# Summary

# 6. DC Machine

## 6.1 Structure

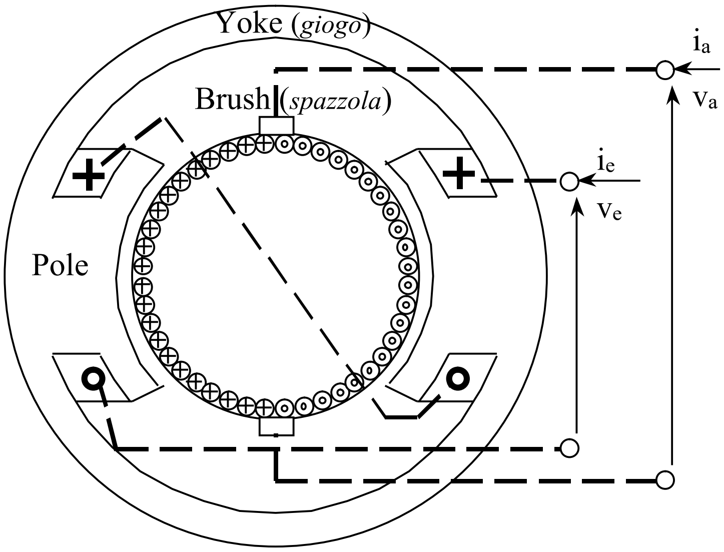



Figure 6-1. Two poles, separately excited, DC machine cross-section

Structurally, a d.c. machine consists of:

- an external stator with salient poles which acts as an inducer
- an internal rotor which acts as an armature.

Around the polar body, it will have a coil into which a current is flowing; it is made by turns connected in series, forming the excitation or "main" field winding.

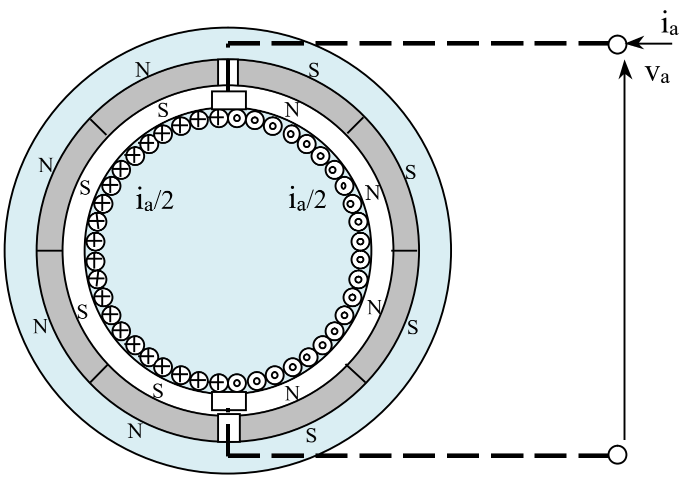



Figure 6-2. Two poles PM DC machine cross-section

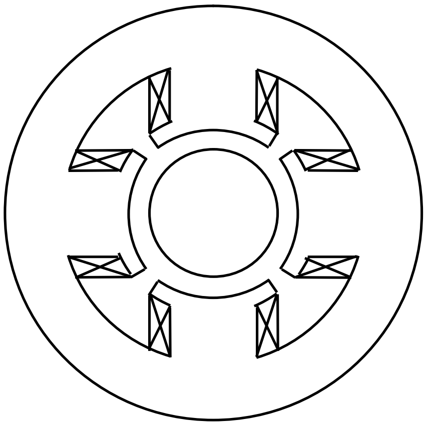


Figure 6-3. Four poles, separately excited, DC machine cross-section

The armature (*armatura*) consists of a cylinder made of laminated ferromagnetic material with distributed slots on the periphery. In these slots a closed coil is wound. On the rotor another device is also mounted; it is called "commutator" (*commutatore a lamelle*).

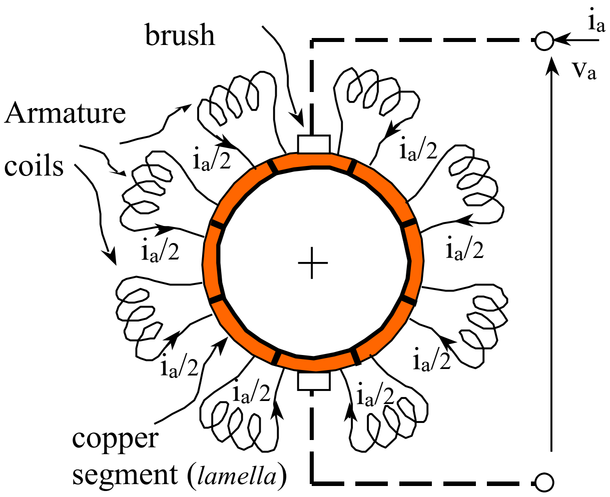



Figure 6-4. Commutator with 8 copper segments and 2 brushes

To understand the operation of the machine it should initially refer to a simplified armature winding made by a single turn rotating on itself, surrounded by a uniform magnetic field.

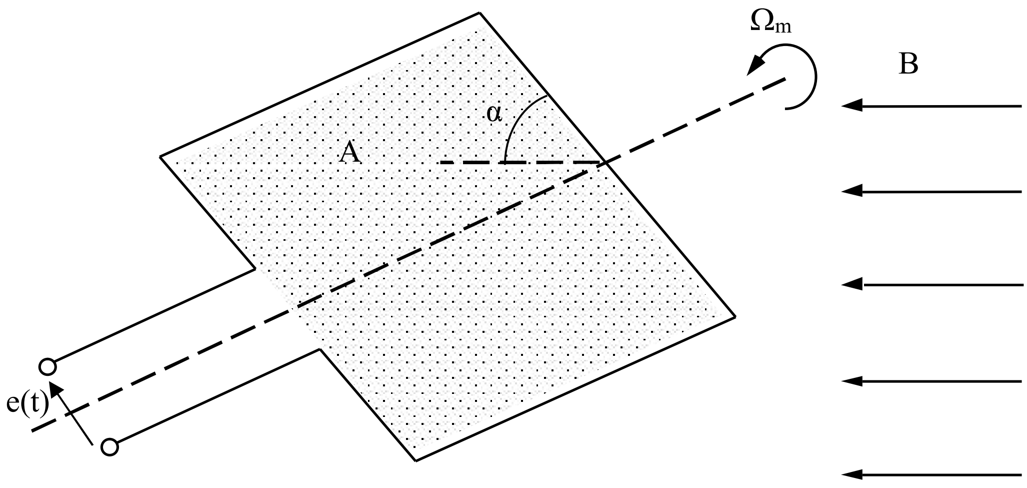



Figure 6-5. single turn armature winding

If we assume that the plane of the coil forms, at any given time, an angle α with the direction of the vector B (flux density), we have that the magnetic flux linked with the coil is (*A* refers to the surface of the coil itself):

$$\psi = B \cdot A \cdot \sin\alpha$$

If we now assume that the coil rotates with constant speed $\Omega_m$, we have that, at its terminals, an induced electromotive force (*forza elettromotrice*) is measurable; its value is equal to:

$$e(t) = \frac{d\psi}{dt} = \frac{d}{dt}(B \cdot A \cdot \sin(\Omega_m t)) = \Omega_m \cdot B \cdot A \cdot \cos(\Omega_m t)$$

where $\alpha$ is replaced with $\Omega_m t$, being the angular speed constant.

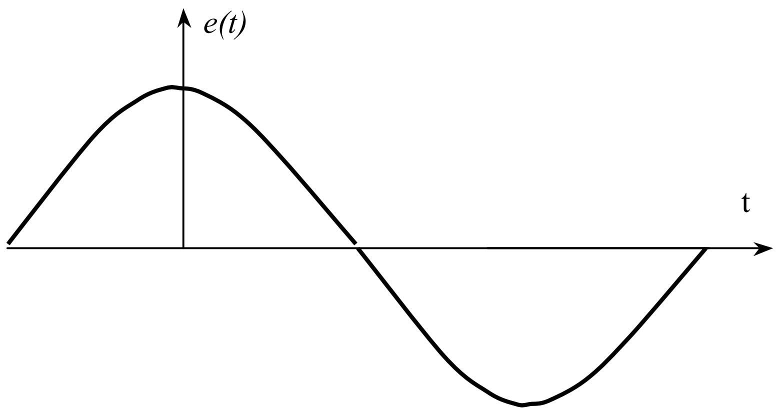



Figure 6-6. electromotive force waveform

If now we suppose to replace the terminals with two semi-rings and to include two electrical contacts (brushes) properly fixed in space, it is easy to see that the two contacts collect a rectified electromotive force consisting of two half sine wave.

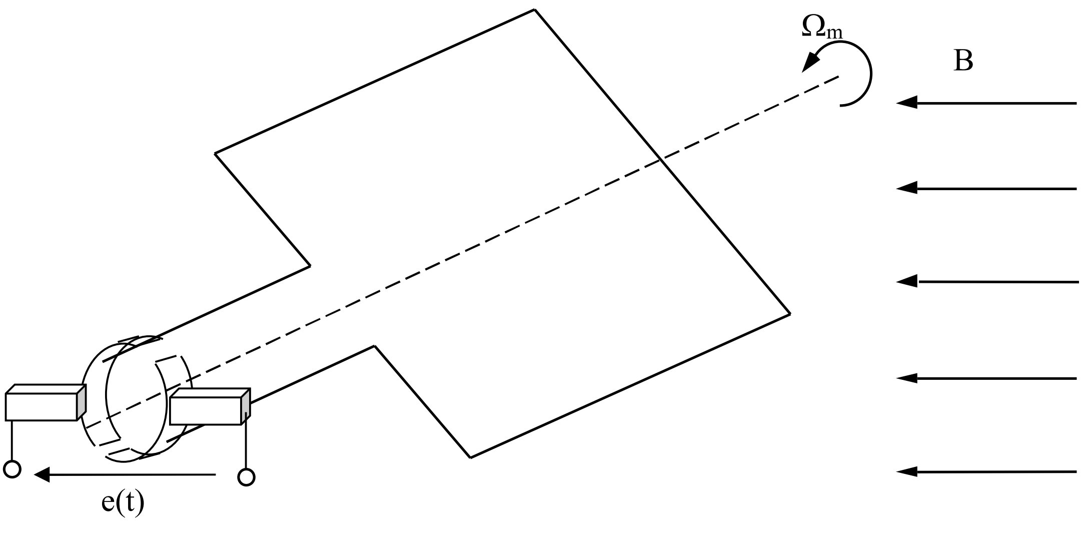



Figure 6-7. Semi-rings and brushes

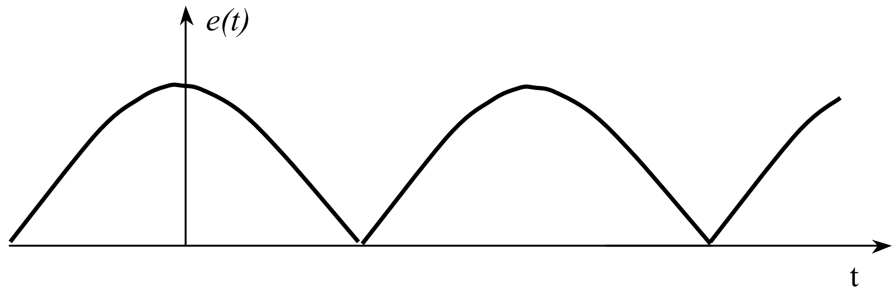



Figure 6-8. electromotive force waveform (two semi-rings and two brushes)

By inserting an increasing number of turns, you get a rectification effect, more and more effective so to reach a nearly constant value. At the same time the ring, consisting simply of two semi-rings, has to be made, as the turns increase, by thicker and thicker segments. Their task is to give the proper voltage of the coil on the electrical contacts (brushes). The set of segments that provides the electromotive force to the brushes is the so-called commutator.

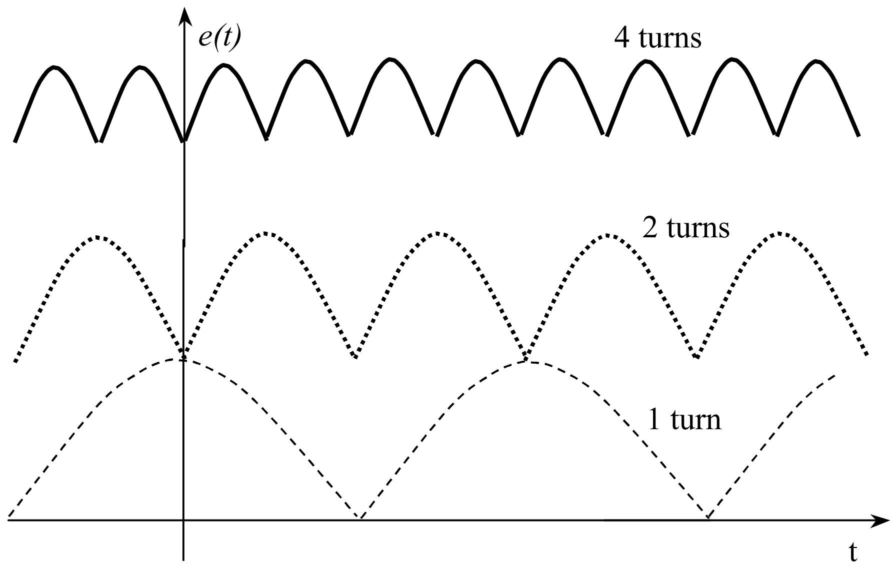



Figure 6-9. electromotive force waveform with two brushes and a commutator with: two semi-rings (1 turn), 4 segments (2 turns), 8 segments (4 turns)

To summarize: the principle of operation of the machine is based on the following points:

- there is a winding (excitation), realized on the stator, that generates a constant excitation magnetic field
- there is, also, a winding mounted on the rotor (armature), connected to a device with some segments, called commutator, which, through the brushes mounted on the inter-polar axis, realizes the rectification of ac electromotive forces into a rather constant voltage (dc voltage), proportional to the mechanical speed and to the flux $\psi_{ae}$, in some way linked with the armature winding, but supported by the only excitation current $i_e$.

Another way to find the expression of the total electromotive force may be the following one.

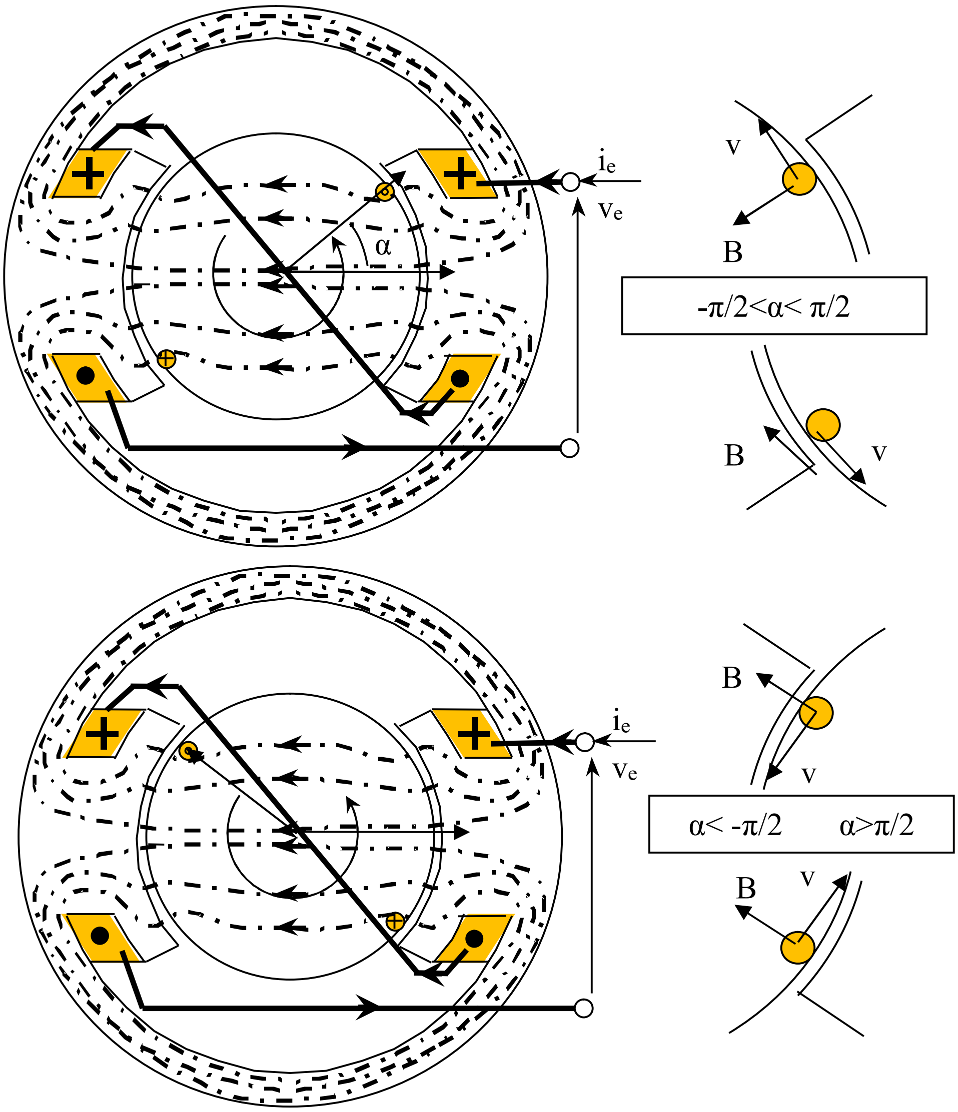



Figure 6-10. Two poles, separately excited, DC machine cross-section

Consider the angle $\alpha$ equal to zero when the winding is horizontal. Due to the high value of the permeability of the ferromagnetic material respect to the air, the flux density is orthogonal to the surface of the rotor. In this condition, in the upper right side of the winding (L is its length) an electromotive force is induced, in the direction which goes outside the plane. In the lower left side of the winding the electromotive force goes inside the plane. In the winding, the total electromotive force is the sum of the two contributes. This is true when the angle $\alpha$ is between $-\pi/2$ and $\pi/2$. For $\alpha$ higher than $\pi/2$ or lower than $-\pi/2$, the electromotive forces change their sign. The linear speed v may be considered constant while the flux density B is constant along the airgap (constant) and lower outside.

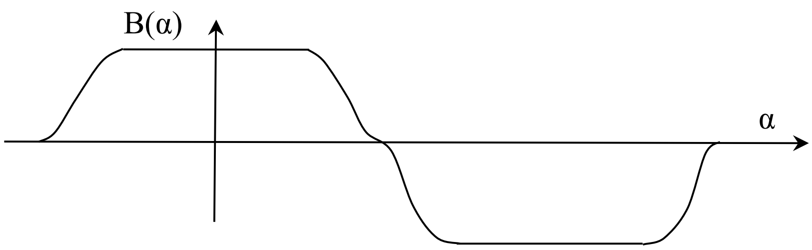



Figure 6-11: Flux density along the airgap due to the excitation current only

Constant linear speed means constant angular speed (v=ω r) and α=ω t.

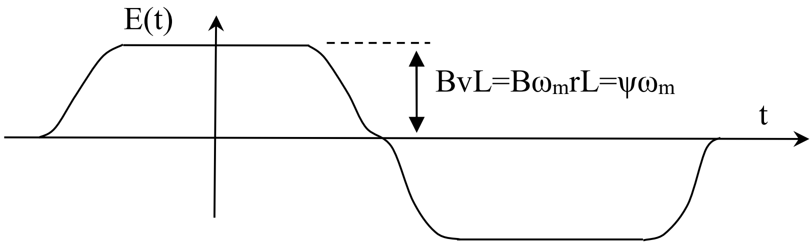



Figure 6-12: Electromotive force along the airgap due to the excitation current only

By means of the semi-rings, it is possible to rectify the electromotive force. The following steps are similar to the previous one.

## 6.2 Steady state model and basic equations

At steady state and with constant voltages, at its terminals the machine looks like an electromotive force which is connected in series with a resistance that takes into account the power losses, by Joule effect, in the armature conductors.

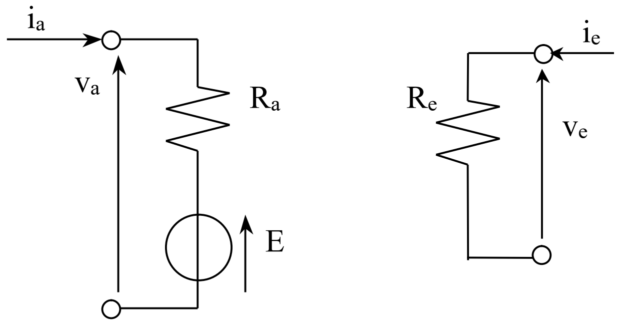



Figure 6-13. Steady state equivalent circuit of a DC machine

The armature and excitation steady state equivalent circuits are shown in Figure 6-13. The steady state equations are:

$$v_a = R_a \cdot i_a + E$$
$$v_e = R_e \cdot i_e$$

with the following flux/current relationship:

$$\psi_a = L_a \cdot i_a$$
$$\psi_e = f(i_e) = L_e(i_e) \cdot i_e$$
$$\psi_{ae} = g(i_e) = L_{ae}(i_e) \cdot i_e$$

The electromotive force $E$, as established above, is proportional to the flux $\psi_{ae}$ linked with the armature windings, function of the excitation current $i_e$, and to the rotation speed $\Omega_m$ of the machine:

$$E = k_e \cdot \psi_{ae} \cdot \Omega_m$$

The torque expression may be obtained by an energy balance, at steady state. As regards to the excitation circuit, you get:

$$v_e \cdot i_e = R_e \cdot i_e^2$$

The term on the left is the electric power entering the system through the excitation terminals; at steady state and with constant voltages, it is constant. The term on the right is equal to the Joule losses.

As regards to the armature circuit, it is:

$$v_a \cdot i_a = R_a \cdot i_a^2 + E \cdot i_a$$

The term on the left is the electric power entering the system through the armature terminals; at steady state and with constant voltages, it is constant. The first term on the right is equal to the Joule losses (in the armature resistance), while the second one represents the power transmitted between stator and rotor.

$$P_e = E \cdot i_a = k_e \cdot \psi_{ae} \cdot \Omega_m \cdot i_a$$

Thanks to the principle of energy conservation and neglecting any friction, it can be said that the transmitted electrical power coincides with the mechanical power and then you get:

$$P_m = T_e \cdot \Omega_m = P_e = k_e \cdot \psi_{ae} \cdot \Omega_m \cdot i_a$$

The torque is:

$$T_e = k_e \cdot \psi_{ae} \cdot i_a$$

## 6.3 No load operation

Without any connection (open circuits, i.e. no electrical load), the voltage *E* (back electromotive force: bemf), measured at the armature terminals of a separately excited dc machine, has a non-linear dependency on the excitation current $i_e$.

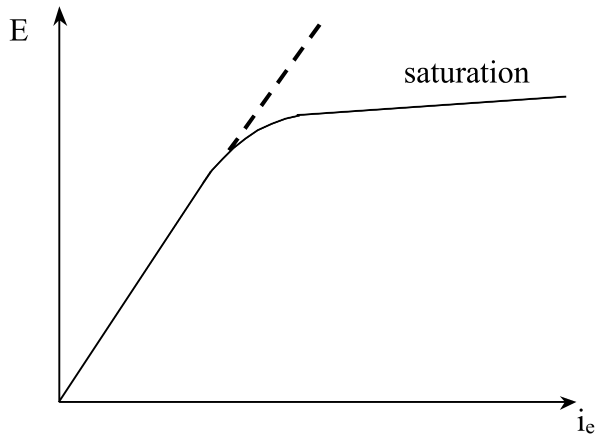



Figure 6-14: Electromotive force versus excitation current

This is due to the non-linear behaviour of the flux, given by the excitation current, and flowing through a very low airgap.

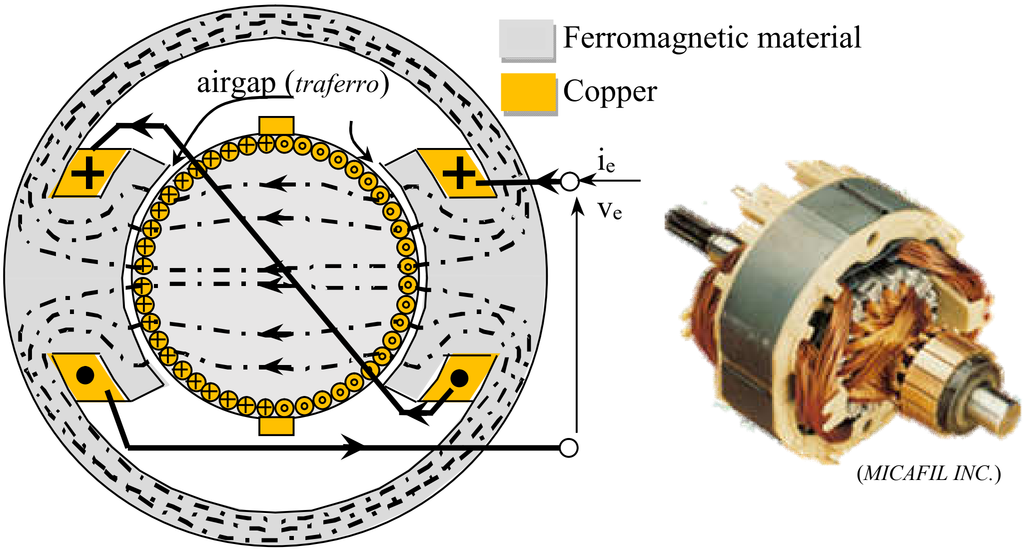



Figure 6-15: Flux lines due to the excitation current

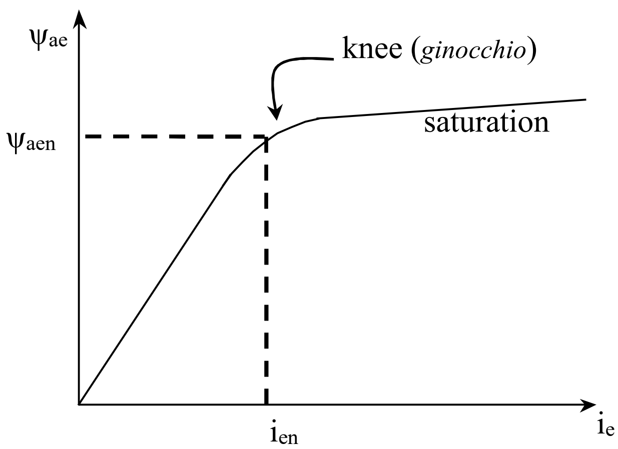



Figure 6-16: Magnetization curve

As discussed in the previous paragraph, the electromotive force $E$ is proportional to the flux linkage $\psi_{ae}$ and the mechanical speed $\Omega_m$: $E = k_e \cdot \psi_{ae} \cdot \Omega_m$.

Another problem may arise. It is due to the armature reaction. The armature current produces a magnetic flux: some of its lines (red ones) flow through the airgap and the excitation pole.

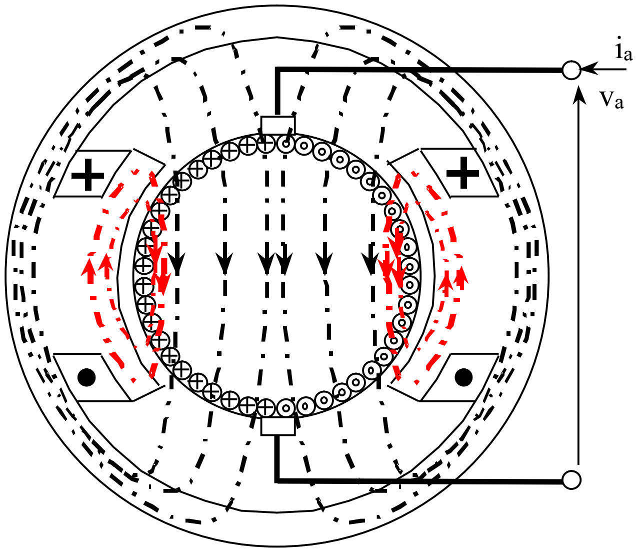



Figure 6-17:

Suppose to cut and stretch the machine (linearization).

The magnetomotive force ($mmf_a$) due to the armature current has a triangular shape (distributed windings along the airgap). The flux density is different due to the variable airgap

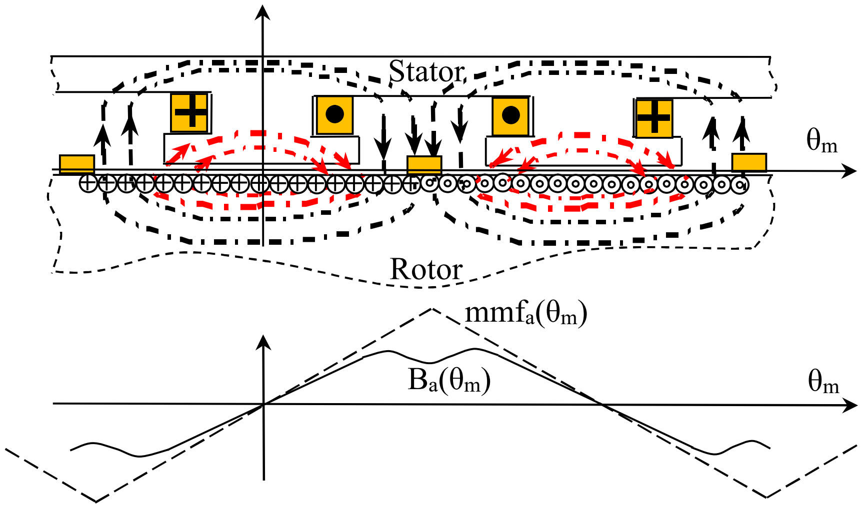



Figure 6-18: Flux lines and Flux density along the airgap due to the armature current only

The magnetomotive force ($mmf_e$) due to the excitation current has a square wave shape (concentrated winding). The flux density is constant under the stator pole (constant airgap); outside the pole, it decreases.

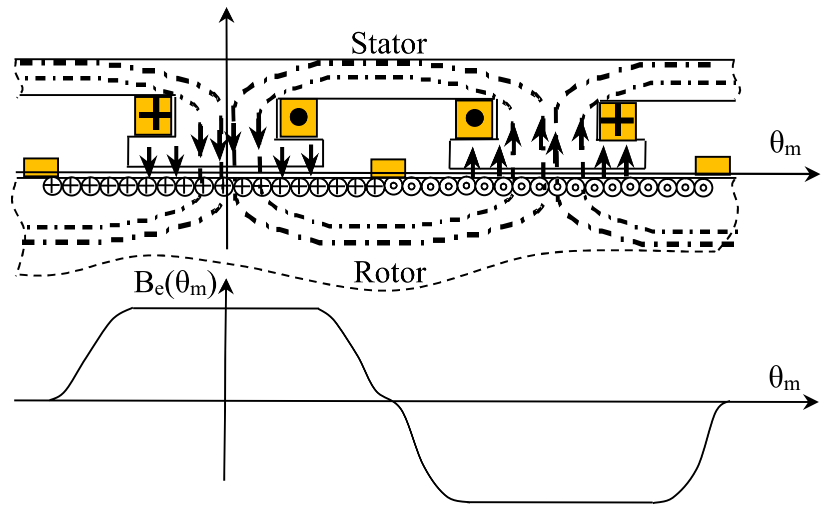



Figure 6-19: Flux lines and Flux density along the airgap due to the excitation current only

Therefore, along the air-gap, the flux density is given by the superposition of the effect of the excitation current and the armature one.

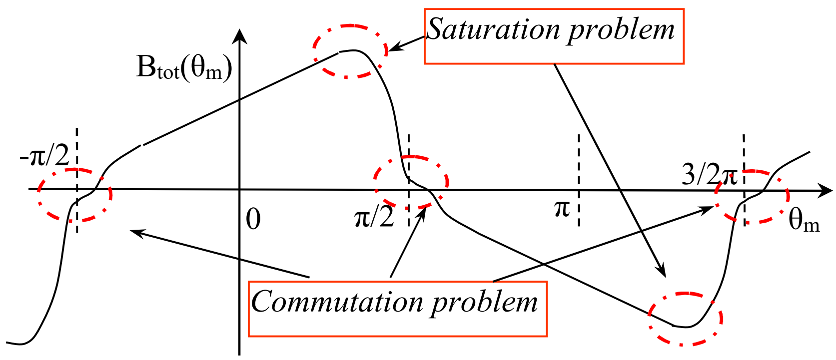



Figure 6-20: Flux density along the air-gap due to both armature and excitation currents

Two problems may arise: the saturation of a portion of the stator pole reduces the excitation magnetic flux given the same excitation current; secondly, a flux density different from zero in the position of the brushes (commutation) may produce sparks and flashes.

## 6.4 Field-circuit connections of a DC machine

As regards the connection between the armature and excitation windings, you may have different types of machines: separately-excited (or with separate excitation), series, shunt, compound (see Figure 6-21).

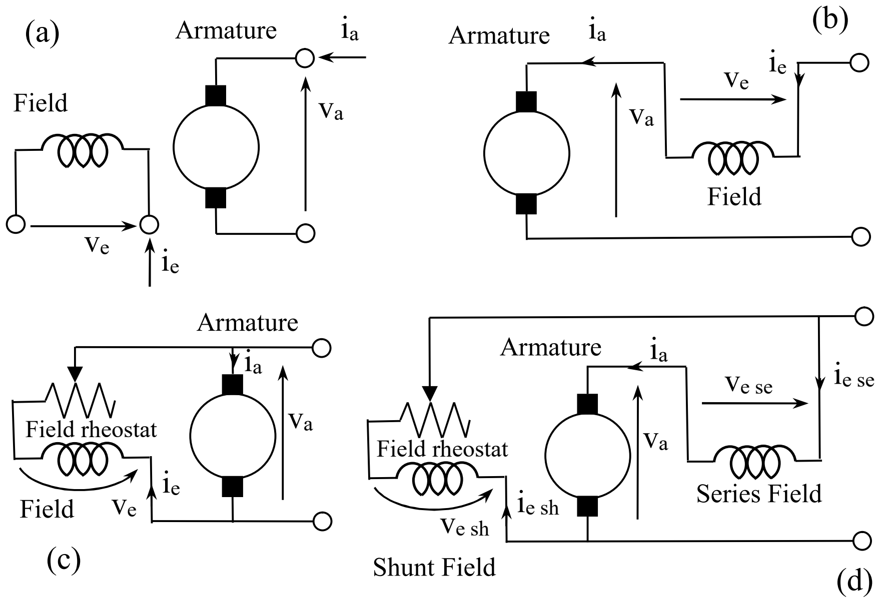



Figure 6-21: Field-circuit connections: (a) separately-excited, (b) series, (c) shunt, (d) compound

In a separately excited dc machine, the armature is fed by a different supply system from that of the excitation winding.

In a series excitation dc machine, the armature winding is in series with the excitation winding (that is $i_a = i_e$).

In a shunt connection dc machine, the armature winding is in parallel to the excitation winding (that is $v_a = v_e$).

The compound dc machine is a combination of a series and shunt connection.

The above-mentioned types of machine may operate both as generator and as a motor.

## 6.5 DC Generator

Suppose to consider a DC generator (the versus of the armature current changes). The steady-state equivalent circuit of the armature winding is made by a DC ideal generator E (emf), in series with a resistance ($R_a$, the resistance of the armature winding).

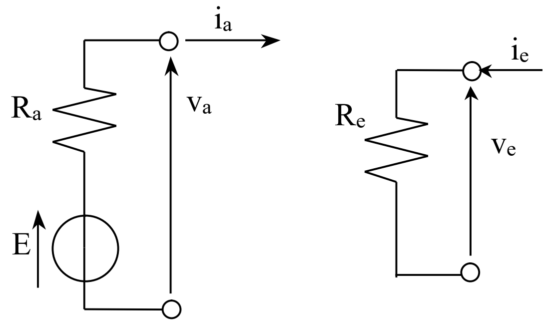



Figure 6-22: Steady-state equivalent circuit of the armature and field windings for a DC generator

The steady-state equations (generator) are:

$$v_a = E - R_a \cdot i_a$$
$$v_e = R_e \cdot i_e$$

The electromotive force $E$, as established above, is proportional to the flux $\psi_{ae}$ linked with the armature winding, function of the excitation current $i_e$, and to the rotation speed $\Omega_m$ of the machine:

$$E = k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m$$

### 6.5.1 Separately-excited

Consider a condition with constant excitation current $i_e$ and constant speed $\Omega_m$. It means that E is constant.

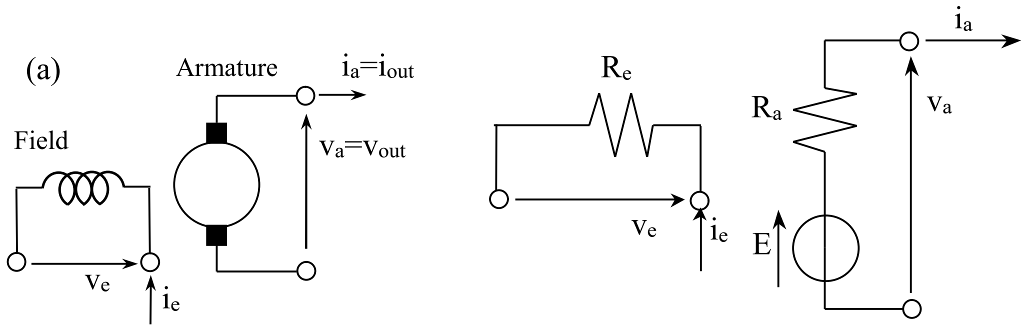



Figure 6-23: Separately-excited DC generator

The relationship between the armature voltage $v_a$ and the armature current $i_a$ is ideally linear: $v_a = E - R_a \cdot i_a$. The same for the external characteristic: $v_{out} = E - R_a \cdot i_{out}$ The intersection of the characteristic with the horizontal axis is called "short-circuit current" $i_{out\_sc}$. Usually, the voltage drop on the armature resistance due the rated current is some percent of the rated voltage (2%-10%). Therefore, the short-circuit current $i_{out\_sc}$ is ten-fifty times the rated current $i_{an}$.

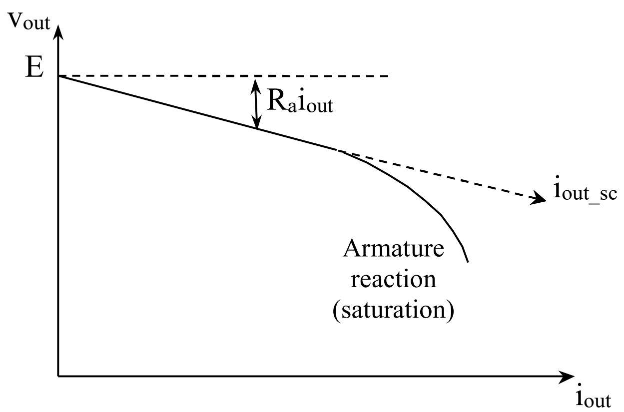



Figure 6-24: external (Volt-ampere) characteristic of a separately-excited generator

For high values of the armature current, however, the armature reaction saturates the excitation pole. In this condition (saturation), with the same value of the excitation current, the magnetic flux due to this current is lower than that of a non-saturated condition. So, the electromotive force E is lower than before and consequently, the output voltage. The characteristics moves up with speed or/and excitation current increasing (the excitation current is limited by saturation problem).

### 6.5.2 Series connection

Consider the operation with constant speed $\Omega_m$.

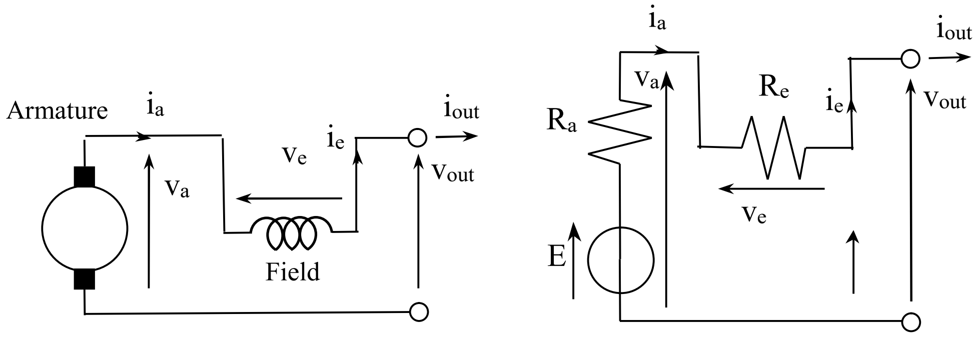



Figure 6-25: Series connection DC generator

The excitation current $i_e$ is equal to the armature current $i_a$ (due to the series connection).

If you suppose a linear relationship between the magnetic flux and the excitation current ($\psi_{ae}=L_{ae}\, i_e$ when $i_e<i_{en}$), the expression of $E$ becomes:

$$E = k_e \cdot L_{ae} \cdot i_e \cdot \Omega_m = k_e \cdot L_{ae} \cdot i_a \cdot \Omega_m = K_2 \cdot i_a \cdot \Omega_m$$

but

$$v_a = E - R_a \cdot i_a$$

so

$$v_a = K_2 \cdot i_a \cdot \Omega_m - R_a \cdot i_a = (K_2 \cdot \Omega_m - R_a) \cdot i_a$$

The output voltage is $v_{out} = v_a - v_e = (K_2 \cdot \Omega_m - R_a) \cdot i_a - R_e \cdot i_e = (K_2 \cdot \Omega_m - R_a - R_e) \cdot i_{out}$.

The relationship between the output voltage $v_{out}$ and the output current $i_{out}$ is linear, starting from zero (the slope depends on the mechanical speed $\Omega_m$).

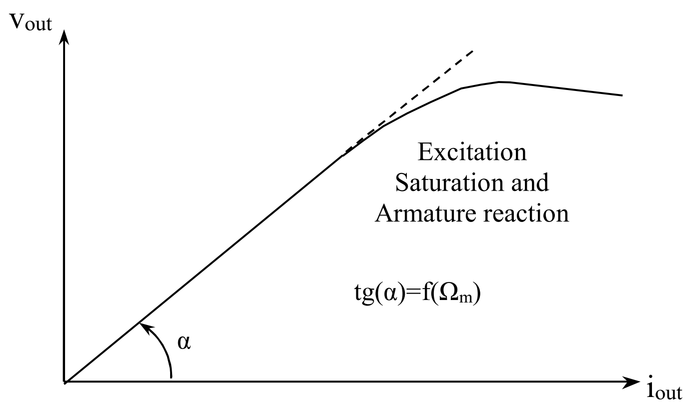



Figure 6-26: external characteristic of a series dc generator

With the increase of the excitation current, the ferromagnetic material goes towards the saturation condition. The magnetic flux may be considered constant and equal to $\psi_{ae\_sat}$.

Now

$$E = k_e \cdot \psi_{ae\_sat} \cdot \Omega_m = K_3 \cdot \Omega_m$$

and

$$v_a = K_3 \cdot \Omega_m - R_a \cdot i_a$$

The output voltage is $v_{out} = v_a - v_e = K_3 \cdot \Omega_m - R_a \cdot i_a - R_e \cdot i_e = K_3 \cdot \Omega_m - (R_a + R_e) \cdot i_{out}$.

which represents a characteristic similar to the separately-excited one.

### 6.5.3 Shunt connection

Consider a condition with constant speed $\Omega_m$ and no-load ($i_{out}$=0)

The equivalent circuit is shown in the Figure 6-27

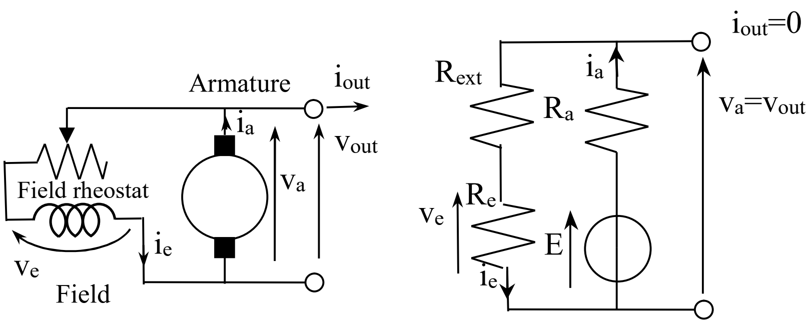



Figure 6-27: equivalent circuit of a shunt DC generator with no-load

Consider $R_{ext}$=0.

Due to the series connection between $R_a$ and $R_e$, it is:

$$i_a = \frac{E}{R_a + R_e}$$

but

$$E = k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m$$

where the relationship between flux and excitation current is non-linear.

If there is a residual flux density (the characteristic of E does not start from 0 with a mechanical speed different from zero and an initial current equal to 0), there is a point (operation point) in which the Volt-ampere characteristic of the armature ($v_a$=E-$R_a i_a$) matches the Volt-ampere characteristic of the excitation resistance $R_e$. This is true for the non-linear waveform of $E$ (otherwise, no matching point could exist).

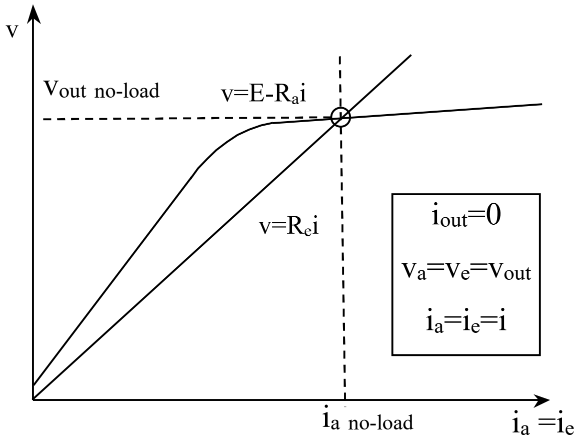



Figure 6-28: matching between the Volt-ampere characteristics of the armature and excitation windings

You may change the output voltage by means of a variable resistance (Field Rheostat in Figure 6-21) put in series to the excitation resistance.

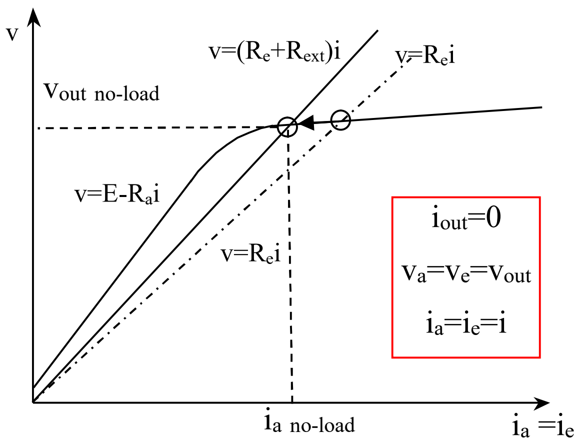



Figure 6-29: change of the operation point by means of an external resistance in series with the excitation winding

Consider, now, a load connected to the machine (represented by an ideal current generator $I_{load}$) ($R_{etot}=R_e+R_{ext}$).

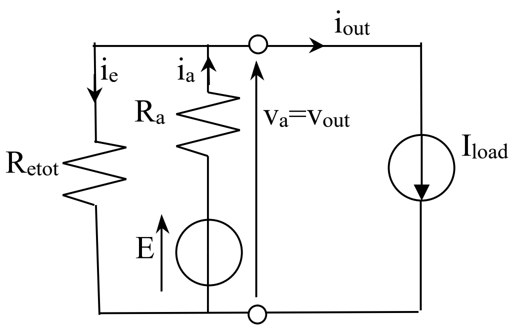



Figure 6-30: equivalent circuit of a shunt dc machine connected to a load

The output voltage is the voltage on the three branches of the circuit (Gx are the conductances=1/Rx):

$$v_{out} = \frac{E \cdot G_a - I_{load}}{G_a + G_{etot}}$$

As the machine is working in saturation condition, consider the following approximation (see Figure 6-31):

$$E = E_o + K \cdot i_e$$

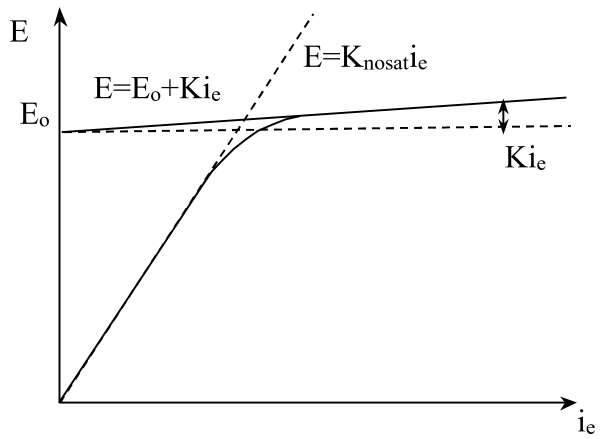



Figure 6-31: simplified external characteristic of the armature winding

So

$$v_{out} = \frac{(E_o + Ki_e) \cdot G_a - I_{load}}{G_a + G_{etot}}$$

but

$$i_e = v_{out} \cdot G_{etot}$$

and

$$v_{out} = \frac{(E_o + Kv_{out} \cdot G_{etot}) \cdot G_a - I_{load}}{G_a + G_{etot}}$$

and (collecting $v_{out}$)

$$v_{out} = \frac{E_o \cdot G_a - I_{load}}{G_a + G_{etot} - KG_{etot}G_a}$$

In the real condition, with the increasing of the output current, the output voltage decreases, with the excitation current, going outside the saturation (in the linear part).

The output voltage decreases more than in the saturation condition.

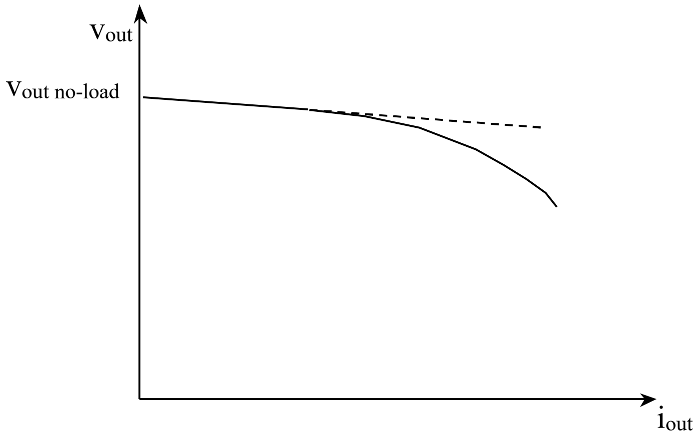



Figure 6-32: external characteristic of a shunt dc machine

### 6.5.4 Compound connection

A dc generator with a compound connection has an external characteristic given by the combination of the two previous types of machine (series and shunt).

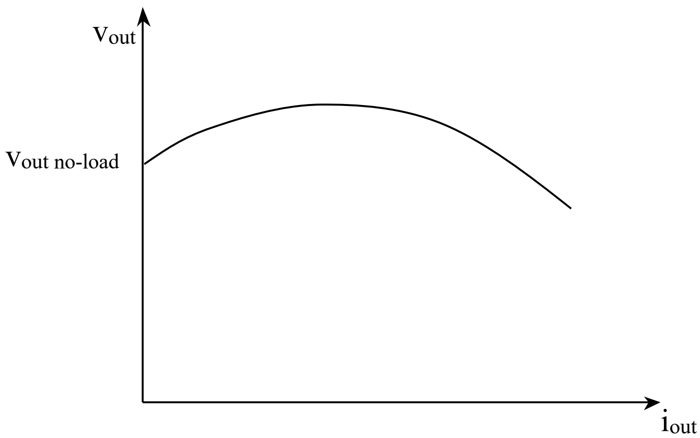



Figure 6-33: external characteristic of a compound dc machine

### 6.5.5 Comparison

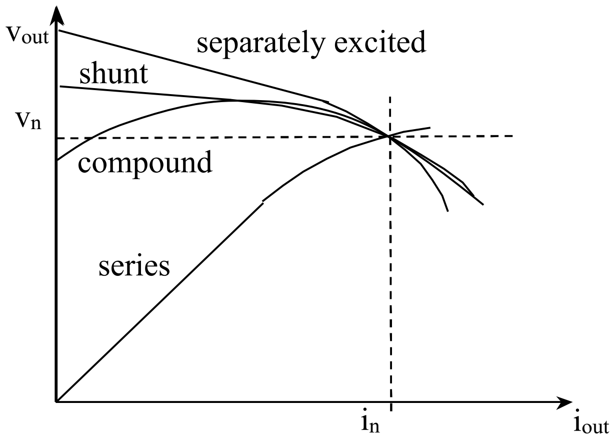



Figure 6-34: Volt-ampere (external) characteristics of dc generators ($i_n$ and $v_n$ are the rated value of the output current and voltage)

## 6.6 DC Motor

Suppose to consider a DC motor. The steady-state equivalent circuit of the armature winding is made by a DC ideal generator E (emf) in series with a resistance ($R_a$, the resistance of the armature winding).

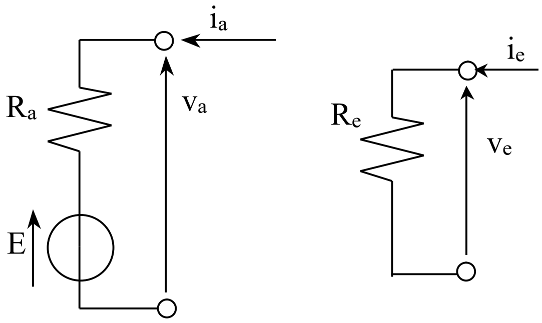



Figure 6-35: Steady-state equivalent circuit of the armature and field windings for a DC motor

The steady-state equations (motor operation) are:

$$v_a = E + R_a \cdot i_a$$
$$v_e = R_e \cdot i_e$$

The electromotive force *E* is the same as the previous of a DC generator:

$$E = k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m$$

The expression of the electromagnetic torque $T_e$ is:

$$T_e = k_e \cdot \psi_{ae}(i_e) \cdot i_a$$

### 6.6.1 Separately excited

The armature current is given by:

$$i_a = \frac{v_a - E}{R_a}$$

but

$$E = k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m$$

so

$$i_a = \frac{v_a - k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m}{R_a}$$

and the torque

$$T_e = k_e \cdot \psi_{ae}(i_e) \cdot i_a = k_e \cdot \psi_{ae}(i_e) \frac{v_a - k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m}{R_a}$$

Consider a condition with constant excitation current $i_e$ and constant speed $\Omega_m$.

The relationship between the electromagnetic torque $T_e$ and the mechanical speed $\Omega_m$ is linear.

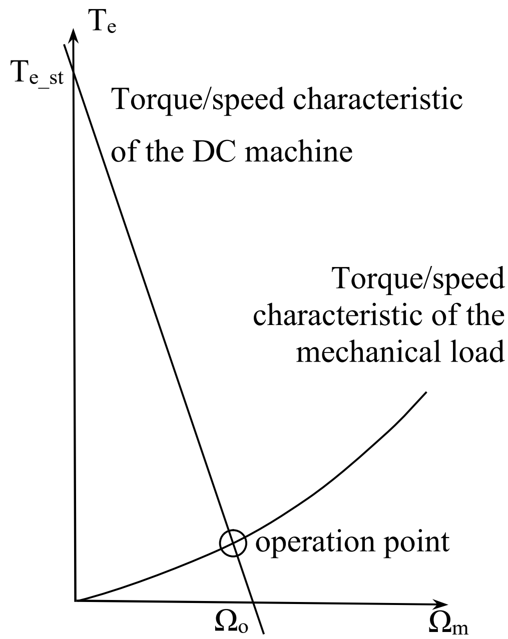



Figure 6-36: Torque-speed characteristic (*caratteristica meccanica*) of a separately excited DC machine

The torque at zero speed is called Starting Torque or Locked Rotor Torque $T_{e\_st}$:

$$T_{e\_st} = k_e \cdot \psi_{ae}(i_e) \frac{v_a}{R_a}$$

Usually its value is very high due to the low value of the armature resistance. It is proportional both to the armature voltage and to the flux linkage $\psi_{ae}$ (function of the excitation current $i_e$).

The speed at zero torque (no mechanical load) is called No-Load speed $\Omega_o$:

$$\Omega_o = \frac{v_a}{k_e \cdot \psi_{ae}(i_e)}$$

It is proportional to the armature voltage and inverse proportional to the field flux $\psi_{ae}$ (function of the excitation current $i_e$).

The intersection between the Torque/speed characteristic of the machine and of the mechanical load defines the operation point (in the figure the mechanical load is represented by a fan).

The most common used strategy is to maintain constant the excitation current (in order to make a good use of the ferromagnetic materials) and to control the torque by means of the armature current (and consequently of the armature voltage), until the maximum armature voltage is reached.

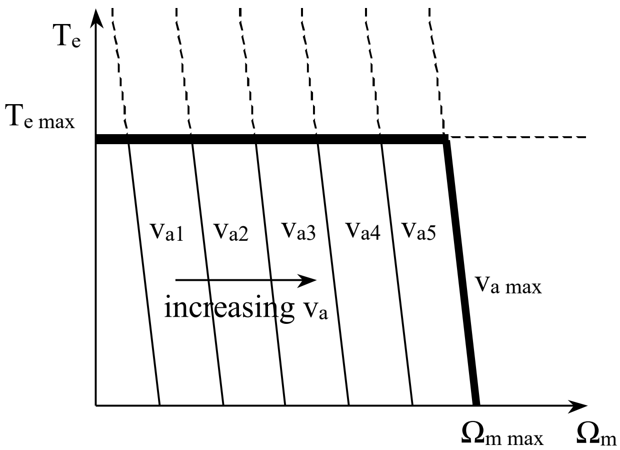



Figure 6-37: Torque-speed characteristic when armature voltage increases

Given a value of the excitation current, the torque is proportional to the armature current, limited by thermal problem. So the torque is limited too.

In order to go over the maximum speed reached in this condition (also called base speed $\Omega_b$), you may decrease the field flux $\psi_{ae}$. The starting torque deceases, but the no-load speed increases.

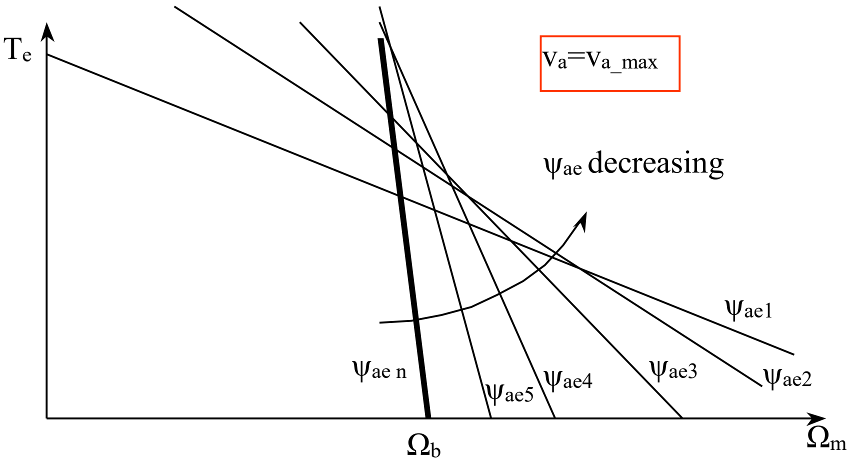



Figure 6-38: Torque-speed characteristic when flux decreases

This mode of operation is called "field weakening".

Another way to control the torque is to add an external resistance in series to the armature winding (like in old fashion tram/train application).

Consider a constant armature voltage and a constant excitation current.

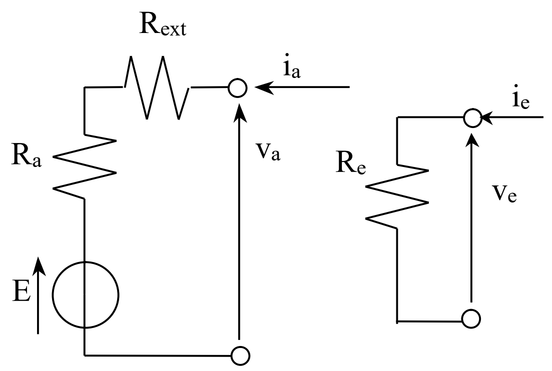
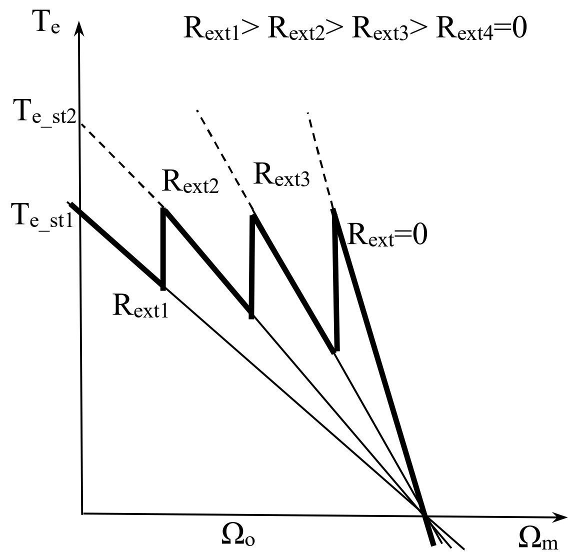



Figure 6-39: Equivalent circuit with an external resistance in series with the armature winding

$$T_{e\_st} = k_e \cdot \psi_{ae}(i_e)\frac{v_a}{R_a + R_{ext}}$$

Supposing you have a finite number of steps, you can start with a high value of resistance (limited starting torque and limited armature current, same no-load speed) and, during the acceleration, you can decrease the resistance towards the zero value. The no-load speed does not change but the armature current (and the electromagnetic torque is limited). This strategy was used before the advent of static power converter and its drawback is mainly the presence of losses in the external resistance.




### 6.6.2 Series connection

Suppose to approximate the non-linear relationship, between flux linkage $\psi_{ae}$ and excitation current $i_e$, to a piecewise linear waveform

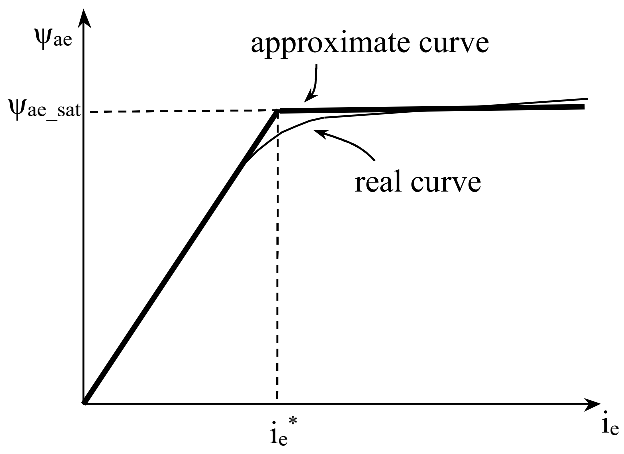



Figure 6-40: Approximate relationship between flux linkage and excitation current

At high current (excitation and armature currents are equal each other), the operation point is into the saturation condition (constant flux linkage), for low current into the non-saturated one (flux linkage is proportional to the excitation current: $\psi_{ae}=L_{ae}\ i_e$).

So, for current ($i=i_a=i_e$) less or equal to $i_e^*$:

$$E = k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m = k_e \cdot L_{ae} \cdot i_e \cdot \Omega_m = k_e \cdot L_{ae} \cdot i_a \cdot \Omega_m = K_2 \cdot i_a \cdot \Omega_m$$

$$v_a = E + R_a \cdot i_a = K_2 \cdot i_a \cdot \Omega_m + R_a \cdot i_a$$

$$i_a = \frac{v_a}{K_2 \cdot \Omega_m + R_a}$$

This means that high speed corresponds to low current (the speed is at the denominator).

The speed at the knee ($i_a=i_e^*$) may be called $\Omega_m^*$

$$\Omega_m^* = \frac{v_a - R_a i_e^*}{K_2 i_e^*}$$

Again (for low current)

$$T_e = k_e \cdot \psi_{ae}(i_e) \cdot i_a = k_e \cdot L_{ae} \cdot i_a^2 = K_2 \cdot i_a^2 = K_2 \cdot \left( \frac{v_a}{K_2 \cdot \Omega_m + R_a} \right)^2$$

The torque decreases with the mechanical speed as $1/\Omega_m^2$

On the other hand, for high current (and, consequently, low speed):

$$T_e = k_e \cdot \psi_{ae}(i_e) \cdot i_a = k_e \cdot \psi_{ae\_sat} \cdot i_a = K_3 \cdot i_a = K_3 \cdot \frac{v_a - K_3 \cdot \Omega_m}{R_a}$$

in a similar way to the separately excited machine (the torque depends linearly on the mechanical speed)

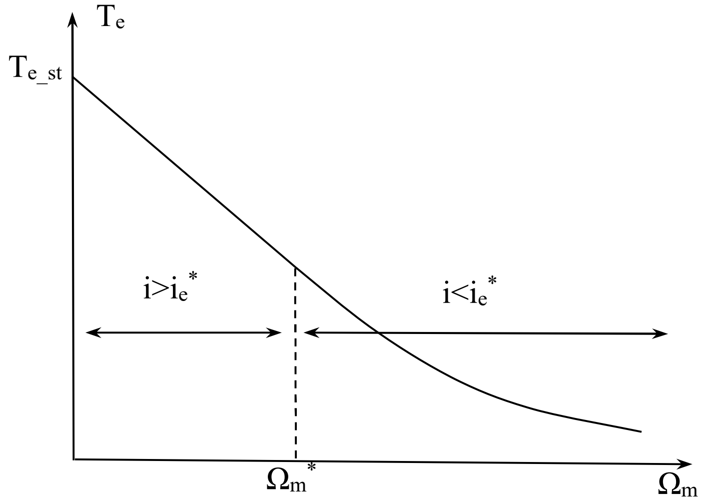



Figure 6-41: Torque-speed characteristic of a series DC machine

## 6.7 PM DC machine

Main feature of this machine is the presence of permanent magnets on the stator that creates a constant flux density B and, accordingly, a constant excitation flux and a constant flux linked with the armature windings ($\Psi_{apm}$ instead of $\psi_{ae}$). The mechanical torque is thus a function of the only armature current. You can write:

$$T_e = k_e \cdot \Psi_{apm} \cdot i_a = K_{ePM} \cdot i_a$$

Similarly, the electromotive force is a function of speed only:

$$E = k_e \cdot \Psi_{apm} \cdot \Omega_m = K_{ePM} \cdot \Omega_m$$

From these equations, we can easily obtain the torque versus mechanical speed characteristics (in steady state condition) for this type of machine.

Starting from the electrical relationship:

$$v_a = E + R_a \cdot i_a$$
$$i_a = \frac{v_a - E}{R_a}$$

we have:

$$T_e = K_{ePM} \cdot \frac{v_a - E}{R_a} = K_{ePM} \cdot \frac{v_a - K_{ePM}\Omega_m}{R_a}$$

This equation shows that the relationship between torque and speed is linear (with a constant value of the armature voltage), so you can get a family of curves, of equal slope, as a function of the supply voltage $v_a$. The slope of these curves, because of the typical values of the variables involved (the value of the armature resistance $R_a$ is usually low), is very high.

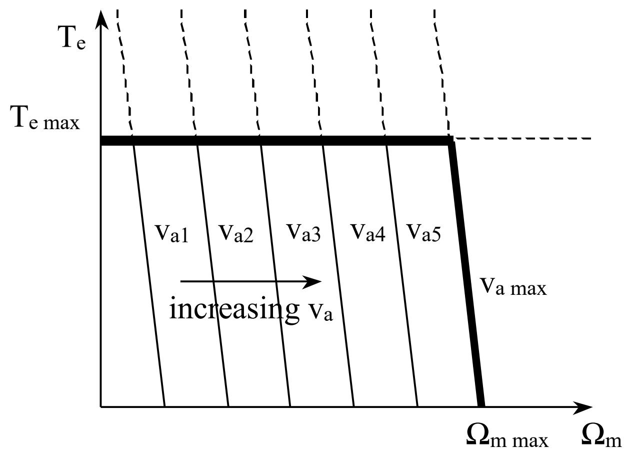



Figure 6-42: Torque/speed characteristics as a function of the armature voltage for a PM DC machine

In the case of a permanent magnet machine, then, the maximum speed corresponds to the no load speed (zero load torque), obtained at the maximum value of the supply voltage. There is no way to decrease the flux linkage.

## 6.8 Series Universal Machine

The series DC machine may be fed by an AC supply system. This machine is called "Universal machine" and is often used for appliances applications (vacuum cleaner, washing machine,...). In an AC supply system, the voltage is sinusoidal. At steady state, the currents are sinusoidal. A series connection assures that the excitation current is equal to the armature current. Suppose a current $i = I_m \sin(\omega t + \varphi)$. The flux linkage $\psi_{ae}$ will be sinusoidal, as well.

$$T_e = k_e \cdot \psi_{ae}(i_e) \cdot i_a = k_e \cdot \psi_{mae} \sin(\omega t + \varphi) I_m \sin(\omega t + \varphi) = k_e \cdot \psi_{mae} I_m \sin(\omega t + \varphi)^2 =$$
$$= k_e \cdot \psi_{mae} I_m \frac{1 - \cos(2\omega t + 2\varphi)}{2} = \frac{k_e \cdot \psi_{mae} I_m}{2} - \frac{k_e \cdot \psi_{mae} I_m \cos(2\omega t + 2\varphi)}{2}$$

Therefore, the electromagnetic torque is sinusoidal but with an average value different from zero, which is able to start the machine and maintain it in rotation.