

Generation

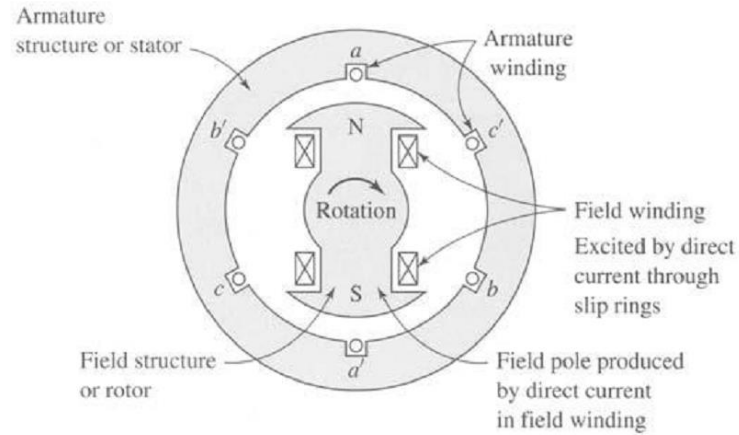


Figure A.1 Elementary two-pole, three-phase generator.

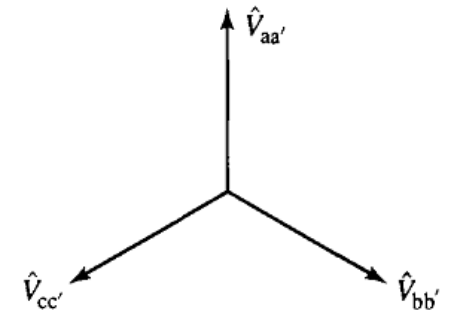
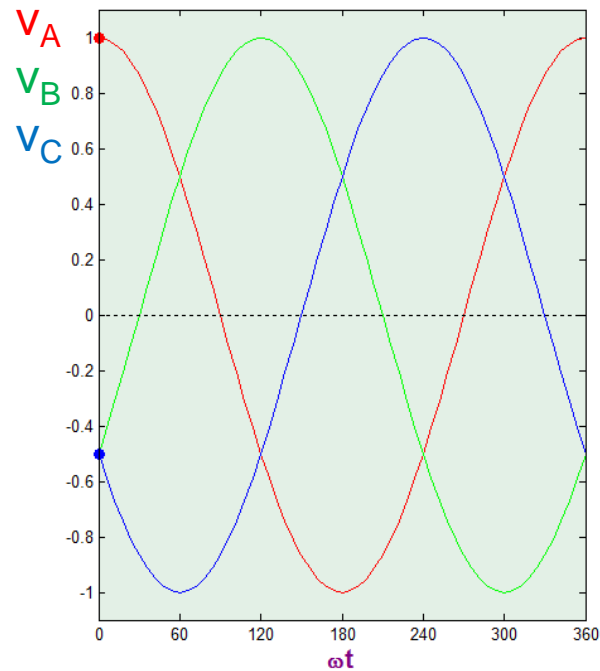
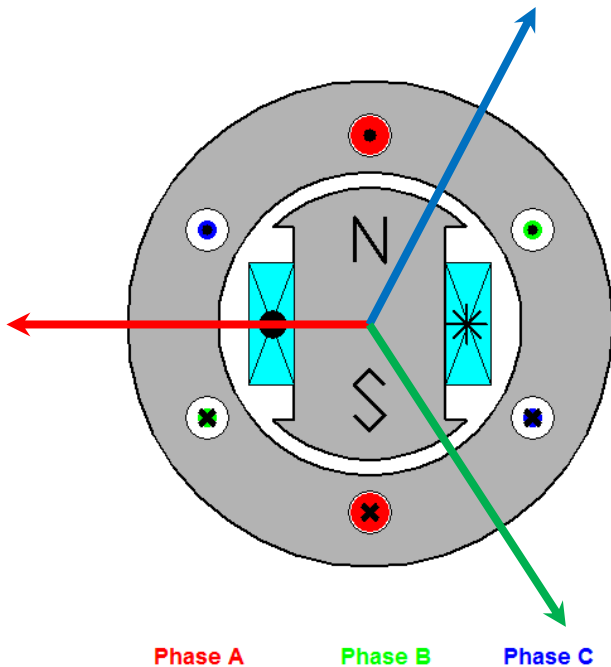


Figure A.4 Phasor diagram of generated voltages.

<http://www.ece.umn.edu/users/riaz/animations/alternator.html>

Connection

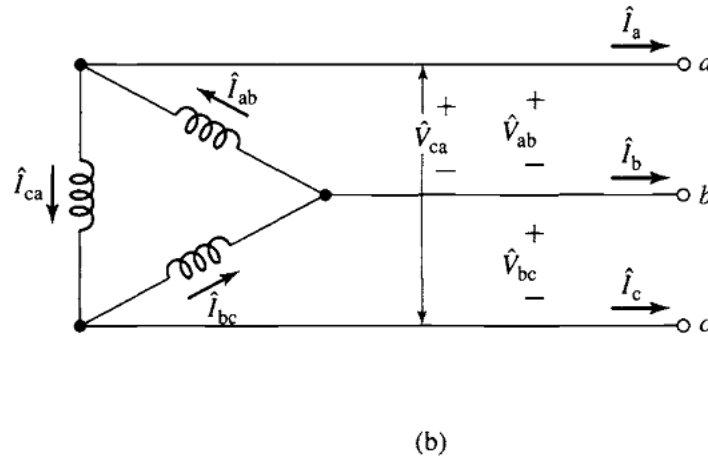
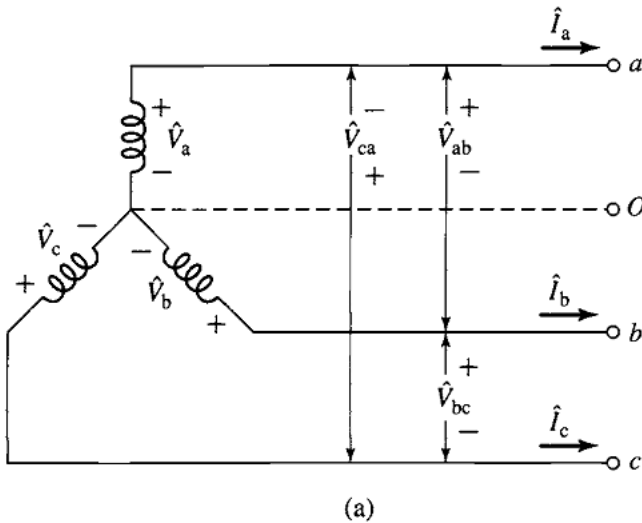
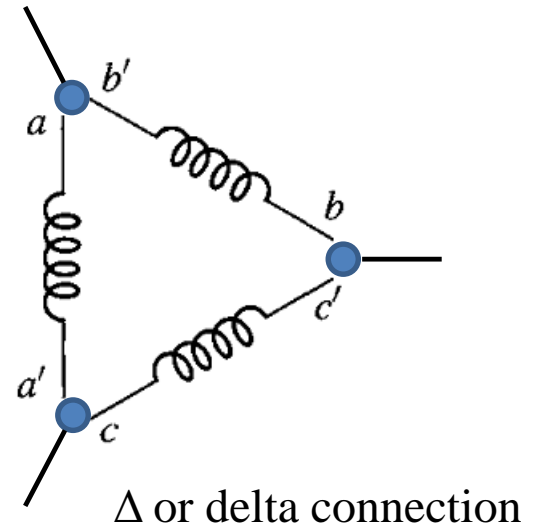
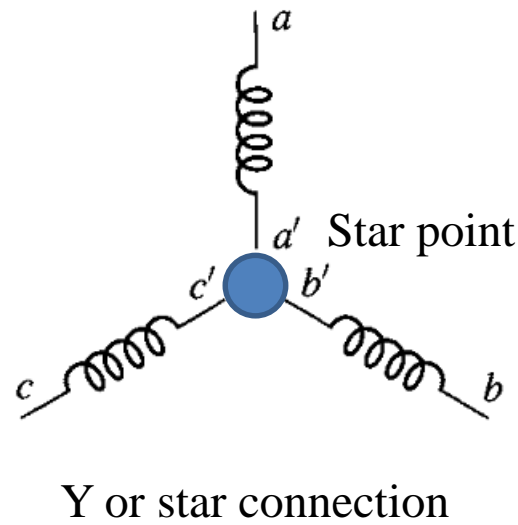


Figure A.5 Three-phase connections: (a) Y connection and (b) Δ connection.

$$\hat{V}_{ab} = \hat{V}_a - \hat{V}_b = \sqrt{3} \hat{V}_a \angle 30^\circ \quad \hat{V}_{bc} = \sqrt{3} \hat{V}_b \angle 30^\circ \quad \hat{V}_{ca} = \sqrt{3} \hat{V}_c \angle 30^\circ$$

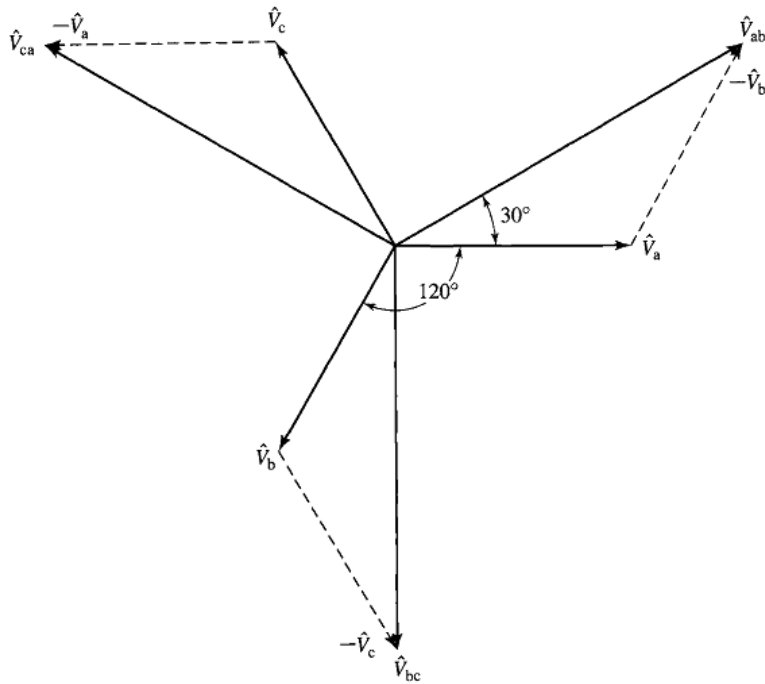


Figure A.6 Voltage phasor diagram for a Y-connected system.

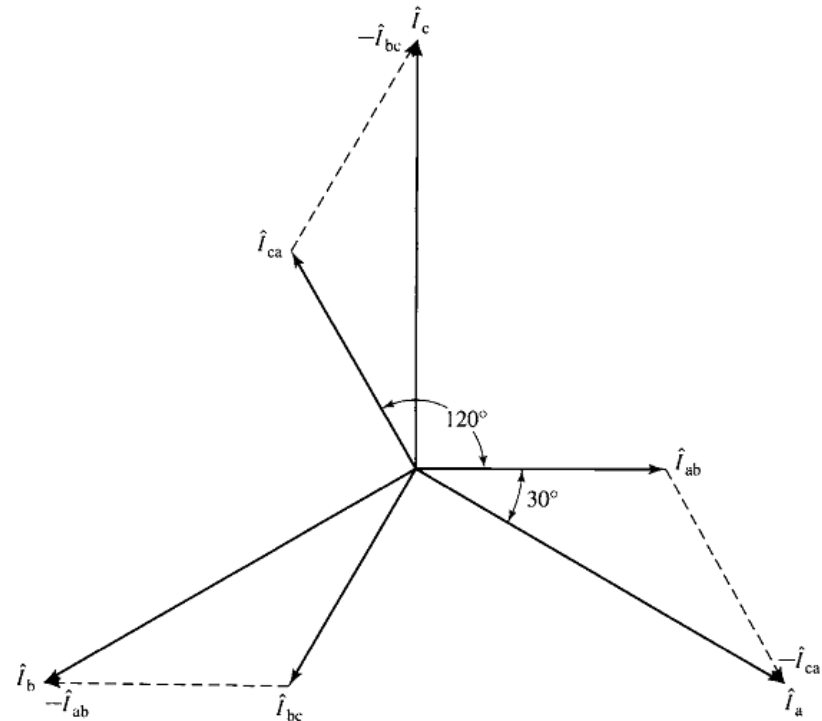


Figure A.7 Current phasor diagram for Δ connection.

$$\hat{I}_a = \hat{I}_{ab} - \hat{I}_{ca} = \sqrt{3} \hat{I}_{ab} \angle -30^\circ \quad \hat{I}_b = \sqrt{3} \hat{I}_{bc} \angle -30^\circ \quad \hat{I}_c = \sqrt{3} \hat{I}_{ca} \angle -30^\circ$$

$$\begin{aligned}
 v_a(t) &= \sqrt{2} V_{\text{rms}} \cos \omega t & i_a(t) &= \sqrt{2} I_{\text{rms}} \cos (\omega t + \theta) \\
 v_b(t) &= \sqrt{2} V_{\text{rms}} \cos (\omega t - 120^\circ) & i_b(t) &= \sqrt{2} I_{\text{rms}} \cos (\omega t + \theta - 120^\circ) \\
 v_c(t) &= \sqrt{2} V_{\text{rms}} \cos (\omega t + 120^\circ) & i_c(t) &= \sqrt{2} I_{\text{rms}} \cos (\omega t + \theta + 120^\circ)
 \end{aligned}$$

$$\begin{aligned}
 p_a(t) &= v_a(t)i_a(t) = V_{\text{rms}}I_{\text{rms}}[\cos (2\omega t + \theta) + \cos \theta] \\
 p_b(t) &= v_b(t)i_b(t) = V_{\text{rms}}I_{\text{rms}}[\cos (2\omega t + \theta - 240^\circ) + \cos \theta] \\
 p_c(t) &= v_c(t)i_c(t) = V_{\text{rms}}I_{\text{rms}}[\cos (2\omega t + \theta + 240^\circ) + \cos \theta]
 \end{aligned}$$

Note that the average power of each phase is equal

$$\langle p_a(t) \rangle = \langle p_b(t) \rangle = \langle p_c(t) \rangle = V_{\text{rms}}I_{\text{rms}} \cos \theta$$

The total instantaneous power for all three phases is

$$\begin{aligned}
 p(t) &= p_a(t) + p_b(t) + p_c(t) = 3V_{\text{rms}}I_{\text{rms}} \cos \theta \\
 Q_p &= V_{\text{rms}}I_{\text{rms}} \sin \theta & Q &= 3Q_p
 \end{aligned}$$

The voltamperes per phase $(\text{VA})_p$ and total three-phase voltamperes VA are

$$(\text{VA})_p = V_{\text{rms}}I_{\text{rms}} = I_{\text{rms}}^2 Z_p \quad (\text{A.22})$$

$$\text{VA} = 3(\text{VA})_p \quad (\text{A.23})$$

Balanced three-phase circuits: single-line diagrams

$$P = 3P_p = 3V_p I_p \cos \theta$$

Since $V_{1-1} = \sqrt{3}V_p$, Eq. A.26 becomes

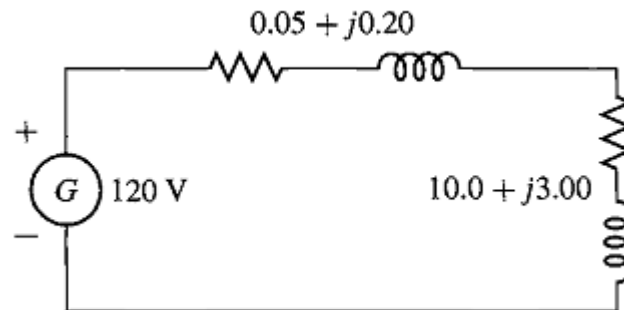
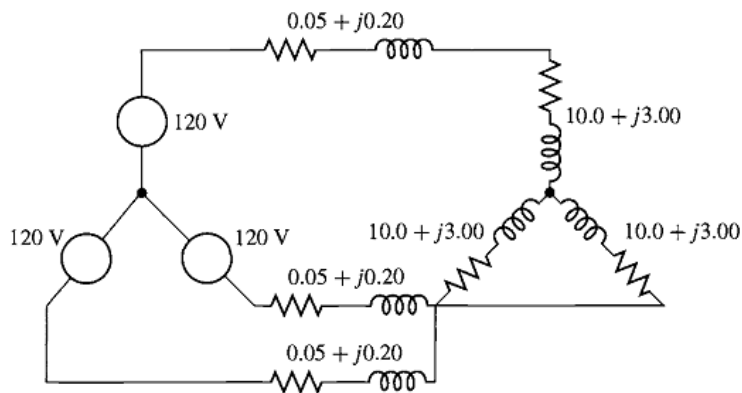
$$P = \sqrt{3}V_{1-1}I_p \cos \theta$$

Similarly,

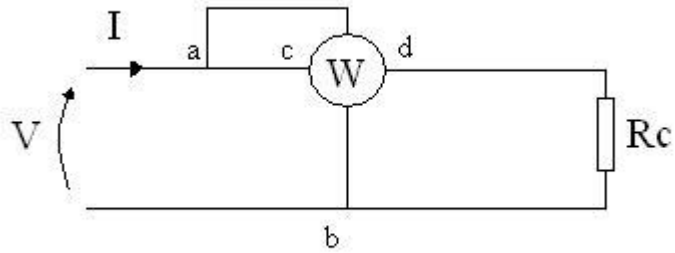
$$Q = \sqrt{3}V_{1-1}I_p \sin \theta$$

and

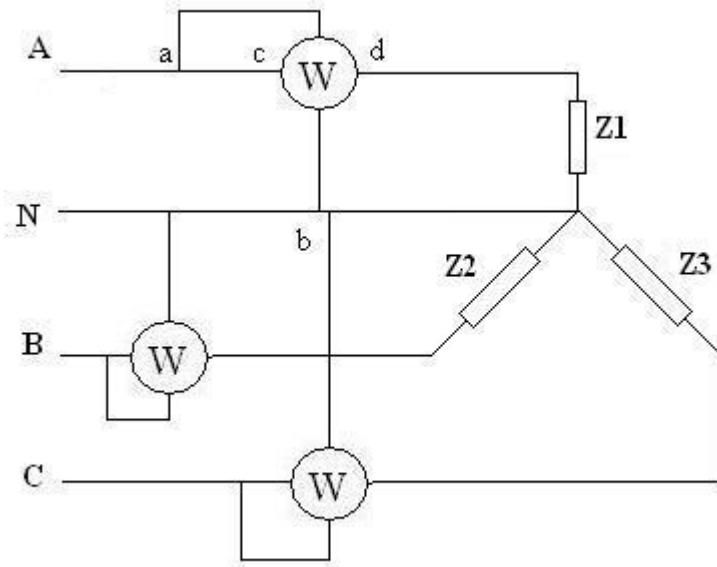
$$VA = \sqrt{3}V_{1-1}I_p$$



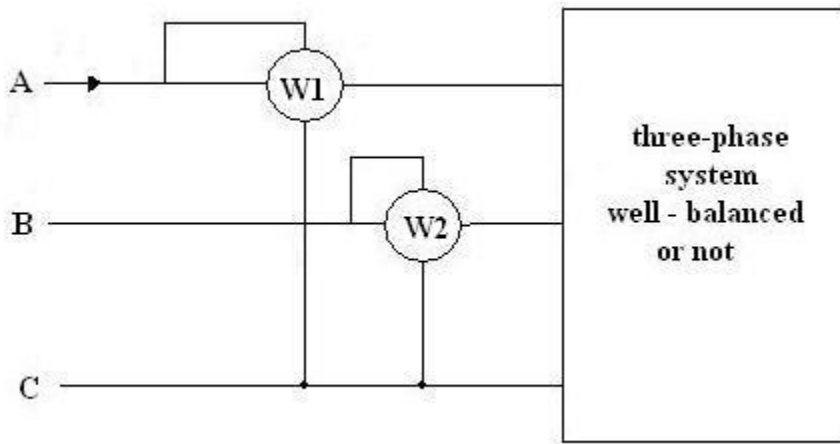
Aron connection



Single-phase wattmeter



Three-phase wattmeter



Aron connection