Greek Alphabet

Α α	alpha	/ˈælfə/	Ν ν	nu	/ˈnjuː/
Β β	beta	/ˈbiːtə/	Ξ ξ	xi	/ˈksaɪ/
Γ γ	gamma	/ˈgæmə/	Ο ο	omicron	/ˈɒmɨkrɒn/
Δ δ	delta	/ˈdɛltə/	Π π	pi	/ˈpaɪ/
Ε ε	epsilon	/ˈɛpsɨlɒn/	Ρ ρ	rho	/ˈroʊ/
Ζ ζ	zeta	/ˈziːtə/	Σ σ	sigma	/ˈsɪgmə/
Η η	eta	/ˈiːtə/	Τ τ	tau	/ˈtaʊ/
Θ θ	theta	/ˈθiːtə/	Υ υ	upsilon	/ˈʊpsɨlɒn/
Ι ι	iota	/aɪˈoʊtə/	Φ φ	phi	/ˈfaɪ/
Κ κ	kappa	/ˈkæpə/	Χ χ	chi	/ˈkaɪ/
Λ λ	lambda	/ˈlæmdə/	Ψ ψ	psi	/ˈpsaɪ/
Μ μ	mu	/ˈmjuː/	Ω ω	omega	/ˈoʊmɨgə/

Symbols

Abbreviation	*Variable*	*Unit*	*Abbreviation*	*Variable*	*Unit*
v	voltage	[V]	J	current density	[A/m^2]
i	current	[A]	E	electric field	[V/m]
$e(t)$	induced voltage e.m.f.	[V]	H	magnetic field intensity	[A/m]
R	resistance	[Ω]	B	magnetic flux density	[T]
G	conductance	[S]	$\mathcal{R}$	reluctance	[H^{-1}]
L	inductance	[H]	Λ ($\mathcal{P}$)	permeance	[H]
C	capacitance	[F]	M ($\mathcal{F}$)	magnetomotive force m.m.f.	[A·turns]
X	reactance	[Ω]	Φ (φ)	magnetic flux	[Wb]
B	susceptance	[S]	ψ (λ)	flux linkage	[Wb]
Z	impedance	[Ω]	n_p	number of pole pairs	
Y	admittance	[S]	T	torque	[Nm]
p	instant power	[W]	F	force	[N]
P	active power	[W]	J	inertia	[kg m^2]
A	apparent power	[VA]	W (U)	energy	[J]
Q	reactive power	[Var]	f	frequency	[Hz]
$Re()$	real part of		ω	angular speed/frequency	[rad/s]
$Im()$	imaginary part of		v	linear speed	[m/s]
$\underline{a}$	complex conjugate				
j	imaginary unit				

Symbol	Variable	Unit	Symbol	Variable	Unit
v	voltage (tensione)	[V]	J	current density (densità di corrente)	[A/m^2]
i	current (corrente)	[A]	E	electric field (campo elettrico)	[V/m]
e(t)	induced voltage, electromotive force (e.m.f.) (tensione indotta, forza elettromotrice (fem))	[V]	H	magnetic field intensity (campo magnetico)	[A/m]
R	resistance (resistenza)	[Ω]	B	magnetic flux density (induzione magnetica)	[T]
G	conductance (conduttanza)	[S]	θ (R)	reluctance (riluttanza)	[H^{-1}]
L	inductance (induttanza)	[H]	Λ (P)	permeance (permeanza)	[H]
C	capacitance (capacità)	[F]	M (F)	magnetomotive force m.m.f. (forza magnetomotrice)	[A·turns]
X	reactance (reattanza)	[Ω]	Φ (φ)	magnetic flux (flusso magnetico)	[Wb]
B	susceptance (suscettanza)	[S]	ψ (λ)	flux linkage (flusso concatenato)	[Wb]
Z	impedance (impedenza)	[Ω]	n_p	number of pole pairs (numero di coppie polari)	
Y	admittance (ammettenza)	[S]	T	torque (coppia)	[Nm]
p	instant power (potenza istantanea)	[W]	F	force (forza)	[N]
P	active power (potenza attiva)	[W]	J	inertia (inerzia)	[kg m^2]
A	apparent power (potenza apparente)	[VA]	W (U)	energy (energia)	[J]
Q	reactive power (potenza reattiva)	[Var]	f	frequency (frequenza)	[Hz]
Re()	real part of (parte reale di)		ω	angular speed/frequency (velocità angolare/pulsazione)	[rad/s]
Im()	imaginary part of (parte immaginaria di)		v	linear speed (velocità lineare)	[m/s]
$\underline{a}$	complex conjugate (complesso coniugato)				
j	imaginary unit (unità immaginaria)				

Passive Circuit Elements

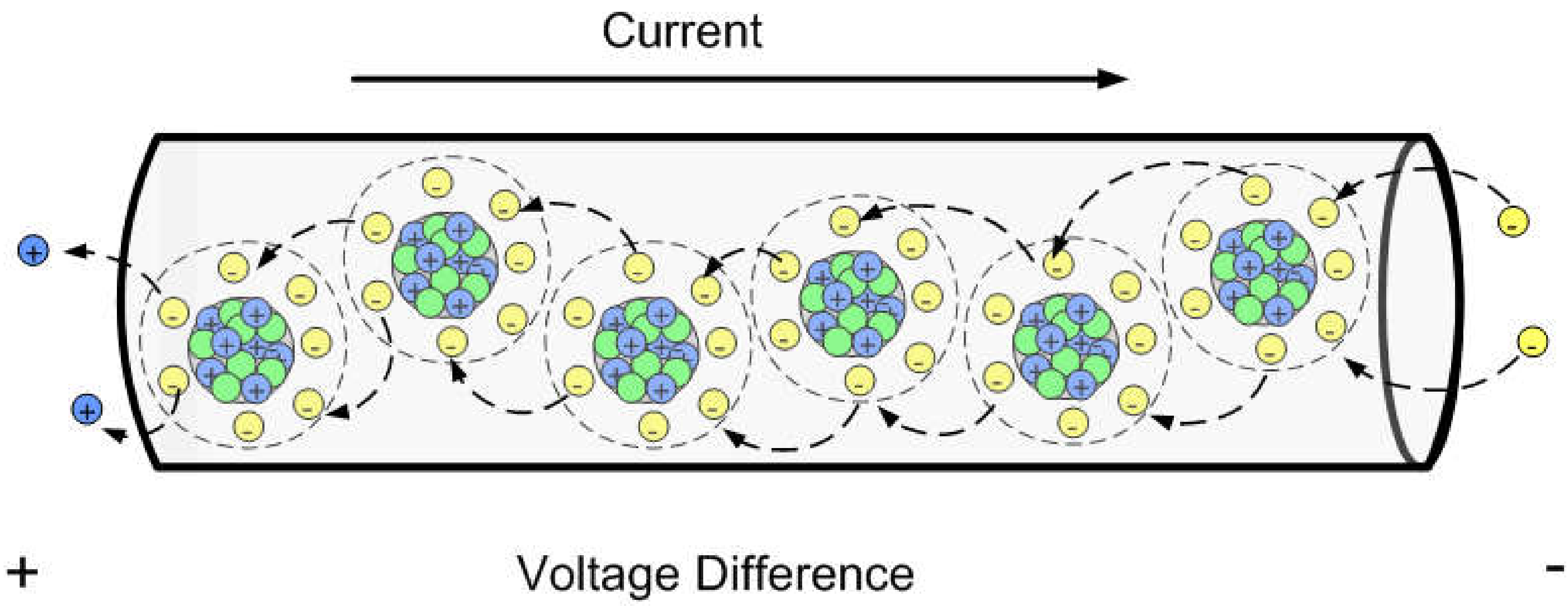
- **Resistors**
- **Capacitors**
- **Inductors**

extract from www.digilentinc.com/eeboard/RealAnalog/text/LectureB.ppt

Passive circuit elements - resistors

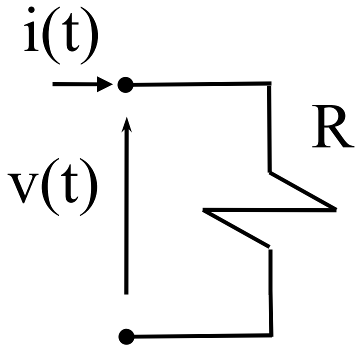
- ***Resistance*** **models the fact that energy is always converted in heat**
- **Electrons moving through a material "collide" with the atoms composing the material**
  - **These collisions impede the motion of the electrons**
  - **Thus, a voltage potential difference is required for current to flow. This potential energy balances the energy lost in these collisions.**

Resistance



Resistors

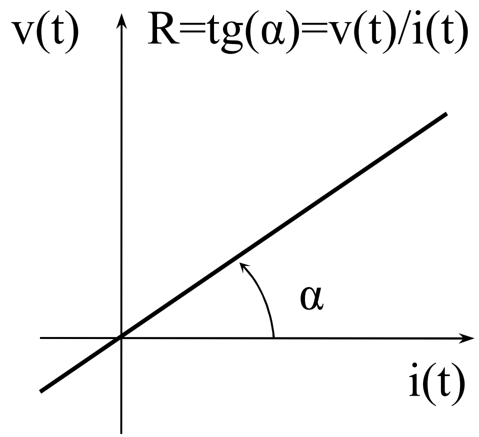
- **Circuit symbol:**



- **Voltage-current relation (Ohm's Law):**

$$v(t) = R \cdot i(t)$$

$$i(t) = G \cdot v(t)$$



- **R is the resistance**
  - **Unit is Ohm [Ω]**
- **G is the conductance = 1/R**
  - **Unit is Siemens [S]**

Resistor Power Dissipation

- **Ohm's Law:** $v(t) = R \cdot i(t) \Longrightarrow i(t) = G \cdot v(t)$

- **Instant power:** $p(t) = v(t) \cdot i(t)$

- **Combining :**

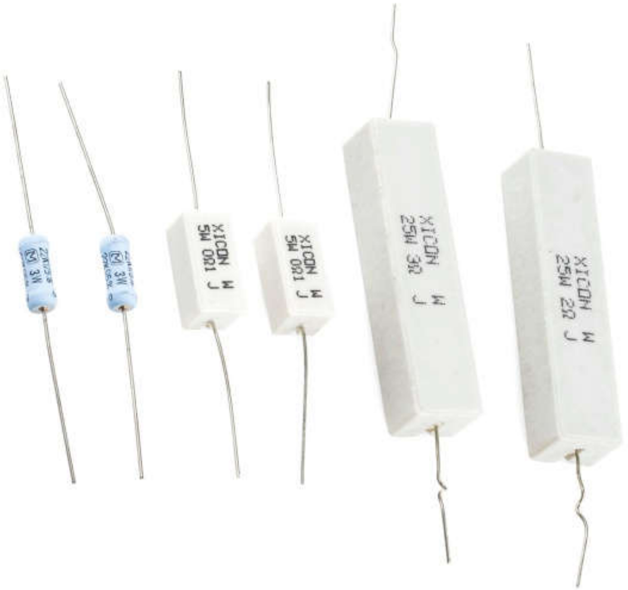
$$p(t) = v(t) \cdot G \cdot v(t) \Longrightarrow p(t) = G \cdot v^2(t) = \frac{v^2(t)}{R}$$

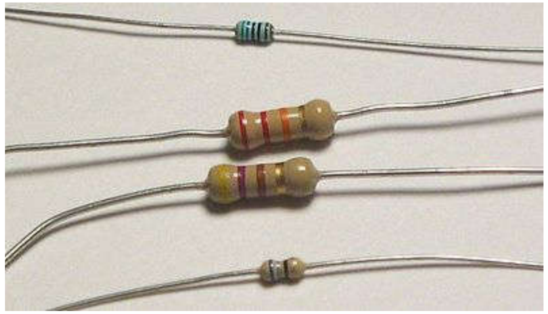
$$p(t) = R \cdot i(t) \cdot i(t) \Longrightarrow p(t) = R \cdot i^2(t)$$

- **Active power: mean value of instant power on a period T**

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T R \cdot i(t)^2 dt = R \cdot \frac{1}{T}\int_0^T i(t)^2 dt = R \cdot I_{rms}^2$$

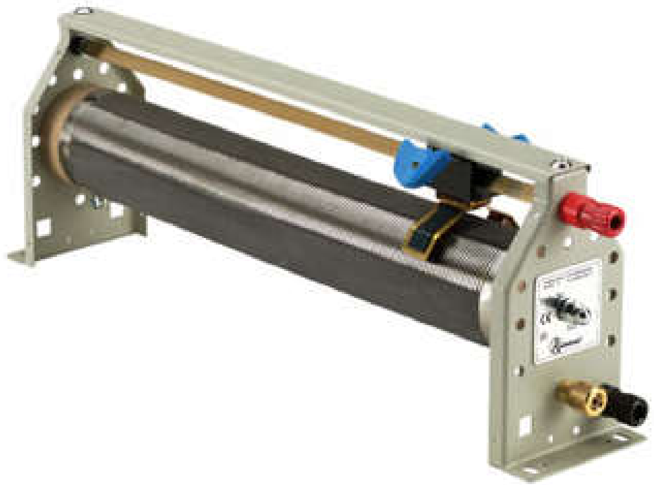
$$P = \frac{V_{rms}^2}{R} = G \cdot V_{rms}^2$$

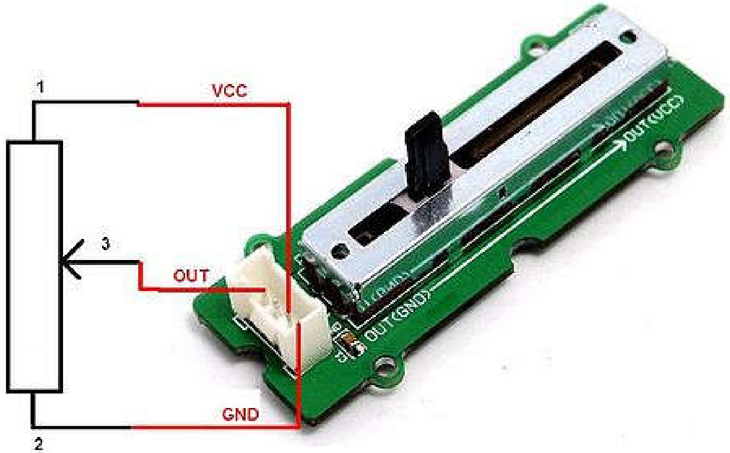


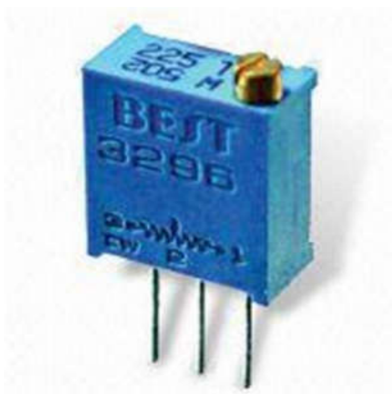


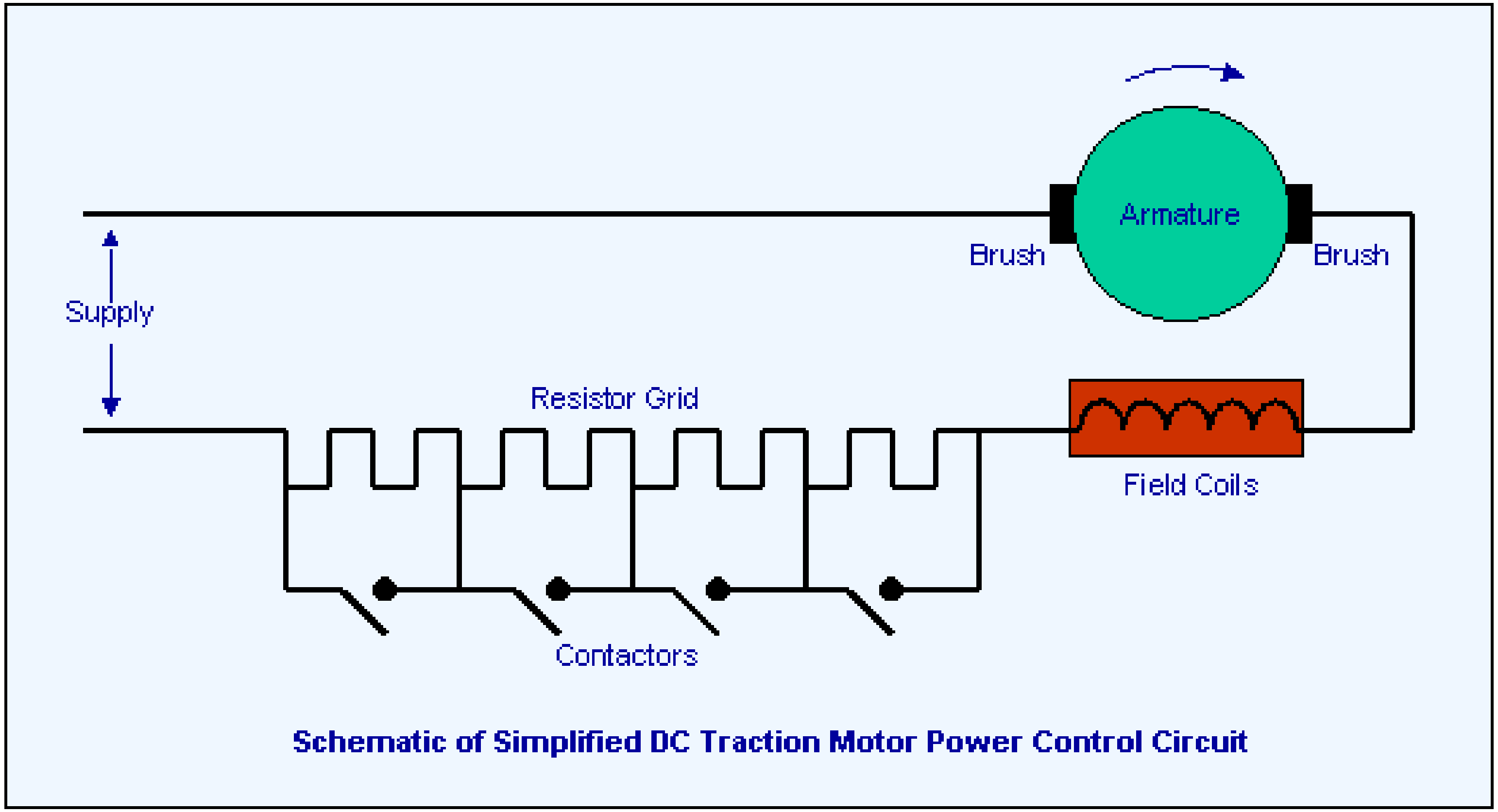

472
2512
SMD Resistor
2R2
103
4R7
8202
473
www.mobilecellphonerepairing.com

Francesco Castelli Dezza

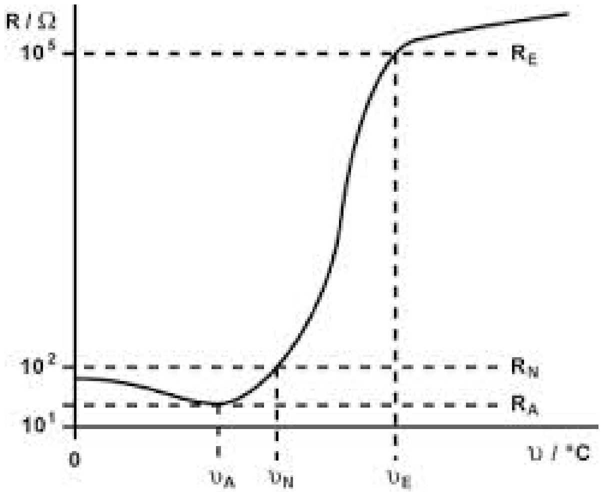


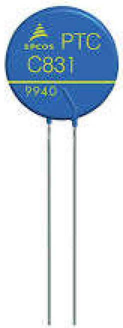


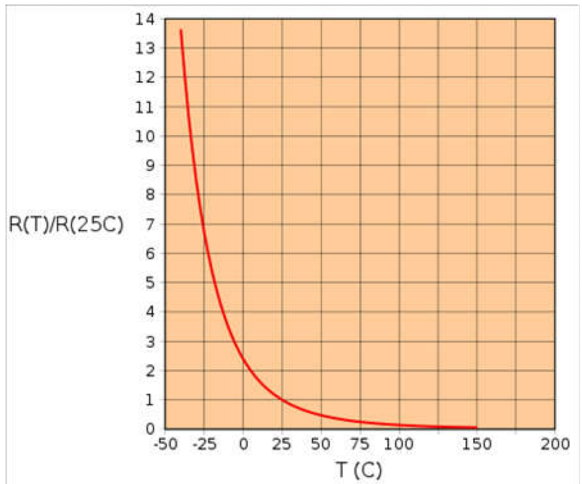


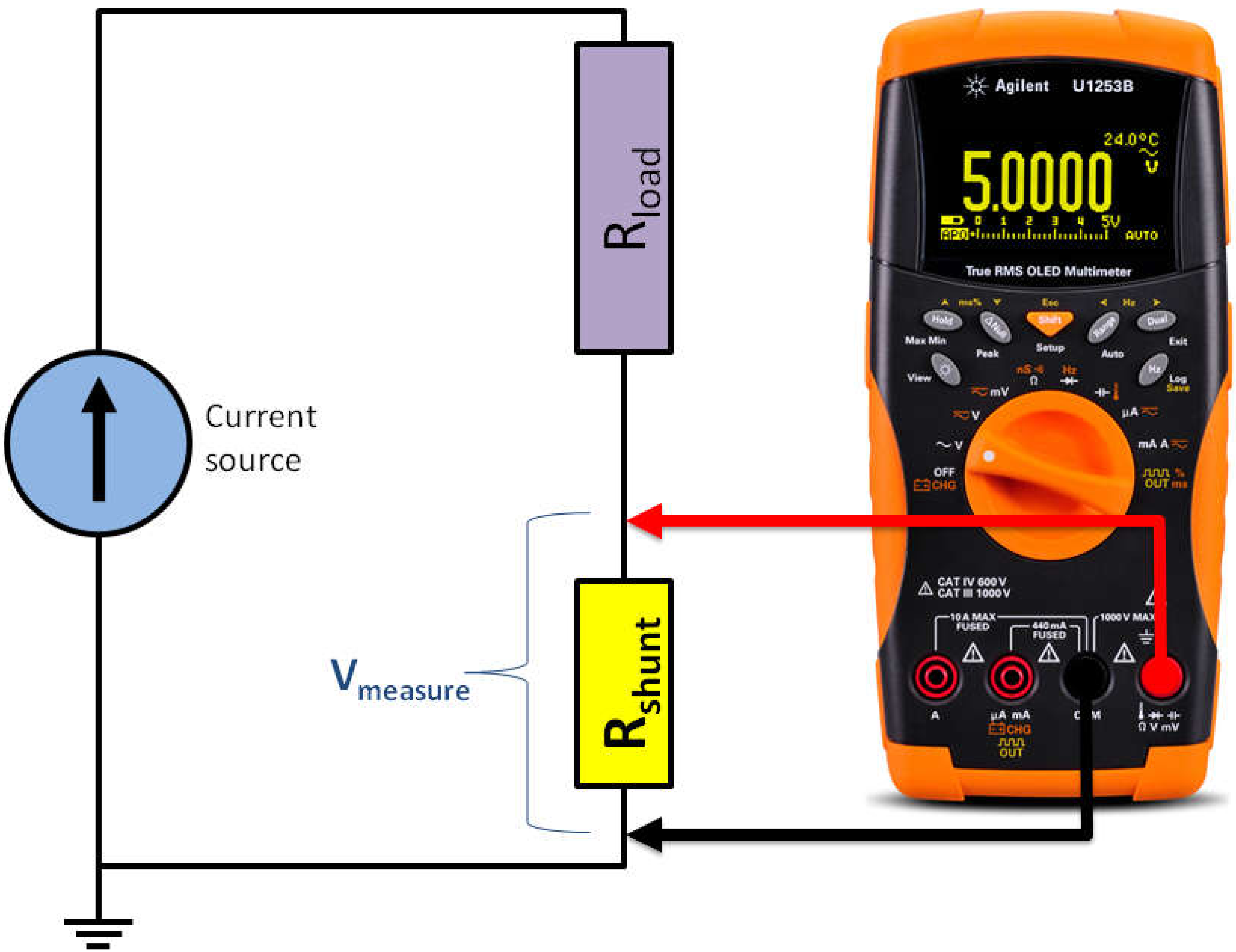


Schematic of Simplified DC Traction Motor Power Control Circuit





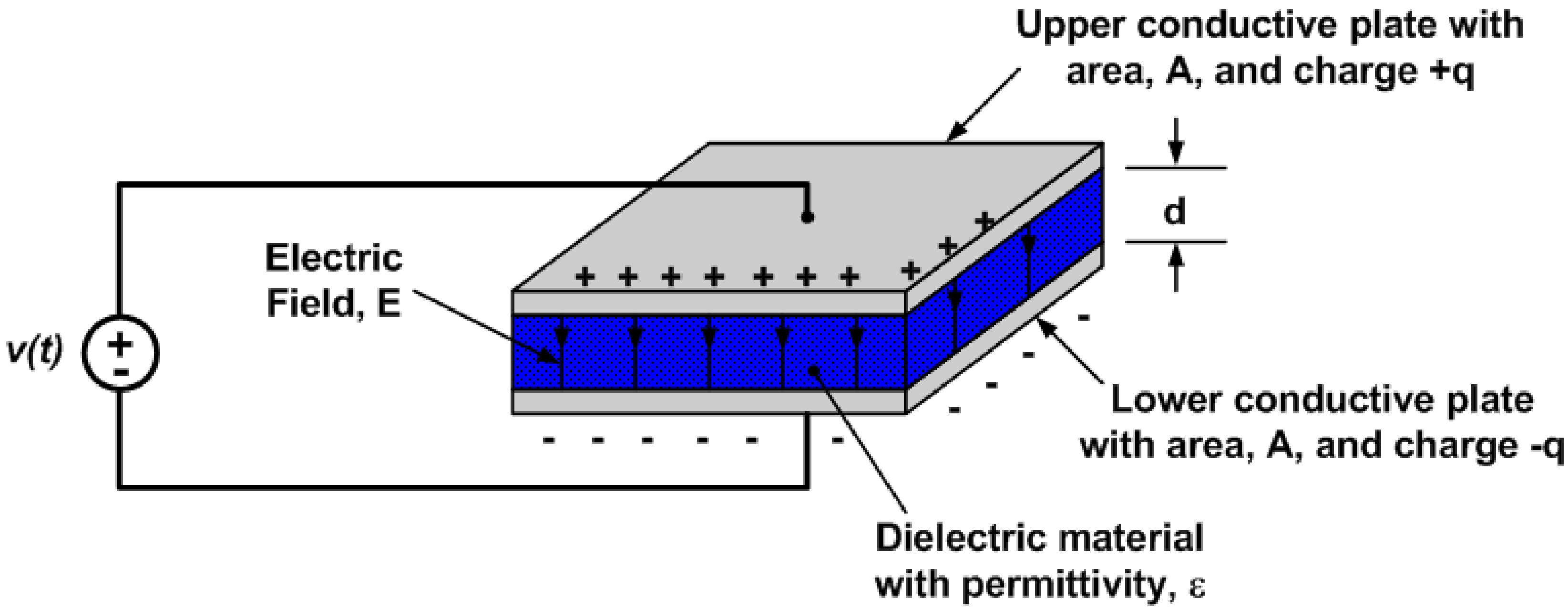




R_{load}
Current source
$V_{measure}$
R_{shunt}
Agilent U1253B
5.0000
True RMS OLED Multimeter

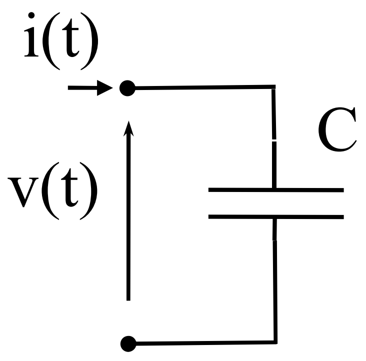
Passive circuit elements – capacitors

- **Capacitors store energy in the form of an electric field**
- **Convert electric energy in dielectric energy (and vice-versa)**
  - **Typically constructed of two conductive materials separated by a non-conductive (*dielectric*) material**



Capacitors

- **Circuit symbol:**



- **C is the capacitance**
  - **Unit is Farad [F]**

- **Voltage-current relation:**

$$i(t) = C\frac{dv(t)}{dt}$$

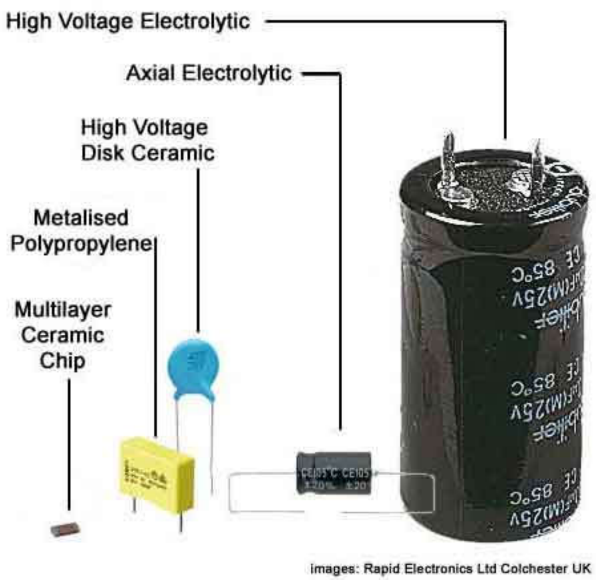
- **Capacitors can store energy (dielectric)**

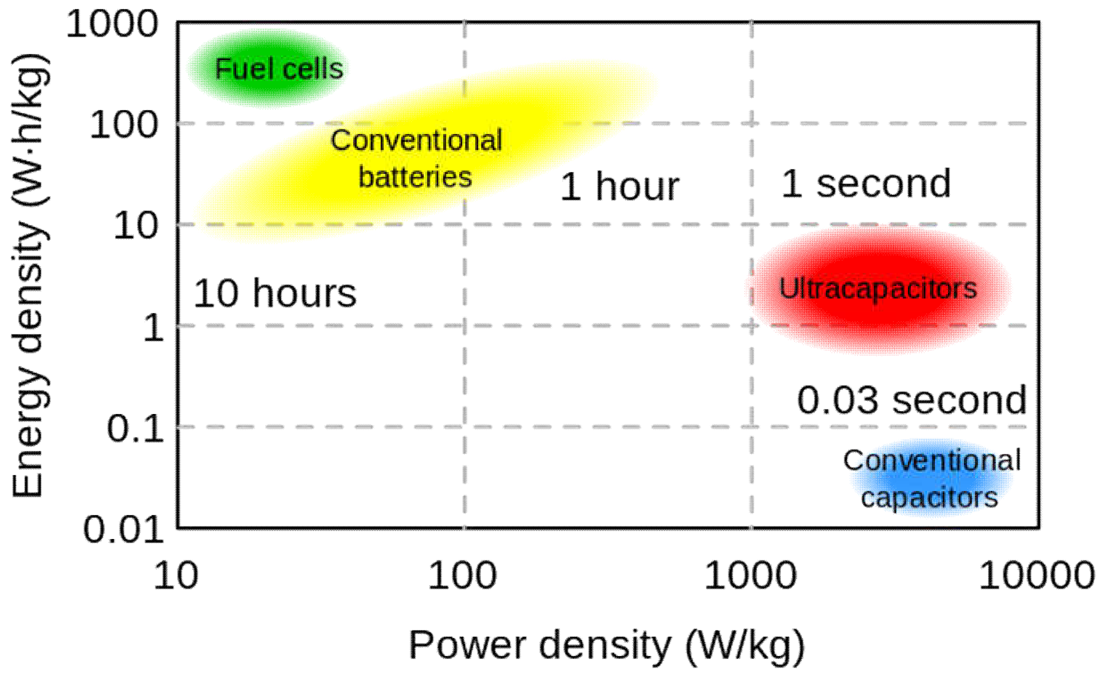
$$W_c = \frac{1}{2}Cv^2$$

Capacitors

- **Notes:**
  - **Capacitors can store energy**
  - **The voltage-current relation is a <u>differential</u> equation**
  - **Capacitance limits <u>rate of change</u> of voltage (filter design)**
  - **If the voltage is constant, the current is zero and the capacitor looks like an open-circuit**

TYPE	CAPACITANCE RANGE	MAXIMUM VOLTAGE	MAXIMUM OPERATING TEMPERATURE (°C)	TOLERANCE (%)	INSULATION RESISTANCE (MΩ)	COMMENTS
Electrolytics Aluminum Tantalum	 1 µF–1 F 0.001–1000 µF	 3–600 V 6–100 V	 85 125	 +100 to −20 ±5 to 20	 <1 >1	Popular, large capacitance, awful leakage, horrible tolerances
Ceramic	10 pF–1 µF	50–1000 V	125	±5 to 100	1000	Popular, small, inexpensive, poor tolerances.
Mica	1 pF–0.1 µF	100–600 V	150	±0.25 to ±5	100,000	Excellent performance; used in high-frequency applications
Mylar	0.001–10 µF	50–600 V		Good	Good	Popular, good performance, inexpensive
Paper	500 pF–50 µF	100,000 V	125	±10 to ±20	100	—
Polystyrene	10 pF–10 µF	100–600 V	85	±0.5	10,000	High quality, very accurate; used in signal filters
Polycarbonate	100 pF–10 µF	50–400 V	140	±1	10,000	High quality, very accurate
Polyester	500 pF–10 µF	600 V	125	±10	10,000	—
Glass	10–1000 pF	100–600 V	125	±1 to ±20	100,000	Long-term stability
Oil	0.1–20 µF	200 V–10 kV			Good	Large, high-voltage filters, long life

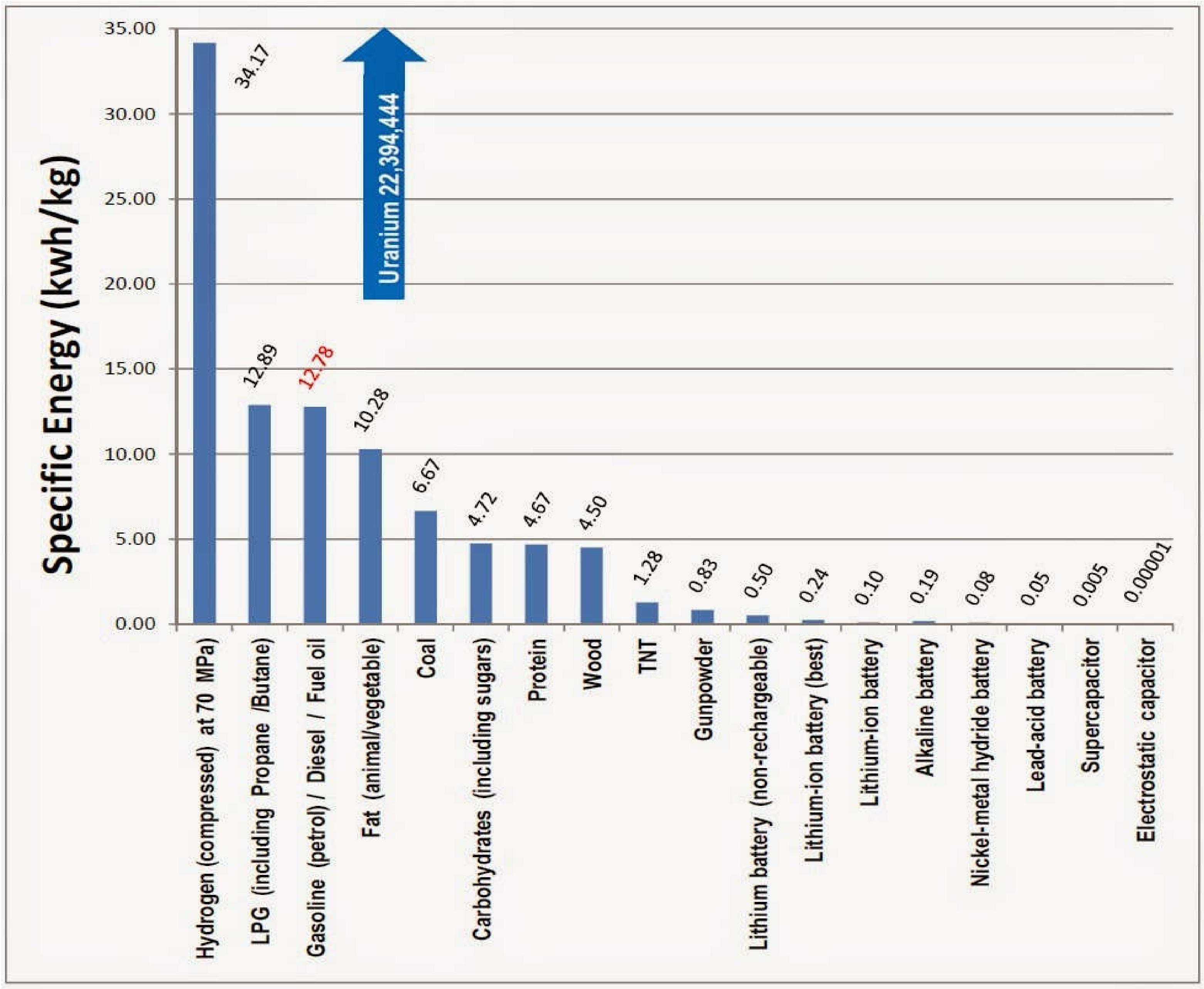




Choose the Ultracapacitor solution that works best for you:

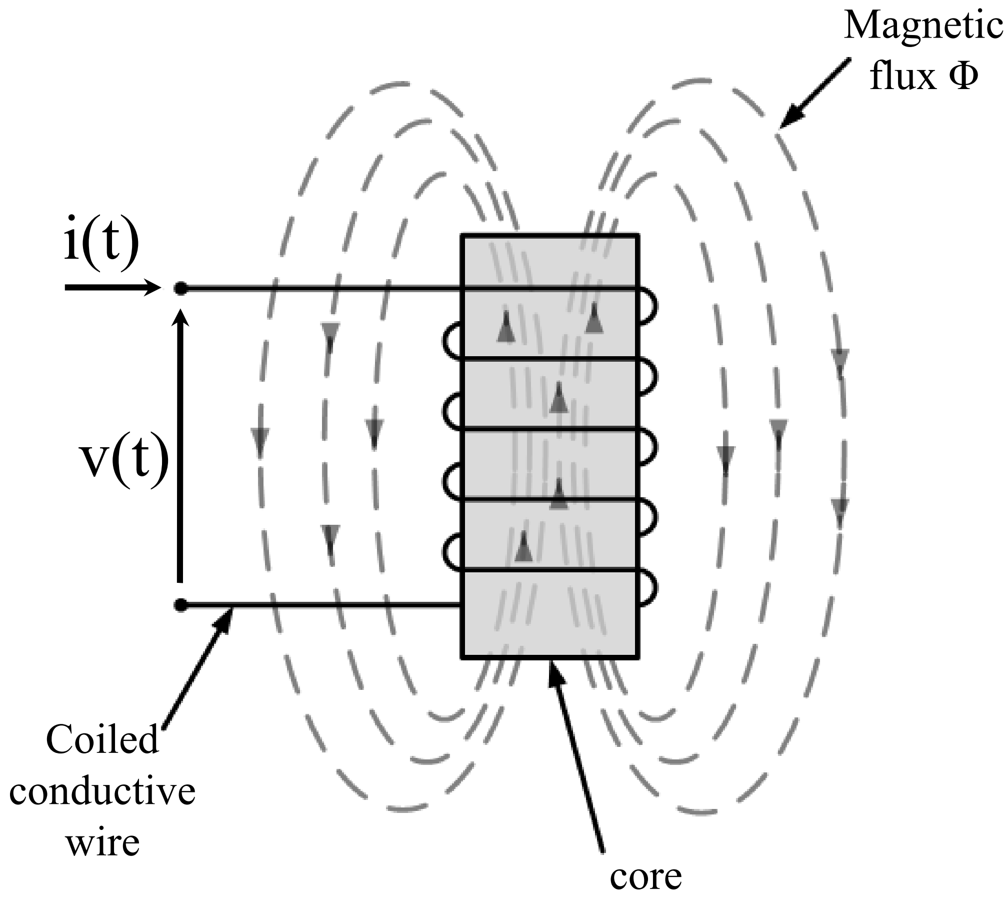
Specifications	BC Series	K2 Series	Modules
Capacitance (F)	310 - 350	650 - 3000	58 - 500
Voltage (V)	2.7	2.7	16 - 125
ESR, DC (mohm)	2.2 - 3.2	0.29 - 0.8	4.1 - 22
Leakage Current (mA)	0.30 - 0.45	1.5 - 5.2	1.5 - 170
Emax (Wh/kg)	5.2 - 5.9	4.1 - 6.0	1.5 - 3.9
Pmax (W/kg)	9,500 - 14,000	12,000 - 14,000	3,600 - 6,800

Available Performance	Lead Acid Battery	Ultracapacitor	Conventional Capacitor
Charge Time	1 to 5 hrs	0.3 to 30 s	10^{-3} to 10^{-6} s
Discharge Time	0.3 to 3 hrs	0.3 to 30 s	10^{-3} to 10^{-6} s
Energy (Wh/kg)	10 to 100	1 to 10	< 0.1
Cycle Life	1,000	>500,000	>500,000
Specific Power (W/kg)	<1000	<10,000	<100,000
Charge/discharge efficiency	0.7 to 0.85	0.85 to 0.98	>0.95
Operating Temperature	-20 to 100 C	-40 to 65 C	-20 to 65 C


Specific Energy (kwh/kg)
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
Uranium 22,394,444
34.17
12.89
12.78
10.28
6.67
4.72
4.67
4.50
1.28
0.83
0.50
0.24
0.10
0.19
0.08
0.05
0.005
0.00001
Hydrogen (compressed) at 70 MPa)
LPG (including Propane /Butane)
Gasoline (petrol) / Diesel / Fuel oil
Fat (animal/vegetable)
Coal
Carbohydrates (including sugars)
Protein
Wood
TNT
Gunpowder
Lithium battery (non-rechargeable)
Lithium-ion battery (best)
Lithium-ion battery
Alkaline battery
Nickel-metal hydride battery
Lead-acid battery
Supercapacitor
Electrostatic capacitor

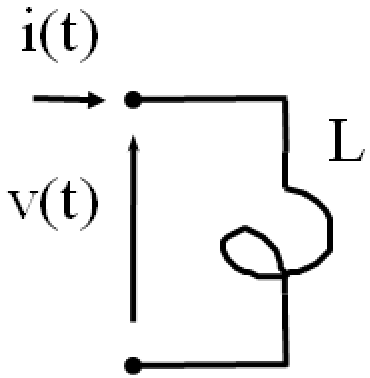
Passive circuit elements - inductors

- **Inductors store energy in the form of a magnetic field**
- **Convert electric energy in magnetic energy (and vice-versa)**
- **Often constructed by coiling a conductive wire around a ferrite core**



Inductors

- **Circuit symbol:**



- **L is the inductance**
  - **Unit is Henry [H]**

- **Voltage-current relation:**

$$v(t) = L\frac{di(t)}{dt}$$

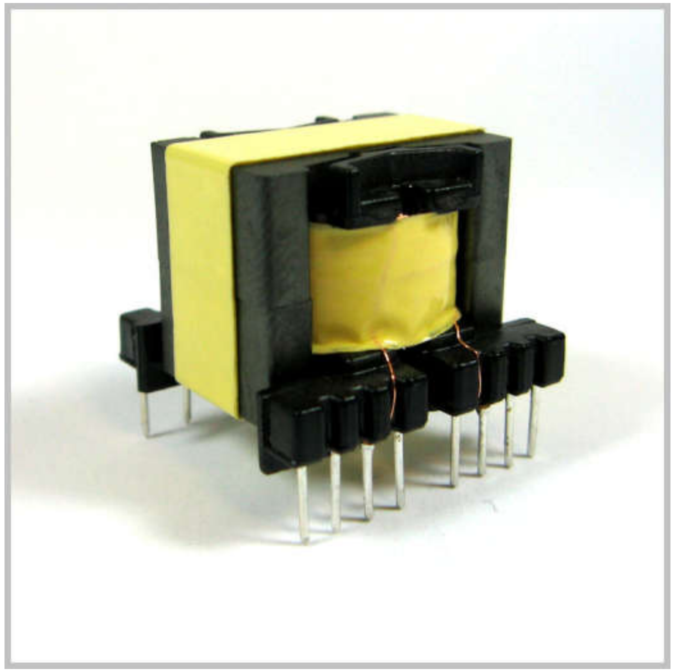
- **Inductors can store energy (magnetic)**

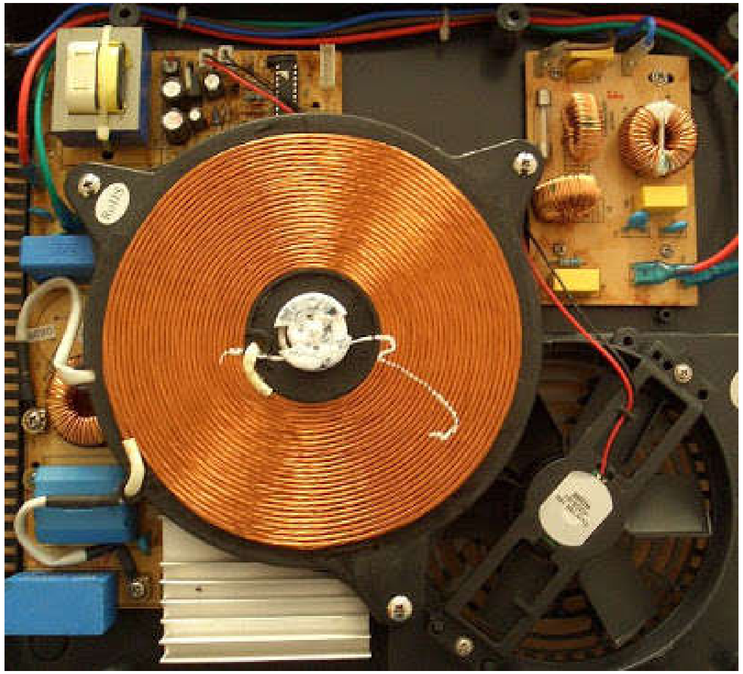
$$W_L = \frac{1}{2}Li^2$$

Inductors

- **Notes:**
  - **Inductors can store magnetic energy**
  - **The voltage-current relation is a <u>differential</u> equation**
  - **Inductance limits <u>rate of change</u> of current (filter design)**
  - **If the current is constant, the voltage difference is zero and the inductor looks like a perfect conductor (short-circuit)**

Francesco Castelli Dezza



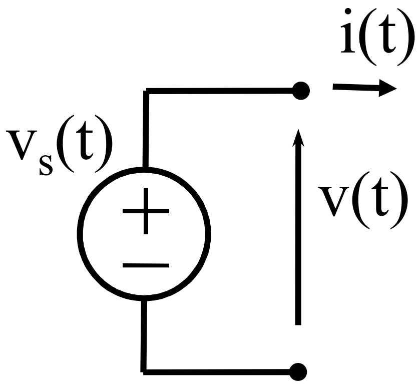


Active Circuit Elements

- **Ideal voltage generator**
- **Ideal current generator**

Ideal voltage generator

- **Circuit symbol:**



- **Unit is Volt [V]**

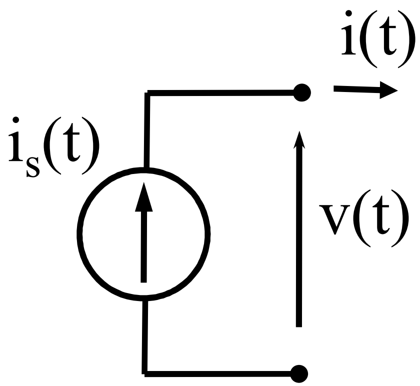
- **Voltage-current relation:**

$$v(t) = v_s(t) \qquad \forall i(t)$$

- **v_s(t) is a known function of the time**
- **Electric energy is converted in other forms (mech., chem.,...) and vice-versa**

Ideal current generator

- **Circuit symbol:**



- **Unit is Ampere[A]**

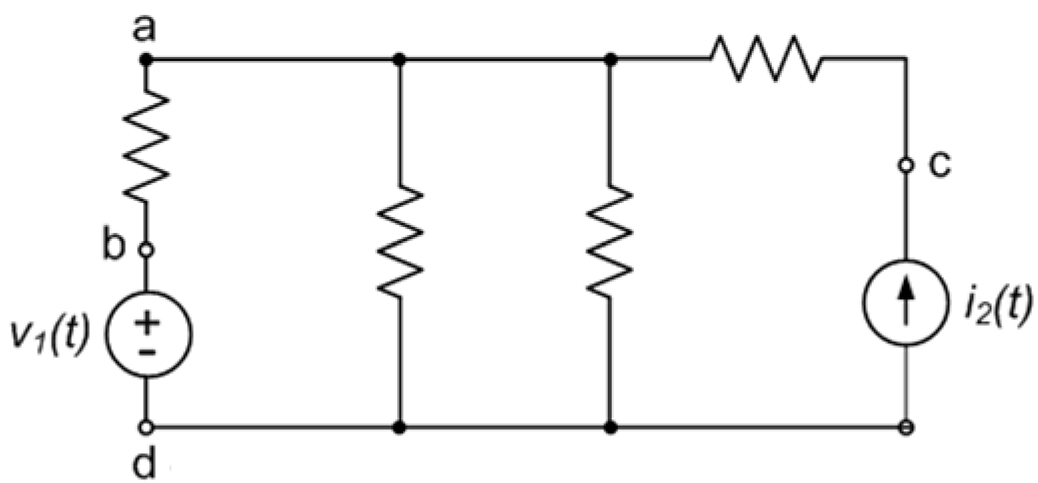
- **Voltage-current relation:**

$$i(t) = i_s(t) \qquad \forall v(t)$$

- **$i_s(t)$ is a known function of the time**
- **Electric energy is converted in other forms (mech., chem.,...) and vice-versa**

Basic Definition – Node

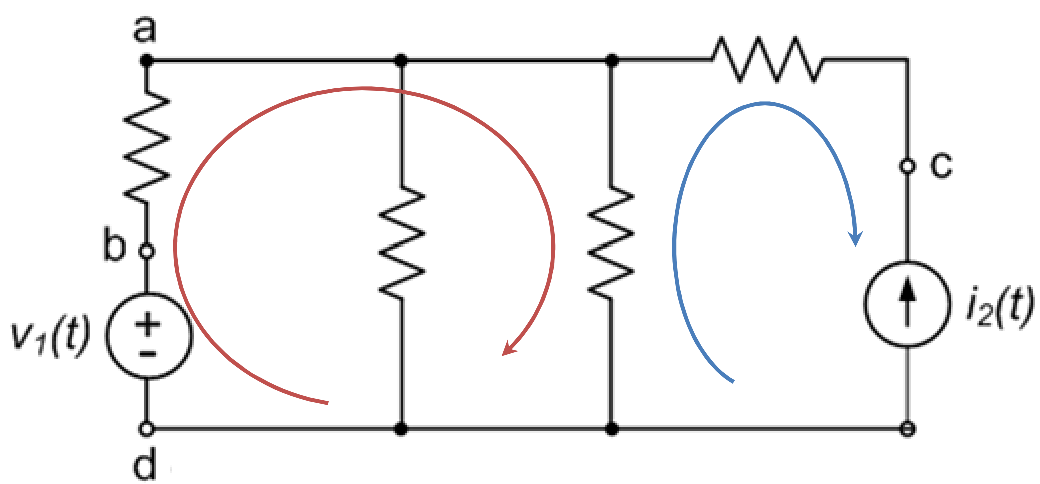
- A *Node* is a point of connection between two or more circuit elements



extract from http://www.digilentinc.com/eeboard/RealAnalog/text/Lecture3.ppt

Basic Definition – Loop (mesh)

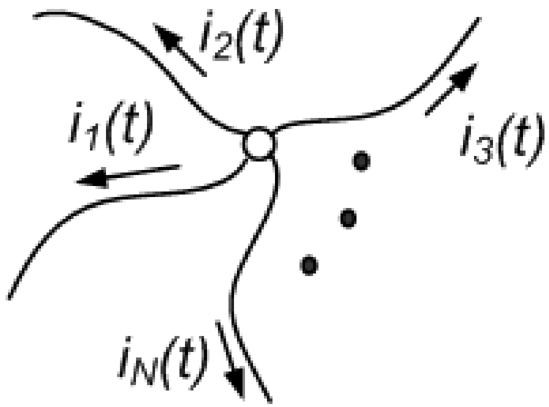
- A ***Mesh*** **is any closed path through the circuit which encounters no node more than once**
  - A ***Loop*** **is a particular mesh with no elements inside**



Kirchhoff's Current Law (KCL)

- **The algebraic sum of all currents entering (or leaving) a node is zero**
  - **Equivalently: The sum of the currents entering a node equals the sum of the currents leaving a node**
  - **Mathematically:**

$$\sum_{k=1}^{N} i_k(t) = 0$$



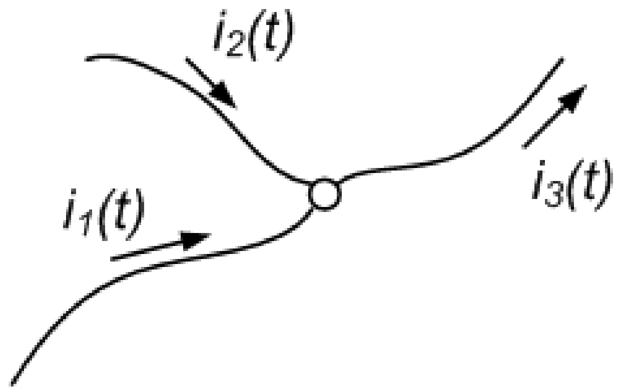
We can't accumulate charge at a node

Kirchhoff's Current Law – continued

- **When applying KCL, the current directions (entering or leaving a node) are based on the <u>assumed</u> directions of the currents**
  - **Also need to decide whether currents <u>entering</u> the node are <u>positive</u> or <u>negative</u>; this dictates the sign of the currents leaving the node**
  - **As long all assumptions are consistent, the final result will reflect the <u>actual</u> current directions in the circuit**

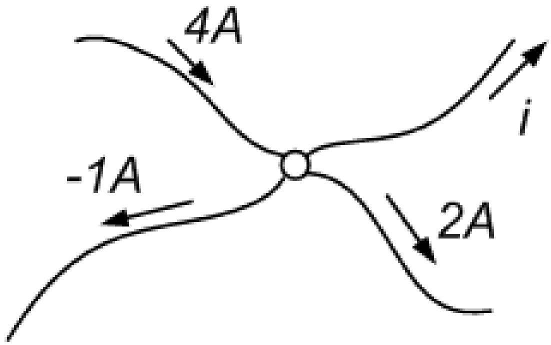
KCL – Example 1

- **Write KCL at the node below:**



KCL – Example 2

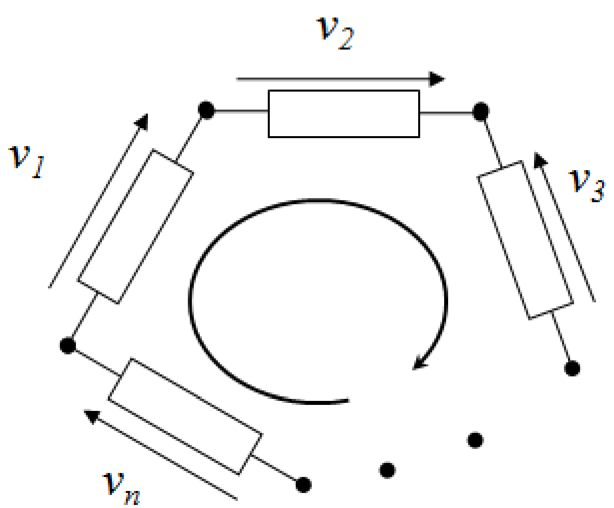
- **Use KCL to determine the current *i***



Kirchhoff's Voltage Law (KVL)

- **The algebraic sum of all voltage differences around any closed loop is zero**
  - **Equivalently: The sum of the voltage rises around a closed loop is equal to the sum of the voltage drops around the loop**
  - **Mathematically:**

$$\sum_{k=1}^{N} v_k(t) = 0$$



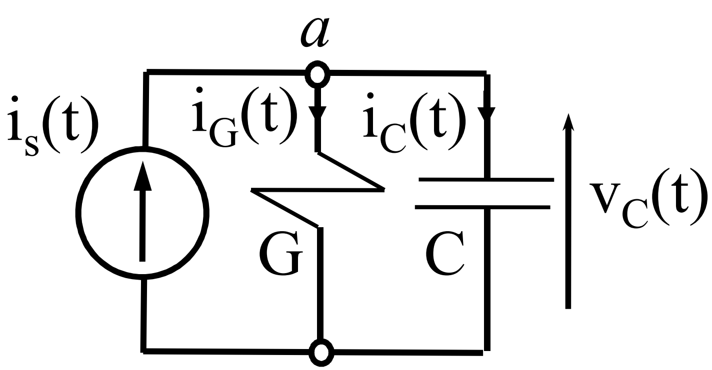
Kirchhoff's Voltage Law – continued

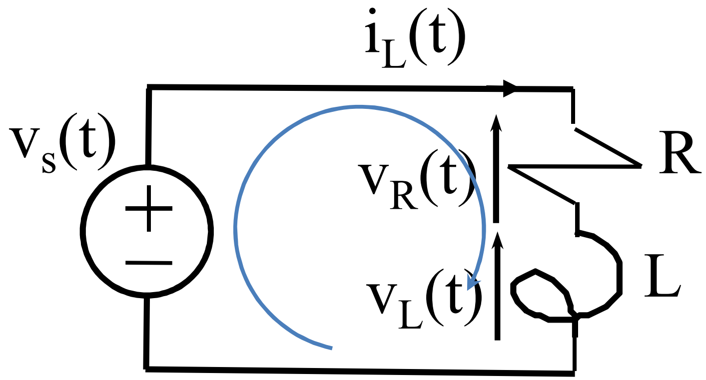
- **Voltage polarities are based on <u>assumed</u> polarities**
  - **If assumptions are <u>consistent</u>, the final results will reflect the <u>actual</u> polarities**
- **To ensure consistency, I recommend:**
  - **Indicate assumed polarities on circuit diagram**
  - **Indicate loop and direction we are traversing loop**
  - **Follow the loop and sum the voltage differences:**
    - **If the voltage difference has the same direction of the loop, treat the difference as <u>positive</u>**
    - **Else, treat the difference as <u>negative</u>**

Circuit analysis – applying KVL and KCL

- **In circuit analysis, we generally need to determine voltages and/or currents in one or more elements**
- **Given *l* elements and *n* nodes, the unknown qualtuities are *2l***
- **We can determine voltages, currents in <u>all</u> elements by:**
  - **Writing a voltage-current relation for each element (Ohm's law, for resistors): *l* equations**
  - **Applying KVL around all loops in the circuit: *l-(n-1)* equations**
  - **Applying KCL at all but one node in the circuit: *n-1* equations**

First Order Transient





KCL at the node *a*:

$$i_s(t) - i_G(t) - i_C(t) = 0$$

$$Gv_C(t) + C\frac{dv_C(t)}{dt} = i_s(t)$$

KVL around the loop:

$$v_s(t) - v_R(t) - v_L(t) = 0$$

$$Ri_L(t) + L\frac{di_L(t)}{dt} = v_s(t)$$

Complete Solution

Voltages and currents in a 1st order circuit satisfy a differential equation of the form (*f(t)* is called the **forcing function**)

$$a\frac{dx(t)}{dt}+bx(t)=f(t)$$

The complete solution is the sum of **particular** solution (forced response or steady state solution) and **complementary** (homogeneous) solution (natural response).

$$x(t)=x_c(t)+x_p(t)$$

The complementary solution has the form (***A*** has to satisfy the **initial** conditions):

$$x_c(t)=Ae^{\lambda t}$$

where λ is the solution to the homogeneous equation (τ is the **time constant**)

$$a\lambda+b=0 \quad \lambda=-\frac{b}{a} \quad \tau=-\frac{1}{\lambda}$$

For an RC circuit, $\tau = C/G = RC$
For an RL circuit, $\tau = L/R$

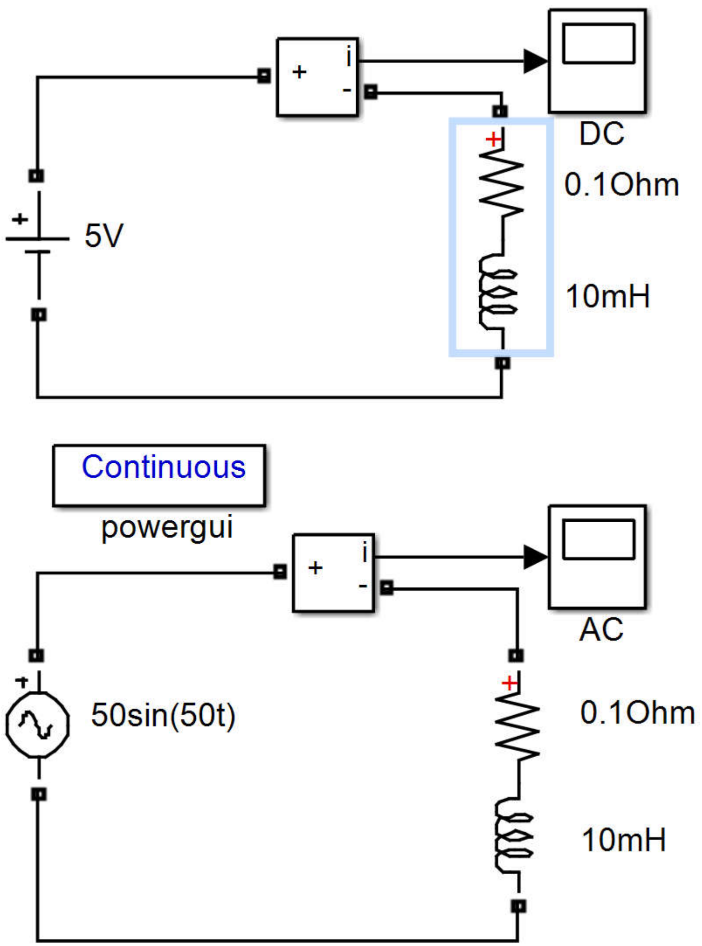
The particular solution

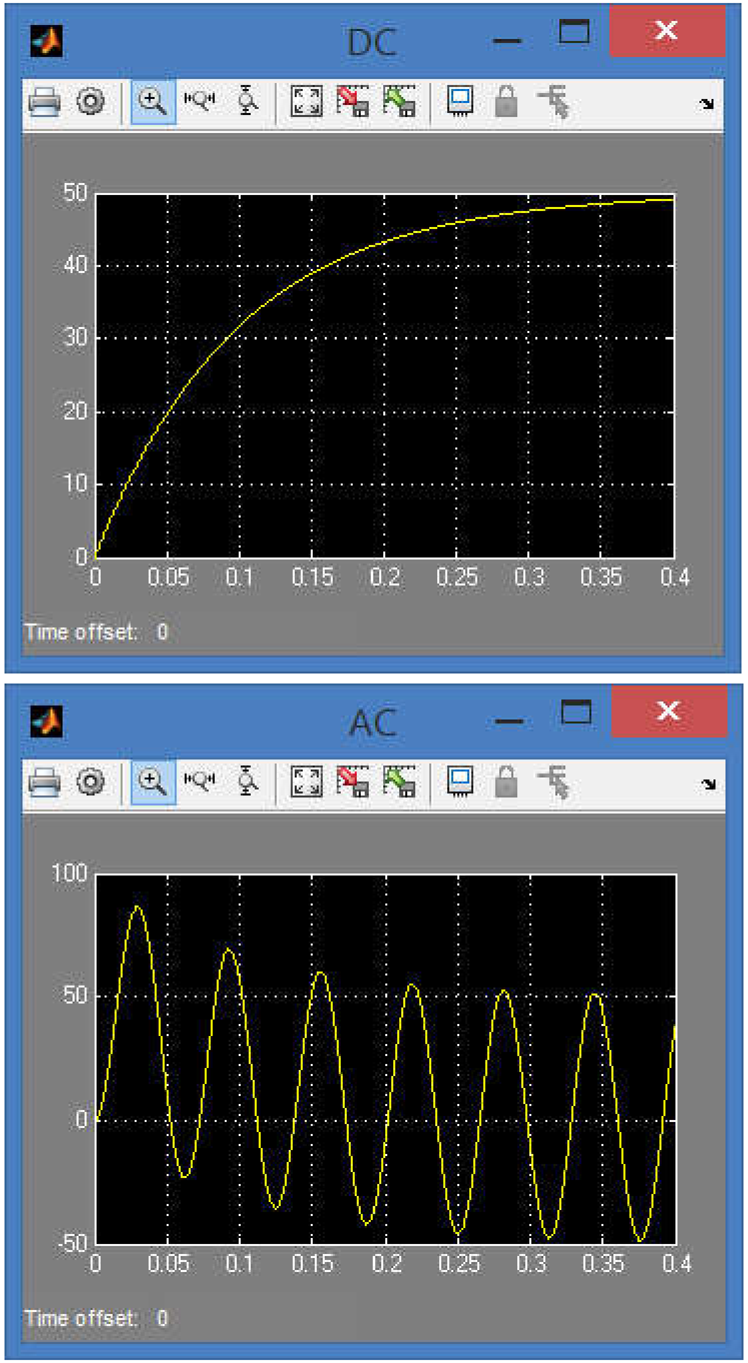
- The particular solution $x_p(t)$ has the same waveform of the forcing term $f(t)$
- If $f(t)$ is constant, then $x_p(t)$ is constant (steady-state solution, constant terms, L is a short circuit, C is an open circuit)
- If $f(t)$ is sinusoidal, then $x_p(t)$ is sinusoidal (sinusoidal steady-state analysis, element L becomes an impedance $Z_L=j\omega L$, C becomes $Z_C=1/(j\omega C)=-j/(\omega C)$)
- Given the initial condition $x_0 = x(0)$

$$x(0) = x_0 = x_c(t) + x_p(t) = Ae^{\lambda 0} + x_p(0)$$

$$A = x_0 - x_p(0) \quad x_c(t) = (x_0 - x_p(0))e^{-\frac{t}{\tau}}$$

Simulink example





Sinusoidal steady-state analysis (a.c.)

- f frequency [Hz]
- ω angular frequency [rad/s]
- φ phase angle (or displacement angle)

- **Phasor:**
  - given the sinusoidal function

$$v(t) = V_{\max}\cos(\omega t + \varphi)$$

  - the corresponding phasor is

$$\overline{V} = V_{rms} e^{j\varphi}$$

  - where

$$V_{rms} = \frac{V_{\max}}{\sqrt{2}}$$

Root mean square definition

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T R\cdot i(t)^2\,dt = R\cdot\frac{1}{T}\int_0^T i(t)^2\,dt = R\cdot I_{rms}^2$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T i(t)^2\,dt \Rightarrow I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2\,dt} \quad V_{rms}^2 = \frac{1}{T}\int_0^T v(t)^2\,dt \Rightarrow V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

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AC voltage-current relations

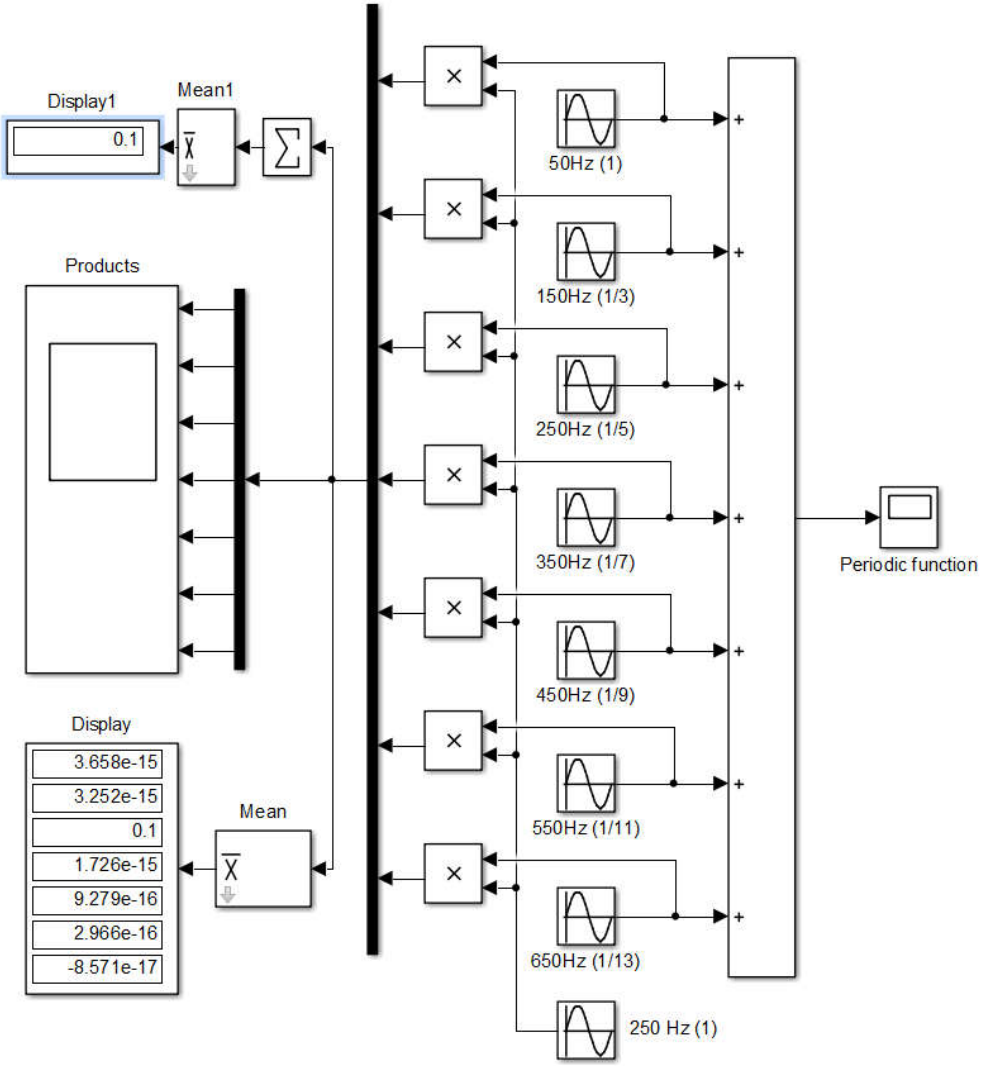
- **Differential operators**

$$\frac{d}{dt} = j\omega \qquad \int dt = \frac{1}{j\omega} = -\frac{j}{\omega}$$

- **R** $\quad \overline{V} = R\overline{I} = \overline{Z_R}\,\overline{I}$
- **L** $\quad \overline{V} = j\omega L\overline{I} = jX_L\overline{I} = \overline{Z_L}\,\overline{I}$
- **C** $\quad \begin{cases} \overline{I} = j\omega C\overline{V} = jB_C\overline{V} = \overline{Y_C}\,\overline{V} \\ \overline{V} = -\dfrac{j}{\omega C}\overline{I} = -jX_C\overline{I} = \overline{Z_C}\,\overline{I} \end{cases}$

- **X reactance**
- **B susceptance**
- **Z impedance**
- **Y admittance**

Fourier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0.1
Mean1
Products
Display
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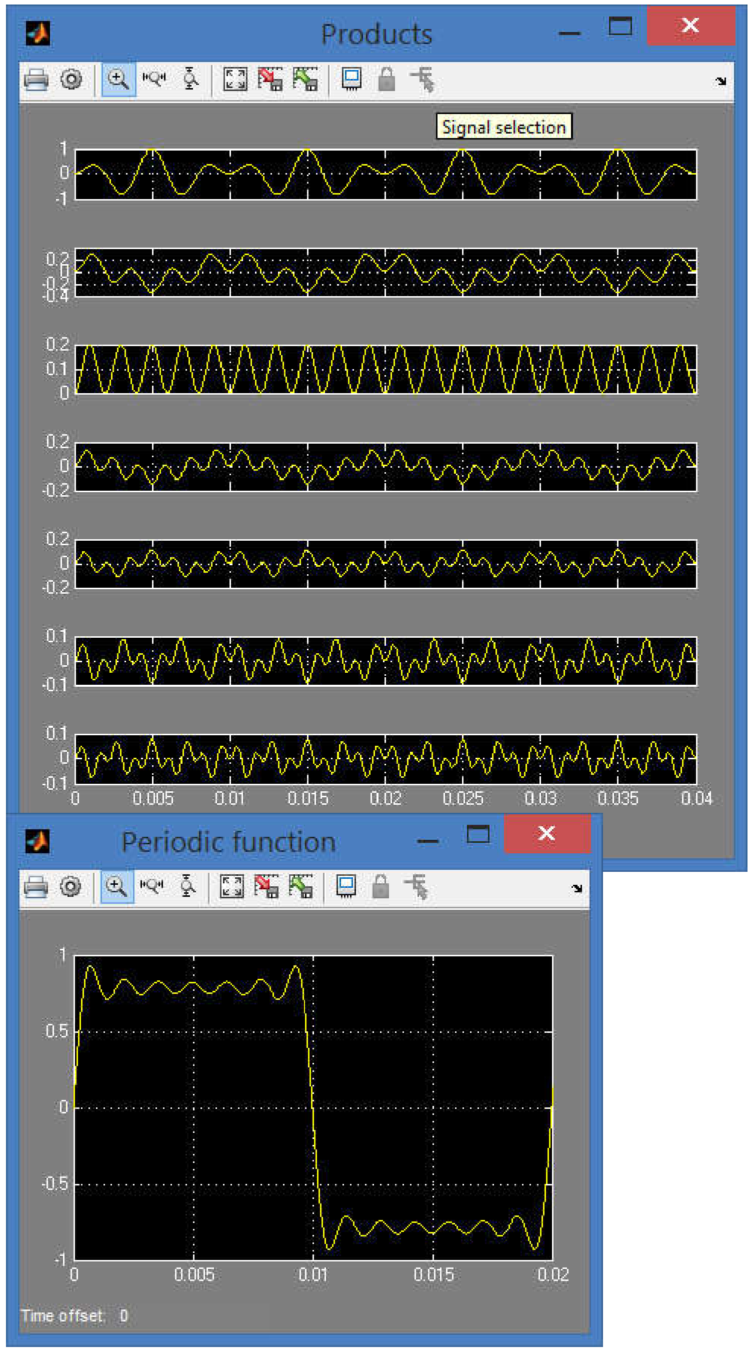
3.252e-15
0.1
1.726e-15
9.279e-16
2.966e-16
-8.571e-17
Mean
50Hz (1)
150Hz (1/3)
250Hz (1/5)
350Hz (1/7)
450Hz (1/9)
550Hz (1/11)
650Hz (1/13)
250 Hz (1)
Periodic function


Products
Signal selection
Periodic function
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