Summary

7. DC brushless

7.1 Synchronous machine model

In the following, the mathematical model of the synchronous machine is briefly recalled.

Consider the machine shown schematically in Figure 7-1.

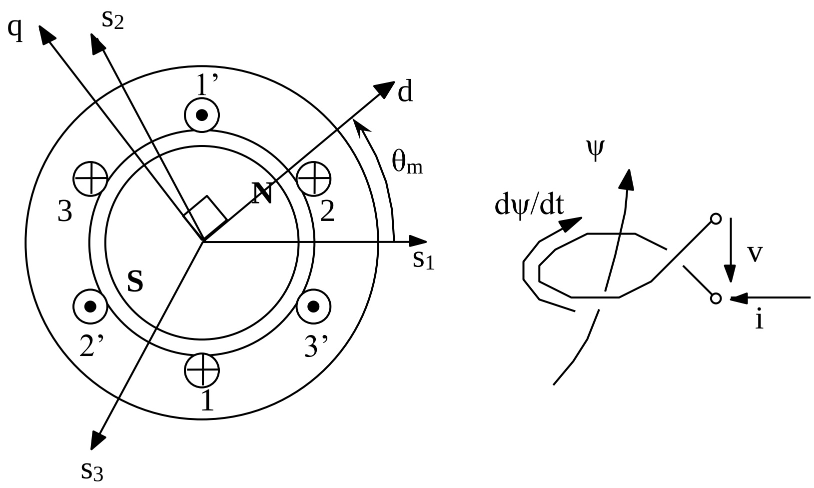

Figure 7-1: Schematic diagram of the brushless motor

It consists of a cylindrical stator with a symmetrical three-phase concentrated winding so as to generate a constant distribution of the magnetomotive force in the air-gap. The rotor flux is created by permanent magnets (for the windings, apply the conventions of Figure 7-1) and there are no damper cages. We also assert that the stator and the rotor are made of laminated material and of infinite permeability. Saturation, hysteresis of iron and anisotropy of the machine due to the slots are neglected.

Starting from the equations for the stator windings and current-flux relationship (for which, given the above assumptions, the superposition principle is valid)

$$v_{s1} = R_s i_{s1} + p\psi_{s1}$$
$$v_{s2} = R_s i_{s2} + p\psi_{s2}$$
$$v_{s3} = R_s i_{s3} + p\psi_{s3}$$
$$\psi_{s1} = L_{ss}(\theta_m)i_{s1} + M_{ss}(\theta_m)i_{s2} + M_{ss}(\theta_m - \frac{2}{3}\pi)i_{s3} + \psi_{pm}(\theta_m)$$
$$\psi_{s2} = L_{ss}(\theta_m - \frac{2}{3}\pi)i_{s2} + M_{ss}(\theta_m)i_{s1} + M_{ss}(\theta_m + \frac{2}{3}\pi)i_{s3} + \psi_{pm}(\theta_m - \frac{2}{3}\pi)$$
$$\psi_{s3} = L_{ss}(\theta_m - \frac{4}{3}\pi)i_{s3} + M_{ss}(\theta_m - \frac{2}{3}\pi)i_{s1} + M_{ss}(\theta_m + \frac{2}{3}\pi)i_{s2} + \psi_{pm}(\theta_m - \frac{4}{3}\pi)$$

Considering, for sake of simplicity, an isotropic structure of the rotor, the self-inductance L_s does not depend on the mechanical position. Recalling that the sum of the three phase currents is zero due to the connection (isolated star point or triangle), we have

$$\psi_{s1} = L_s i_{s1} + \psi_{pm}(\theta_m)$$
$$\psi_{s2} = L_s i_{s2} + \psi_{pm}(\theta_m - \frac{2}{3}\pi)$$
$$\psi_{s3} = L_s i_{s3} + \psi_{pm}(\theta_m - \frac{4}{3}\pi)$$

with L_s=L_{ss}-M_{ss} (synchronous inductance).
The dynamic equations becomes:

$$v_{s1} = R_s i_{s1} + L_s p i_{s1} + p\psi_{pm}(\theta_m) = R_s i_{s1} + L_s p i_{s1} + e_{s1}(\theta_m)$$
$$v_{s2} = R_s i_{s2} + L_s p i_{s2} + p\psi_{pm}(\theta_m - \frac{2}{3}\pi) = R_s i_{s2} + L_s p i_{s2} + e_{s2}(\theta_m)$$
$$v_{s3} = R_s i_{s3} + L_s p i_{s3} + p\psi_{pm}(\theta_m - \frac{4}{3}\pi) = R_s i_{s3} + L_s p i_{s3} + e_{s3}(\theta_m)$$

The main difference between an AC and a DC brushless lies in the realization of the windings: distributed (AC) as shown in Figure 7-2 or concentrated (DC, Figure 7-3). In the first case the induction distribution along the air-gap, due to the stator current, as a function of the generic position θ_s inside the airgap and referred to the stator, is sinusoidal. In the case of a DC brushless, the induction has a square wave trend.

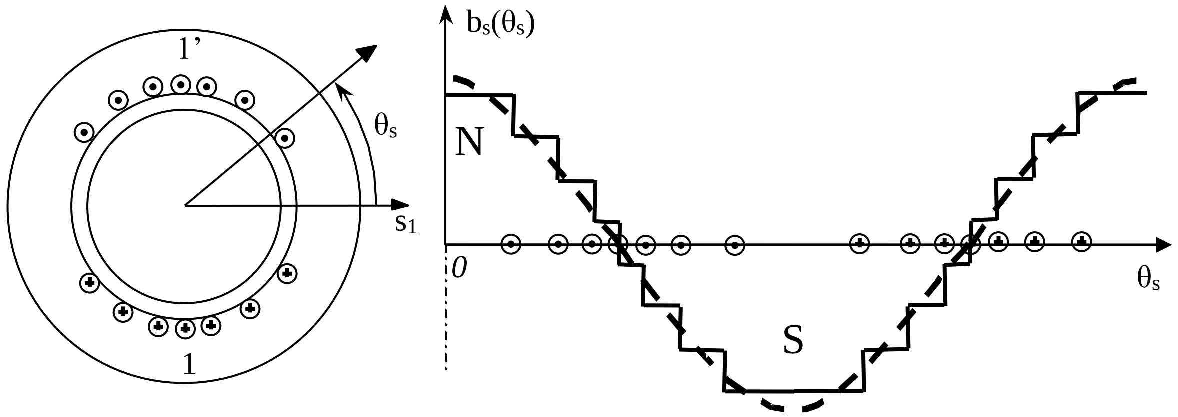

Figure 7-2: Induction waveform along the air-gap in a AC brushless (distributed windings) due to stator current i_{s1}

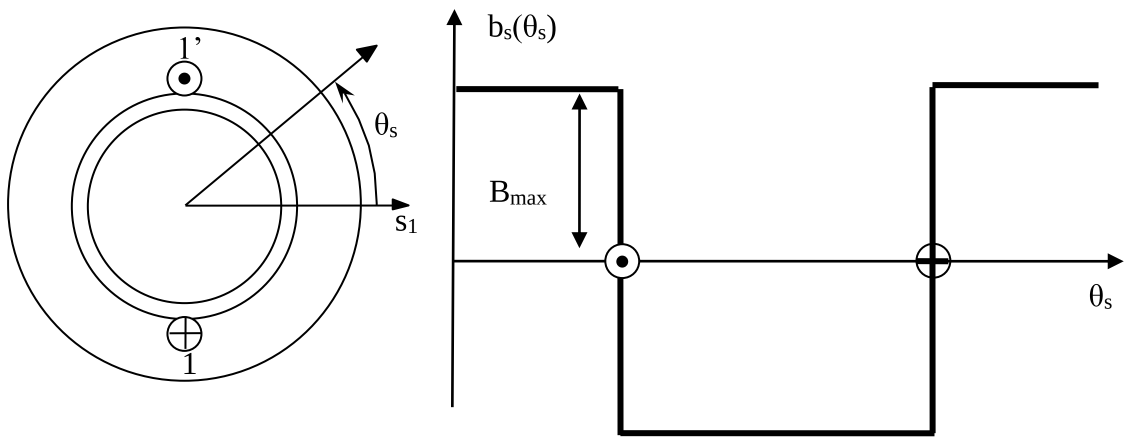

Figure 7-3: Induction waveform along the air-gap in a DC brushless (concentrated windings) due to stator current i_{s1}

Consider, now, the trend of the magnetic induction at the air gap due to the permanent magnets (suppose no stator current). In an isotropic solution, with permanent magnets glued on the surface of the rotor, the induction at the air gap, in function of a generic position θ_r within the air gap and referred to the rotor, still has a trend square wave, regardless of the type of winding the stator.

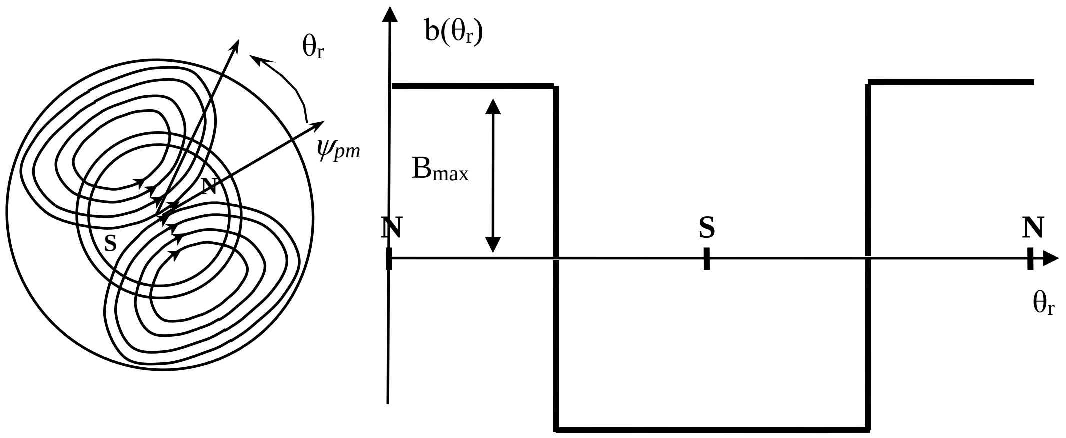

Figure 7-4: Induction waveform along the air-gap in a brushless due to the rotor permanent magnets

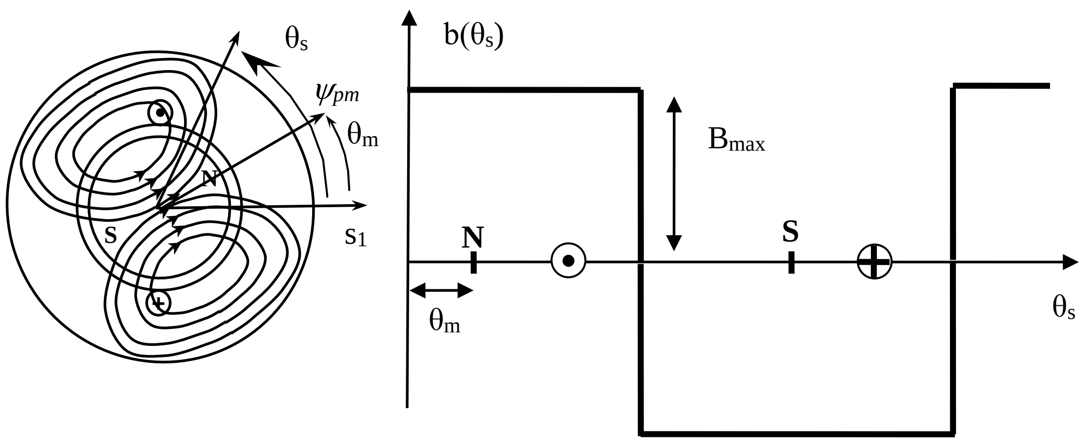

Figure 7-5: Induction waveform along the air-gap due to the rotor permanent magnets, as a function of the position θ_s referred to the stator, given the mechanical potion θ_m

The flux, linked with the stator winding (surface integral of the flux density B), due the permanent magnets, has an alternate waveform. In particular, the flux linkage is maximum when the magnetic axis do the stator winding is aligned with the North of the permanent magnets (Figure 7-6).

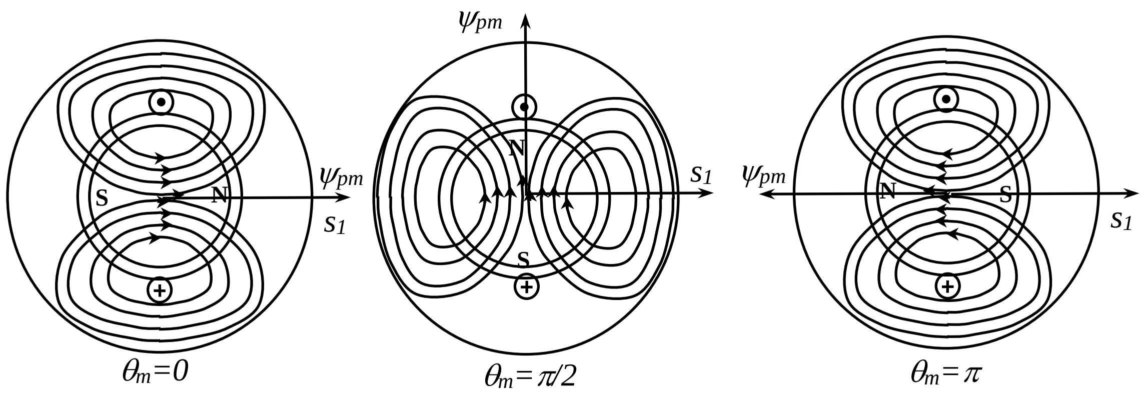

Figure 7-6: Flux linkage (maximum at θ_m=0, zero at $\theta_m=\pm\pi/2$, negative maximum at $\theta_m=\pi$)

When the mechanical angle is equal to 90 ° the flux linkage is zero. The trend between 0° and 180° is linear because the flux is equal to the surface integral of the flux density. Given an angle θ_m the flux has a positive contribution with an angle equal to 180°-θ_m and a negative contribution with an angle equal to θ_m. So (for θ_s between 0 and 180°) we have:

$$dA = r_{ave} l d\theta_s$$

$$\psi_{pm}(\theta_m) = N_s \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} b(\theta_s) r_{ave} l d\theta_s = N_s \int_{-\frac{\pi}{2}}^{-\frac{\pi}{2}+\theta_m} -B_{\max} r_{ave} l d\theta_s + N_s \int_{-\frac{\pi}{2}+\theta_m}^{\frac{\pi}{2}} B_{\max} r_{ave} l d\theta_s = N_s B_{\max} r_{ave} l(\pi - 2\theta_m)$$

where N_s is the number of the stator coils, r_{ave} is the average radius at the airgap and l is the length of the stator.

The waveform is not perfectly triangular because the winding has to occupy at least a slot. In correspondence of the slot there will be a soft fitting between the two slopes.

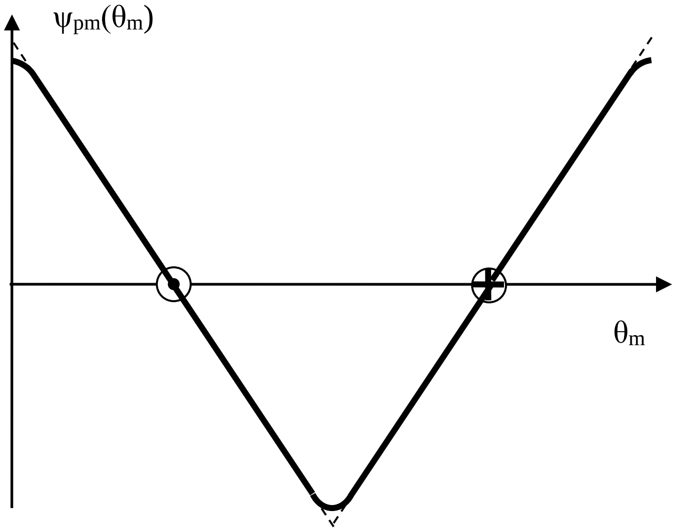

Figure 7-7: Flux linkage as a function of the mechanical angle

The derivative of this flux linkage is a back emf $e_{s1}(\theta_m) = p\psi_{pm}(\theta_m) = \frac{d\psi_{pm}(\theta_m)}{d\theta_m}\frac{d\theta_m}{dt} = \frac{d\psi_{pm}(\theta_m)}{d\theta_m} n_p \Omega_m$. The waveform is trapezoidal. This is why this type of machines is called, also, as trapezoidal back emf brushless machine.

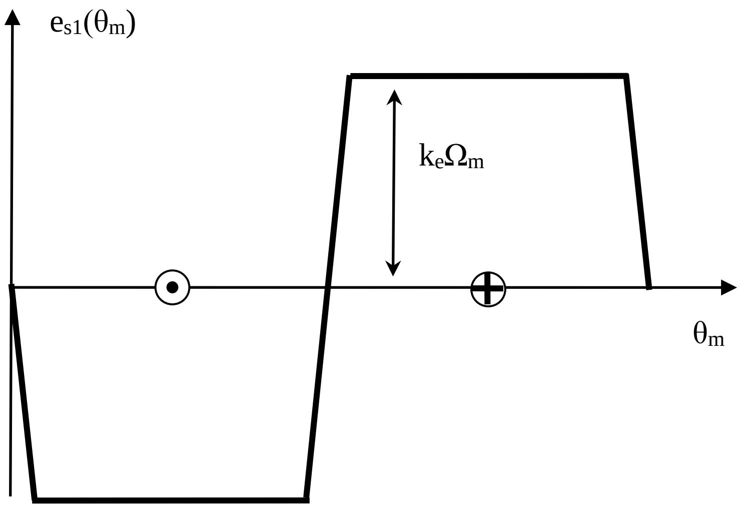

Figure 7-8: The electromotive force e_{s1} as a function of the mechanical position

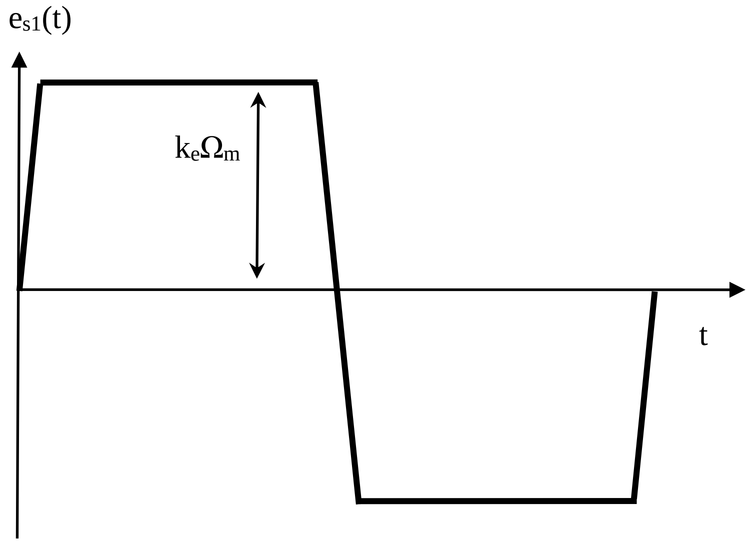

Figure 7-9: The electromotive force e_{s1} as a function of time

Define k_e the derivative $\frac{d\psi_{pm}(\theta_m)}{d\theta_m} n_p$ when it is constant.

Suppose that the time interval during which the emf e_{s1} is constant is, at least, one third of the period T (corresponding angle of 120°). Henceforth, in order to simplify the discussion, we consider that this angle will be 120°.

The energy balance says that:

$$
\begin{aligned}
v_{s1}i_{s1} &= R_s i_{s1}^2 + L_s i_{s1} p i_{s1} + i_{s1} e_{s1}(\theta_m) \\
v_{s2}i_{s2} &= R_s i_{s2}^2 + L_s i_{s2} p i_{s2} + i_{s2} e_{s2}(\theta_m) \\
v_{s3}i_{s3} &= R_s i_{s3}^2 + L_s i_{s3} p i_{s3} + i_{s3} e_{s3}(\theta_m)
\end{aligned}
$$

The total energy entering the system ($v_{s1}i_{s1}+v_{s2}i_{s2}+v_{s3}i_{s3}$) is divided into three terms: the first one represents the Joule losses; the second one represents the variation of the magnetic energy stored in the self inductance L_s while the third is the mechanical power P_m.

If each winding of the machine is supplied by a square wave current (with a value equal to I_d, constant during 1/3 of the period (120°)) the total mechanical power will be constant.

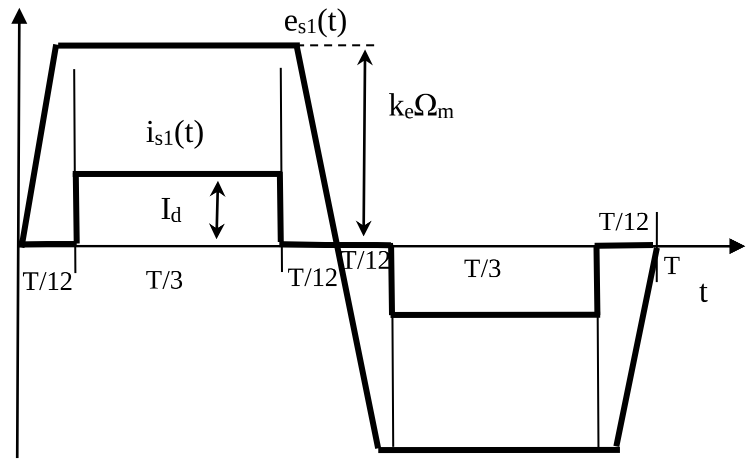

Figure 7-10: The electromotive force e_{s1} and the phase current i_{s1} as a function of time t (T=period)

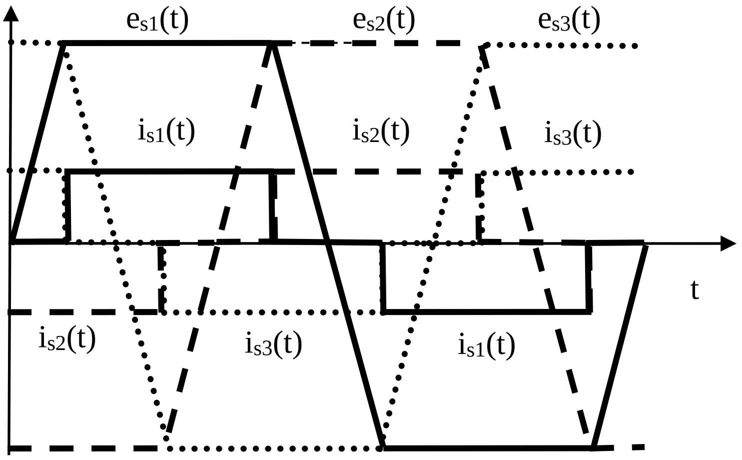

Figure 7-11: The electromotive forces and the phase currents (in the case of constant emf during 120°)

For each instant, there are two currents different from zero, so the mechanical power is:

$$P_m = i_{s1}e_{s1}(\theta_m) + i_{s2}e_{s2}(\theta_m - \frac{2}{3}\pi) + i_{s3}e_{s3}(\theta_m - \frac{4}{3}\pi) = 2k_e\Omega_m I_d$$

So the electromagnetic torque is:

$$T_e = \frac{P_m}{\Omega_m} = 2k_e I_d$$

Finally, there is the equation of the mechanical energy balance (simplified expression):

$$p\dot{\theta}_m = \frac{n_p}{J}(T_e - T_r)$$

7.2 Control technique

Observing Figure 7-12, the total period T may be divided into six sectors of one sixth of T, T_{sect}. During each sector, only two windings are fed. For example during sector 1 the current I_d flows in winding s1 (input) and s2 (output).

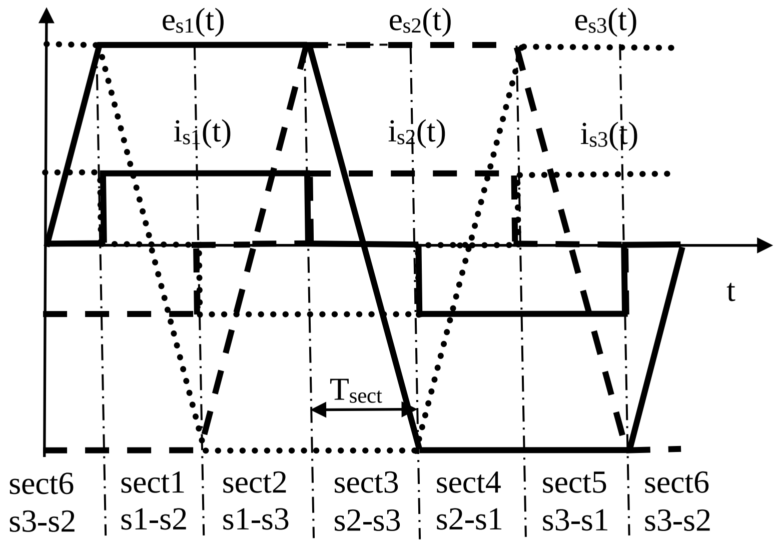

Figure 7-12: Six sectors

For every sector, the equivalent circuit seen by the power converter is always the same; it is necessary to change the terminals x and y as a function of the sector.

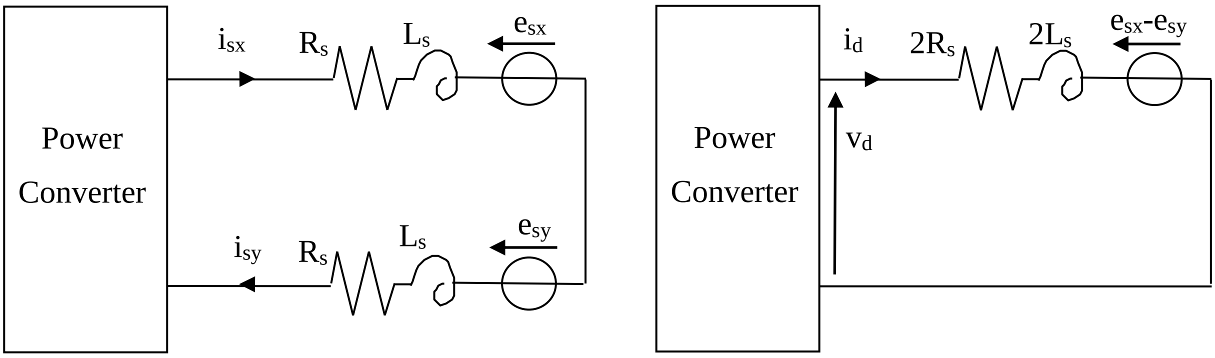

Figure 7-13: Equivalent circuit (two phases on)

During the period T_{sect} the difference e_{sx}-e_{sy} is constant and is given by the series connection of two back emf: $e_{sx}-e_{sy}=2k_e\Omega_m$. In order to have a constant torque, the current i_d has to be controlled as a constant value I_d. So the dynamic equations become (v_d is the output voltage of the converter):

$$v_d = 2R_s i_{s1} + 2L_s p i_{s1} + 2k_e\Omega_m = 2R_s i_{s1} + 2L_s p i_{s1} + E$$

From these considerations it is understood that the condition for which the torque remains constant is that the line-to-line emf has to be constant within the sector, namely 60°, and not a 120° of the the phase emf.

From the control point of view, the system has a dynamic behavior very similar to a PM DC machine:

$$v_a = R_a i_a + L_a p i_a + K_{ePM}\Omega_m = R_a i_a + L_a p i_a + E$$

Just consider $R_a=2R_s$, $L_a=2L_s$, $K_{ePM}=2k_e$. That is why this machine is called DC brushless.

Figure 7-14 shows a possible control scheme. The machine is not real, but it is an equivalent one.

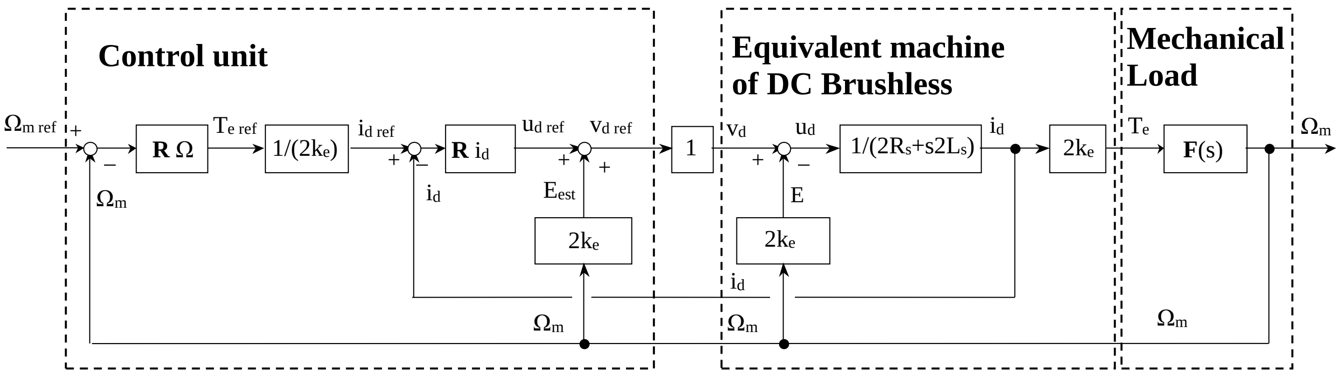

Figure 7-14. Speed control scheme of a DC brushless

7.3 Power converter

The power converter has to be designed to behave like a dc-dc converter for each sector. Three legs may be used. Using two legs at a time, six different configurations may be realized, which correspond to six different dc-dc converter, one for each sector. For example, during the period corresponding to sect1, the first and second legs realize a four quadrant dc-dc converter.

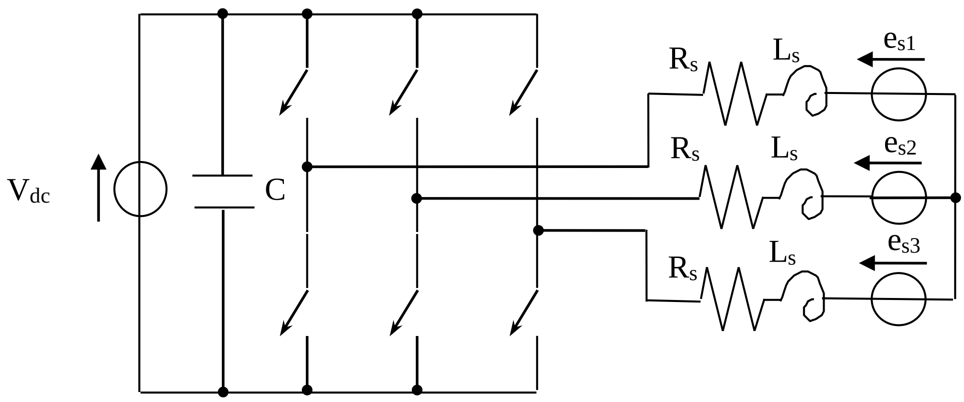

Figure 7-15. Power converter diagram of a DC brushless drive

The problem is to realize the value of $v_{d\ ref}$. The solution is the use of a duty cycle and a simple strategy by which, for each sector, the suitable configuration of the legs is chosen. Another problem is the measurement of the current i_d. This fictitious current is equal to a different phase current (positive or negative) for each sector. For example, during the period corresponding to sect1, i_d is equal to i_{s1}. Two current sensors and an information on the actual sector are enough to calculate the right value of i_d.

The configuration of the legs and the measurement of i_d are made possible by the use of a suitable position sensor. For this type of drive, three Hall effect sensors, with a displacement of 120 electrical degrees each other, are sufficient to recognize the actual sector.

Figure 7-16 shows the control scheme applied to the real machine. You must know the sector so that the power supply will work (choice of legs to drive) and to get the value of the virtual current i_d starting from the phase currents, easily measurable.

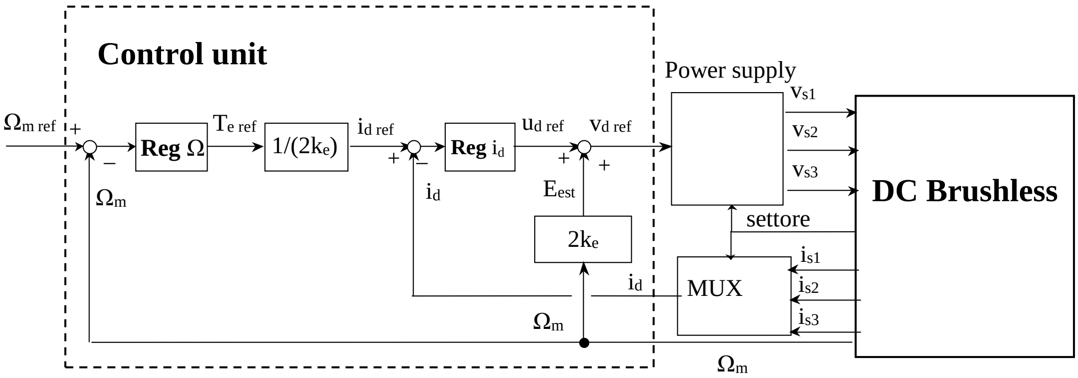

Figure 7-16. Real DC Brushless control scheme, based on the sector value

7.4 Operating regions

The strategy of maintaining the current constant for 120° and equal to Id, incoming in a phase and outgoing in another, depending on the sector in which is located the rotor, may not work for all speeds. It may be noted, in fact, from Figure 7-17, that the total emf (e_{sx}-e_{sy}), line-to-line voltage, equal to $2k_e\Omega_m$, is proportional to the mechanical speed. When this voltage reaches the maximum value that the power supply can provide (in an H-bridge the maximum voltage is V_{dc}, the voltage of the dc bus) less the resistive drop, it is no longer possible to control the current. Given v_{dmax} as the maximum voltage that the converter can provide (less a suitable margin to dynamically control the current), the mechanical speed corresponding to this point is the speed base: $\Omega_b=(v_{dmax}-2R_sI_d)/2k_e$.

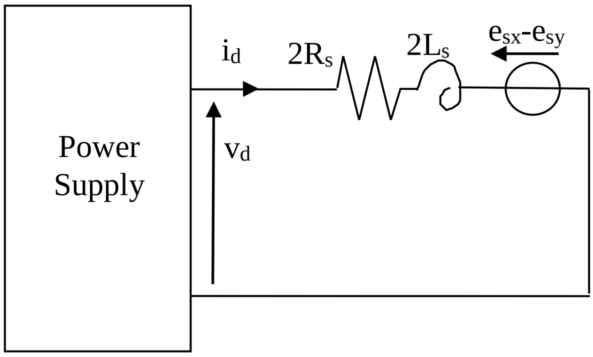

Figure 7-17: Equivalent circuit (two phases on)

In order to go over this speed you must change strategy. Please refer to the sector 1: the Figure 7-18 shows the line-to-line emf e_{s1}-e_{s2} at different mechanical speed.

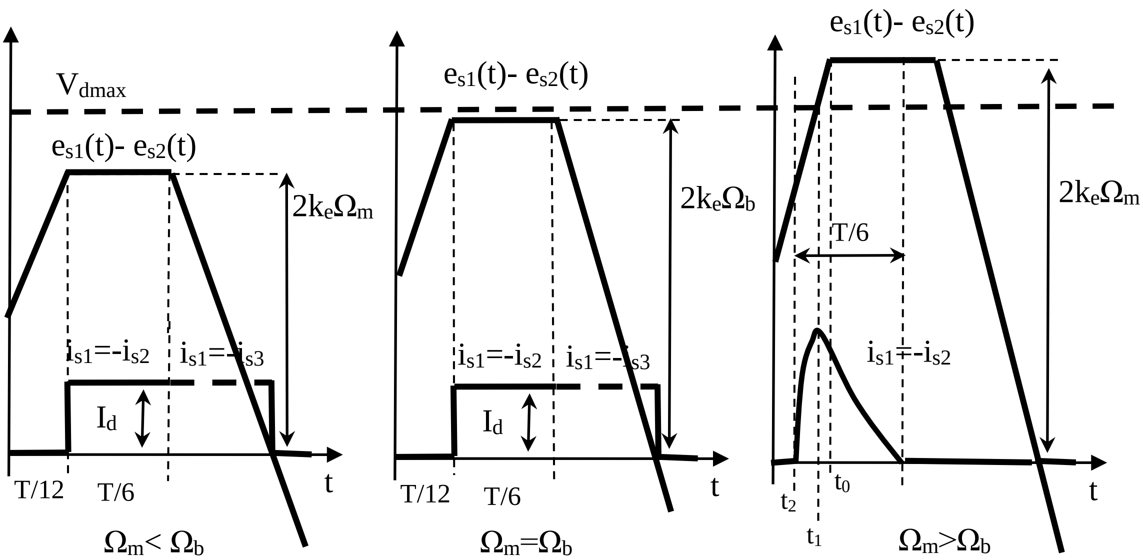

Figure 7-18: Line-to-line emf as the mechanical speed changes

Up to the base speed, it is possible to control the current as the forcing term (v_d-$2k_e\Omega_m$) is greater than zero. Above the base speed, it is necessary to anticipate the change of the sector (in the case of Figure 7-18 from the sector 6 (e_{s3}-e_{s2}) to the sector 1 (e_{s1}-e_{s2})) from instant t_0 to instant t_2, which must be before the instant t_1, instant in which the line-to-line emf equals the maximum voltage V_{dmax}. Looking at the equivalent circuit of Figure 7-17, you notice that, between t_1 and t_2, the forcing term V_{dmax}-(e_{s1}-e_{s2}) is positive. The current can grow up to the instant t1 beyond which it starts to decrease (the forcing term becomes negative) until it reaches zero. Suitably commanding the switches it can be fixed at zero. After 60° (T/6) the transition to the next sector happens.

While under the base speed, the control is usually performed by PWM modulation, over the base speed the operation is a square wave supply.

During this type of operation, the mechanical power, calculated by the energy balance, it is no longer constant but still employs a positive average value. The same behaviour for the torque. Such a torque allows maintaining the mechanical speed above the base speed.

The control scheme totally changes: the speed control acts on the advance of the sector, while maintaining limited the rms value of the stator current to the maximum permissible (for thermal problems). The advance of the instant of change of the sector causes that the first harmonic of the current does not result in phase with the emf but in advance.

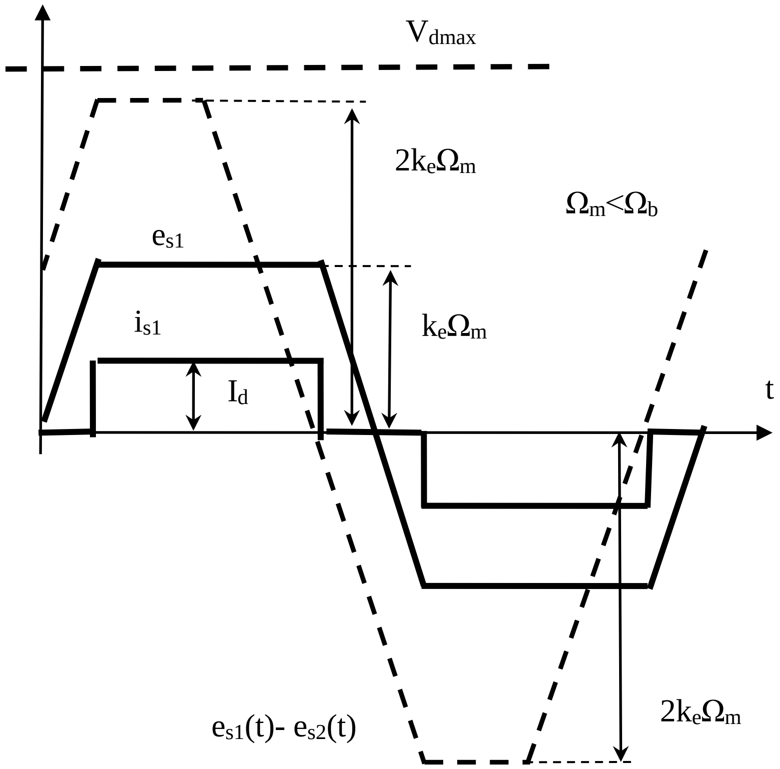

Figure 7-19: Phase current waveform for low speed ($<\Omega_b$)

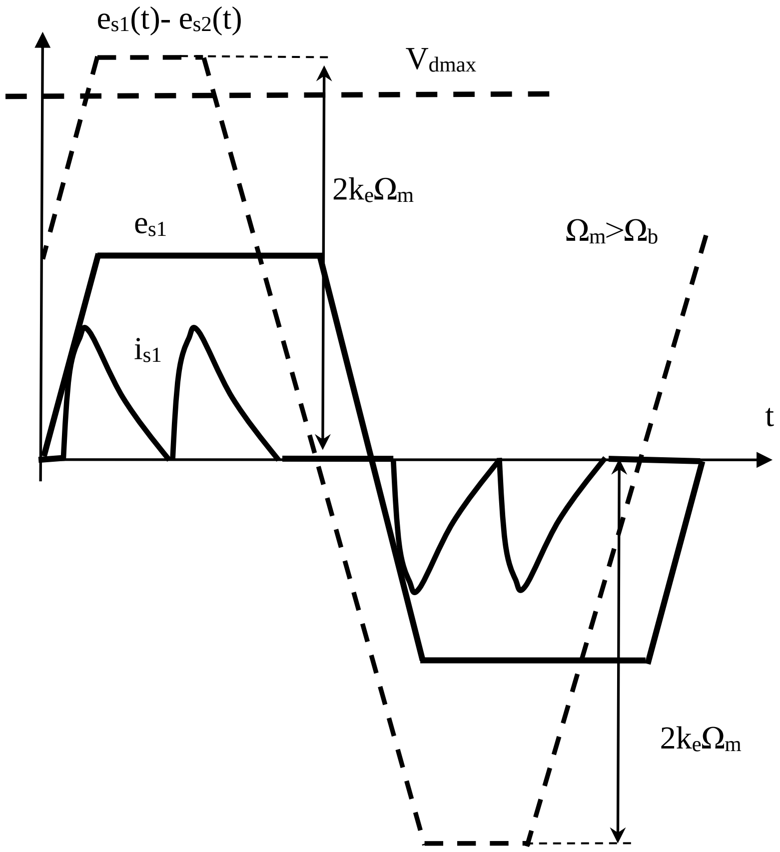

Figure 7-20: phase current waveform when speed exceeds the base speed

It means that, from the phasor point of view (like for AC brushless), a current component acts on the torque (in phase with the emf) and a component in advance of 90°. But since the emf is in advance of 90° with respect to the flux of the permanent magnets, this current is demagnetizing (opposite to the flux of the permanent magnets).

Since the DC brushless is a suitable solution for low cost applications, the permanent magnets used are low cost PM (not rare earth PM), whereby it tends to exploit the region above the base speed only in some cases and by merely a speed which generally does not exceed 20% of the base speed.

7.5 Three phases on

Analyse the change between the second and the third sector. For the presence of inductance, the phase 1 current cannot vanish instantly. In this period, the current i_d that was circulating in the phase 1 and phase 3, begins to flow in phases 2 and 3. Assume that during this period the switches S2H and S3L are controlled in closing. Because the command to S1H has been removed, the current flowing in the phase 1 continues to flow (until it goes to zero) through the freewheeling diode of the switch S1L. The dynamic equivalent circuit of the machine is therefore that shown in Figure 7-21.

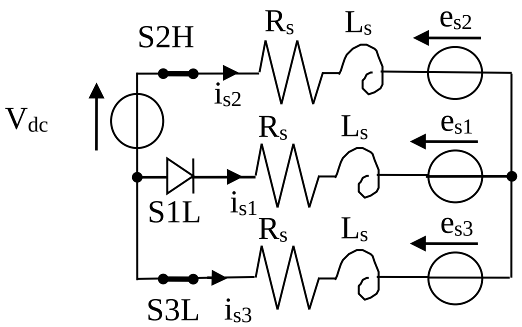

Figure 7-21. Interval of three-phases on

Due to the current Kirchhoff Law, it is:

$$i_{s1} + i_{s2} + i_{s3} = 0$$

While the voltage Kirchhoff Law (applied to the upper mesh) says:

$$R_s i_{s2} + L_s p i_{s2} + e_{s2} - e_{s1} - L_s p i_{s1} - R_s i_{s1} = V_{dc}$$

And for the lower mesh:

$$R_s i_{s1} + L_s p i_{s1} + e_{s1} - e_{s3} - L_s p i_{s3} - R_s i_{s3} = 0$$

Introducing the equation of the node:

$$R_s i_{s1} + L_s p i_{s1} + e_{s1} - e_{s3} - L_s p(-i_{s1} - i_{s2}) - R_s(-i_{s1} - i_{s2}) = 0$$

$$2R_s i_{s1} + 2L_s p i_{s1} + e_{s1} - e_{s3} + L_s p i_{s2} + R_s i_{s2} = 0$$

$$L_s p i_{s2} + R_s i_{s2} = -\left(2R_s i_{s1} + 2L_s p i_{s1} + e_{s1} - e_{s3}\right)$$

Now you can substitute the expression of i_{s2} in the first mesh equation:

$$-2R_s i_{s1} - 2L_s p i_{s1} - e_{s1} + e_{s3} + e_{s2} - e_{s1} - L_s p i_{s1} - R_s i_{s1} = V_{dc}$$

$$-3R_s i_{s1} - 3L_s p i_{s1} + e_{s3} + e_{s2} - 2e_{s1} = V_{dc}$$

$$-3R_s i_{s1} - 3L_s p i_{s1} = V_{dc} + 2e_{s1} - e_{s3} - e_{s2}$$

But at the instant of the change of sector, the emf's are equal to: $e_{s2}=e_{s1}=-e_{s3}=k_e\Omega_m$.

So

$$R_s i_{s1} + L_s p i_{s1} = -\frac{V_{dc}+2k_e\Omega_m}{3}$$

As regards to the current i_{s2} it is:

$$L_s p i_{s2} + R_s i_{s2} = -\left(2R_s i_{s1} + 2L_s p i_{s1} + e_{s1} - e_{s3}\right)$$

$$L_s p i_{s2} + R_s i_{s2} = -\left(-\frac{2}{3}\left(V_{dc}+2k_e\Omega_m\right) + k_e\Omega_m + k_e\Omega_m\right)$$

$$L_s p i_{s2} + R_s i_{s2} = \frac{2V_{dc}-2k_e\Omega_m}{3}$$

It is noted, therefore, that the transients that govern the performance of the currents i_{s1} and i_{s2} are different (in terms of the forcing term). The forcing term is the same (in magnitude) when

$$\frac{V_{dc}+2k_e\Omega_m}{3} = \frac{2V_{dc}-2k_e\Omega_m}{3}$$

$$\Omega_m = \frac{V_{dc}}{4k_e}$$

In particular, for low speed ($\Omega_m<V_{dc}/4k_e$), the current i_{s2} grows faster than i_{s1} decreases. Conversely, for high speed.

While the current i_{s2} grows, the current i_{s1} decreases until it reaches to 0 (KCL). From this instant, the freewheeling diode of the switch S1L goes to the blocking state and the current begins to flow only in the phase 2. The operation returns to be characterized by only two phases on.

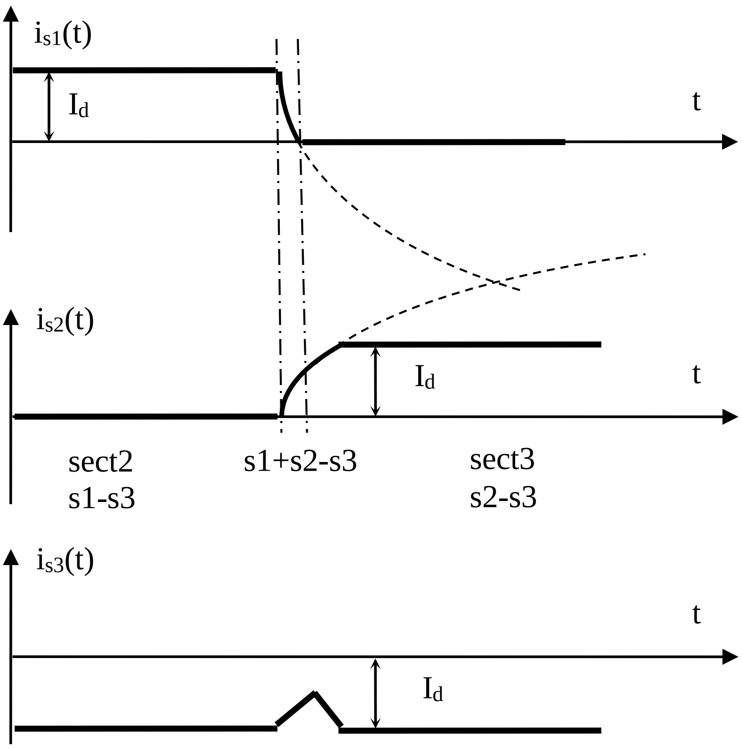

Figure 7-22. Transition between the phase 1 and phase 2 (high speed)

This difference affects the current i_{s3} in such a way as not to remain constant (equal to I_d). At high speed, this transition period (in terms of angles) become significant. In this period the total mechanical power $e_{s1}i_{s1}+e_{s2}i_{s2}+e_{s3}i_{s3}$, is no longer constant but varies six times in the period. Even the torque presents this oscillation, which may solicit mechanical resonances or, at least, produces noise.