motor nameplate and rated values

In order to be able to identify the characteristics of a machine it is necessary to see the nameplate that is located on the frame. This nameplate is a real ID card that allows us to trace the basic features of the machine with regard to the application point of view. The values reported on the plate depend on the type of machine, but they are able to provide the information for a proper connection and use.
As we shall see later, however, the data are not useful for determining the appropriate design of the control, given that certain parameters must be obtained experimentally and are rarely supplied by the manufacturer.
The basic parameters set are called rated data and identify an operation point of the machine, at which the engine can work indefinitely in time without thermal problems (apart from obviously the wear) or damage.

5.1.3.1 Nomenclature notes for rotating machinery

- *stator* part of the machine which remains stationary during operation
- *rotor* part of the machine which rotates during operation
- *inductor winding* winding that creates the main magnetic field
- *induced winding* winding immersed in the field created by the inductor