Summary

2. Space phasors

2.1 The space phasors and the electromagnetic joint

Consider an isotropic rotating machine, with one rotor winding.

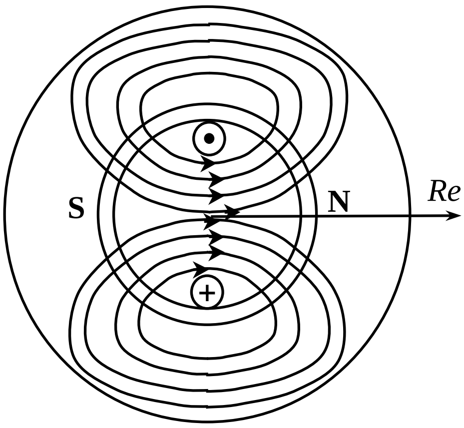

Figure 2-1 Isotropic rotor powered machine – concentrated windings

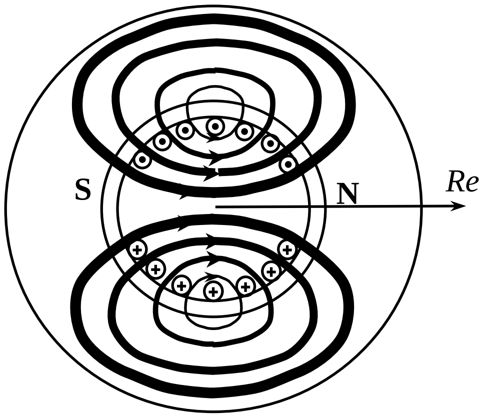

Figure 2-2 Isotropic rotor powered machine – distributed windings

The current, flowing through the rotor winding, is related to a magnetomotive force (mmf) in the air gap, which maintains a magnetic field whose lines are shown in Figure 2-1 and Figure 2-2. The magnetic field is similar to that generated by a permanent magnet that has a North and South as in the figure. The situation along the magnetic air gap has an alternating pattern (see solid line in Figure 2-3 and Figure 2-4)

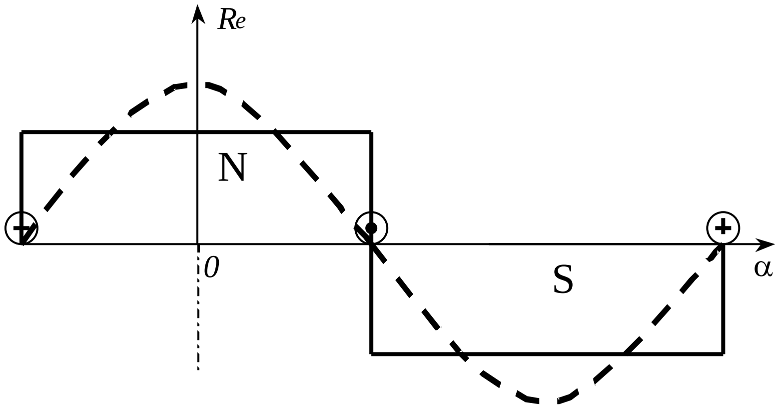

Figure 2-3 Qualitative trend of the magnetomotive force along the air gap – concentrated windings

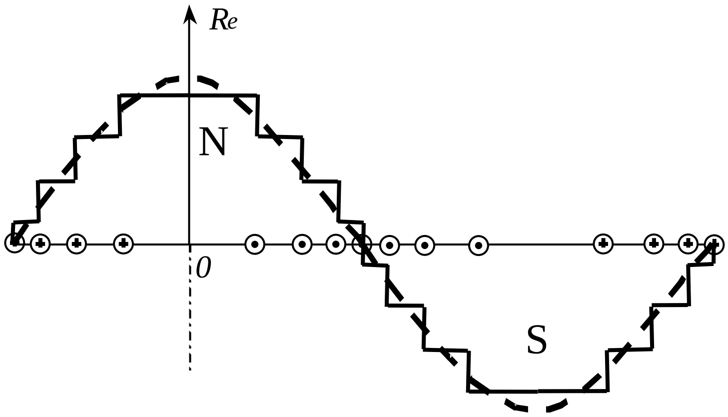

Figure 2-4 Qualitative trend of the magnetomotive force along the air gap – sinusoidally distributed windings

If we consider only the first harmonic (dashed line in Figure 2-3 and Figure 2-4) and neglecting the highest harmonics, the situation in the air gap may be represented by a vector whose direction is along the North polar axis (Figure 2-5).

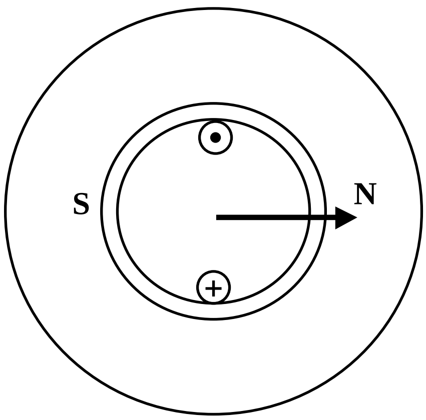

Figure 2-5: Mmf representation by means of a space phasor

The space phasor (whose amplitude is a function of time) allows knowing the value of the corresponding quantity at any location within the air-gap and at each instant. In fact, in order to know the value of the amplitude at a given point of the air gap it is sufficient to project the phasor onto the required direction. It appears that $M(\theta,t) = M(t)\cdot\cos(\theta)$ (Figure 2-6).

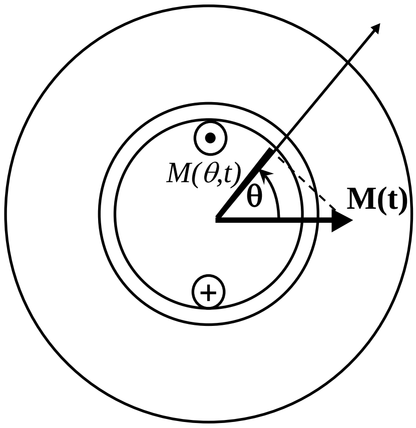

Figure 2-6: Space phasor application

All electrical quantities (voltages and currents) and magnetic quantities (mmf and fluxes) can be represented by space phasors, allowing for easy interpretation of electromagnetic phenomena.

The number of turns *N*, by which the winding is carried out, acts like a transformer. At constant mmf and flux, a greater number of turns implies a lower current value but a higher value of the flux linkage and thus of the induced voltage. The value of the instantaneous electric power remains unchanged.

The distribution of turns in the slots produces an effect (winding factor) that is easy to analyze. Each coil produces a field represented by a space phasor. The vector sum of these fields provides the resulting field (see Figure 2-7)

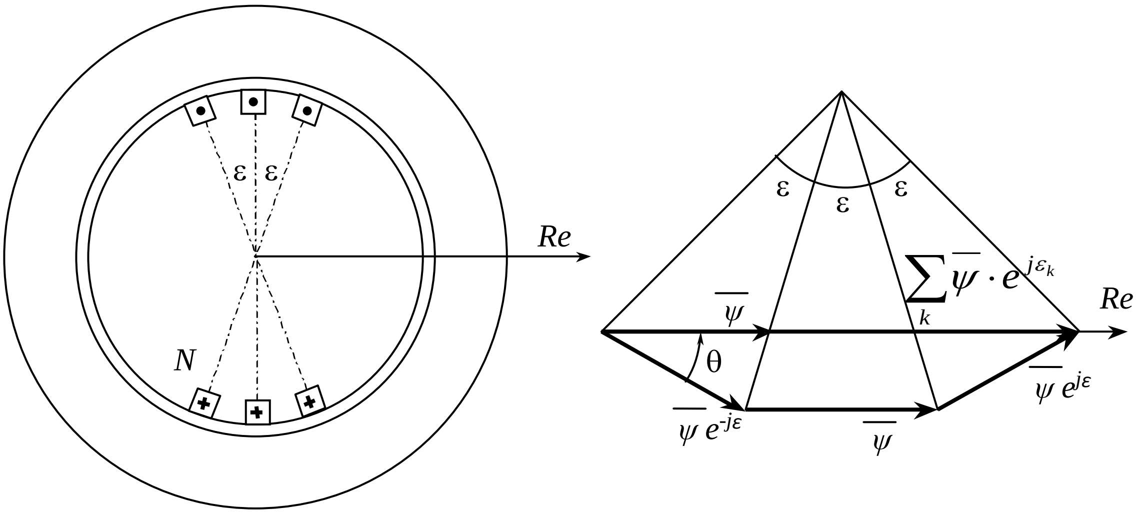

Figure 2-7: Winding factor

The use of space phasor allows for an evaluation of the electromechanical interactions. In fact, even in the presence of the stator winding, the rotor magnetic field will tend to align with the field of stator (both North poles tend to overlap). Otherwise, a torque will arise; its expression is: $T_e = k\cdot\psi_r\psi_s\sin(\theta)$ (see Figure 2-8).

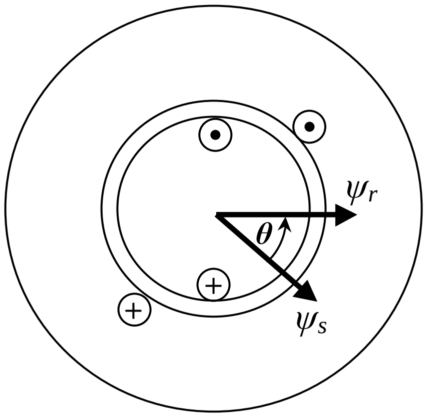

Figure 2-8: Electromagnetic joint

The operating principle of a large part of electrical machines is based on this statement: the torque is different by zero if the rotor magnetic field is not aligned with the stator field. The maximum torque occurs when the fields are orthogonal.
It is now clear that, in order to obtain a constant torque, it is necessary that the two fields remain displaced by a constant angle over time. This would only be possible if the rotor windings and stator have the same speed: in a DC machine this is allowed by the presence of a commutator; in other machines it would not be possible. However, the presence of more windings in the same structure (for example in the stator) provides new degrees of freedom. The magnetic field (stator) is obtained by superimposition of fields generated by the different windings. In the case, for example, of two windings displaced by 90 electrical degrees in space, using suitable values of the currents in the stator windings can force the value and location of the stator flux. In the case where the currents are sinusoidal and displaced each other by one fourth of period, the resulting field rotates at a constant speed and constant amplitude (despite being generated by fixed windings).
The expression of the space phasor of the current as a function of the two currents i_a and i_b is the following:

$$\bar{i} = (i_a + j \cdot i_b)$$

Conversely, knowing the space phasor, the value of winding current can be calculated simply projecting the space phasor along the direction of the corresponding magnetic axis; for example:

$$i_a = Ra(\bar{i})$$

where *Ra* is defined as the projection of the phasor onto the magnetic axis of phase "*a*".
The same effect can be achieved by using three windings, displaced by 120 electrical degrees in space, and three-phase currents, with a time displacement of one third of the period.
It is easy to switch between the three-phase currents to the current space phasor. The formula is as follows:

$$\bar{i} = (i_a + \bar{\alpha} \cdot i_b + \bar{\alpha}^2 \cdot i_c)$$

where $\bar{\alpha} = e^{j\frac{2}{3}\pi}$
On the other hand, it is possible to calculate, knowing the space phasor and assuming that the sum of the phase currents is zero, the value of a phase current using the formula:

$$i_a = \frac{2}{3} Ra(\bar{i})$$

Traditionally (for this course), however, the space phasor is defined as follows:

$$\bar{i} = \sqrt{\frac{2}{3}}(i_a + \bar{\alpha} \cdot i_b + \bar{\alpha}^2 \cdot i_c)$$

$$i_a = \sqrt{\frac{2}{3}} Ra(\bar{i})$$

or better: $i_a = \sqrt{\frac{2}{3}}\,\mathrm{Re}(\bar{i}) \qquad i_b = \sqrt{\frac{2}{3}}\,\mathrm{Re}(\bar{\alpha}^2 \cdot \bar{i}) \qquad i_c = \sqrt{\frac{2}{3}}\,\mathrm{Re}(\bar{\alpha} \cdot \bar{i})$

so as to maintain the same expressions of power and energy both in the phase quantities and in the space phasors (Re means "real part of").

The space phasor, like all vectors, is completely defined by two variables: the amplitude and the argument (or phase angle) or by its components with respect to two orthogonal axes (reference frame).

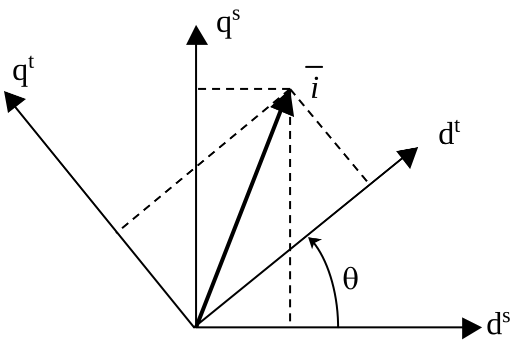

Figure 2-9: Reference frame

The axes of the reference frame are commonly marked with "d" and "q" (where d is the real axis, while q represents the imaginary axis). Given i^s as the phasor with respect to a reference frame "s", the corresponding phasor in a reference frame "t" displaced by an angle θ with respect "s" is:

$$\bar{i}^t = \bar{i}^s \cdot e^{-j\theta}$$

The phasor is always the same, but it is "seen" from a different viewpoint.

The operation of the derivative of a phasor leads to the following relationship:

$$\frac{d\bar{i}^t}{dt} = \frac{d\left(\bar{i}^s \cdot e^{-j\theta}\right)}{dt} = \frac{d\bar{i}^s}{dt} e^{-j\theta} - j\dot{\theta} \cdot e^{-j\theta} \cdot \bar{i}^s = \left(\frac{d\bar{i}^s}{dt} - j\dot{\theta} \cdot \bar{i}^s\right) \cdot e^{-j\theta}$$