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Introduction to Simulation

Dynamical Models and Current Control Methods

Dynamics of Electrical Machines and Drives

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Final exam

- Do one of the suggested exercises in a report-form **(mandatory)**
- If correctly done, you will have one question less at the oral exam
- If not, good luck!

Systems Theory

Dynamical System

Definition

a **dynamical system** describes the evolution of a state over time

we need to specify what we mean for "evolution", "state" and "time":

1. ***continuous time*** $t \in \mathbb{R}$
   the evolution of the state is described by *ordinary differential equations* (ODE). Think of the linear, continuous time system in state space form

$$\dot{x}(t) = Ax(t)$$

2. ***discrete time*** $k \in \mathbb{Z}$
   the evolution of the state is described by a difference equation. Think of the linear discrete time system in state space form

$$x(k+1) = Ax(k)$$

Models for Continuos Systems

Dynamical Model

- A **dynamical model** of a system is a set of *mathematical laws* explaining in a compact form and in quantitative way how the system evolves over time

dynamical model $\Leftrightarrow$ mathematical model

$$v(t) = Ri(t) \qquad i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{dt}} \qquad v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{dt}}$$

- Main questions about dynamical system and their model:
  1. How to built a model (“How X and Y influence each other ?”)
  2. Simulation (“What happens if I apply action Z on the system ?”)
  3. Design (“How to make the system behave the way I want ?”)

Simulation of Dynamical Models

Conflicting Objectives

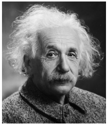
Experiments provide an answer, but have limitations:

- maybe too expensive (e.g.: launch a space shuttle)
- maybe too dangerous (e.g.: a nuclear plant)
- maybe impossible (the system doesn't exist yet!)

Simulating a dynamical model has zero-cost compared to real experiments ...but has conflicting objectives

- *Descriptive enough* to capture the main behavior of the system
- *Simple enough* for analyzing the system

"Make everything as simple as possible, but not simpler." - *Albert Einstein*



A. Einstein
(1879-1955)

MATLAB/Simulink

an Equation Solver (ES)

we are going to use **MATLAB as simulation environment**

it is based on matrix algebra, developed from Prof. Cleve Moler (for his students, as Fortran's interface in early '80s)

C. Moler (founder)

- it's simple to implement control systems using **Simulink**

Good Things

- writing of mathematical equations
- library ready-to-use (e.g. SimPowerSystems)

Bad Things

- manage computational error (e.g. numerical integration)
- simulation time t_s could differ from real time t (e.g. $t_s = 10s \leftrightarrow t = 1h$)

MATLAB

use MATLAB 2016b version (http://software.polimi.it)

Basics for Electrical Drives Simulation

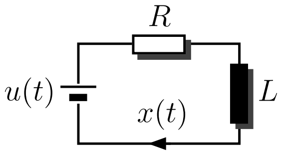
Current Control of an RL load

- electrical machines windings behaves as ***RL*** **(ohmic-inductive) load**
- controlling the *current flow* into windings imply *control the torque*

$$\textit{Kirchhoff's}\ \text{voltage law}: \quad v(t) - L\frac{\mathrm{d}i(t)}{\mathrm{d}t} - Ri(t) = 0$$

1. Rewrite it as a 1^{st} order linear system

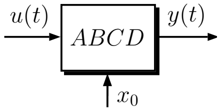
$$\frac{\mathrm{d}i(t)}{\mathrm{d}t} = \frac{1}{L}v(t) - \frac{R}{L}i(t)$$



1. or in a **state-space form**

$$\dot{x}(t) = \underbrace{-R/L}_{\boldsymbol{A}}\, x(t) + \underbrace{1/L}_{\boldsymbol{B}}\, u(t)$$

where $x(t) = i(t)$ and $u(t) = v(t)$



Basics for Electrical Drives Simulation

Evolution of the State (1^{st}order LTI)

1. We want to **observe** and control the currents $x(t) = i(t) \;\rightarrow\; y(t) = x(t)$
2. Voltage $u(t) = v(t)$ is a **forcing signal**
3. Adding the information about **initial conditions** $x_0 \in \mathbb{R}^n$

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \\ x(0) &= x_0 \end{cases}$$

where

$$A = -\frac{R}{L} \quad B = \frac{1}{L} \quad C = 1 \quad D = 0$$

- the **evolution of the state** $x(t) = y(t)$ is

$$x(t) = \underbrace{e^{At}x_0}_{\substack{\text{natural response} \\ \text{(effect of initial condition)}}} + \underbrace{\int_0^t e^{A(t-\tau)}B\,u(\tau)\,\mathrm{d}\tau}_{\substack{\text{forced response} \\ \text{(effect of input signal)}}}$$

- if $A < 0 \;\Rightarrow$ **asyntoptically stable**

Basics for Electrical Drives Simulation

Transfer Functions

- **Laplace transforms** convert integral and differential equations into algebraic equations through the *Laplace operator*
$$F(s) = \mathcal{L}\left[f(t)\right] \quad \leftarrow \quad s = \mathrm{d}/\mathrm{dt}$$
  - It can be demonstrated that

P.S. Laplace
(1749-1827)

$$y(s) = \underbrace{C\,(sI - A)^{-1}\,x_0}_{\substack{\text{Laplace transform} \\ \text{of natural response}}} + \underbrace{\left(C(sI - A)^{-1}B + D\right)u(s)}_{\substack{\text{Laplace transform} \\ \text{of forced response}}}$$

Definition

The **transfer function** $G(s)$ of a continuous-time linear system (A, B, C, D) is

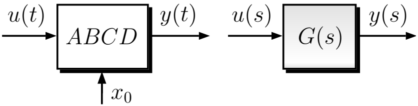
$$G(s) = C(sI - A)^{-1}B + D$$

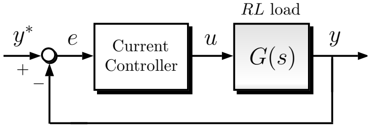
between the Laplace transform $y(s)$ of output and the Laplace transform $u(s)$ of the input signals for the *initial state* $x_0 = 0$

Basics for Electrical Drives Simulation

Current Control PI-based

Note that $y(s) = i(s)$ and $u(s) = v(s)$





RL load - $G(s)$

$$G(s) = \frac{y(s)}{u(s)} = \frac{1}{R + sL}$$

Current Controller - $R(s)$

- P, PI, PID ☑
- Optimal Control

- The RL load to be controlled has a time constant $T_G = L/R$
- The controller settings are chosen according to:
  1. **Time response** → **Bandwidth** ω_c
  2. **Robustness level** → **Phase margin** φ_m
  3. **Sensitivity to disturbances** (and actuations) → $S(s)$

P-Action

Minimum Phase Systems

- Consider a P controller $R(s) \rightarrow u(s) = k_p e(s)$
- The design will be based on the **open-**, **closed-loop** transfer functions

$$L(s) = R(s)G(s) = \frac{k_p}{R + sL} \qquad F(s) = \frac{L(s)}{1 + L(s)} = \frac{k_p}{k_p + R + sL}$$

but $L(s) = u(s)/e(s)$ is included in the category of:

Minimum Phase Systems (mps)

If an LTI system presents: *positive gain*, *poles* $\Re(p) < 0$, *zeros* $\Re(z) < 0$ it is a **minimum phase system**

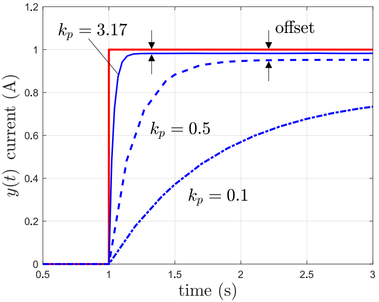
- Then, it is enough an $L(s)$ cutting the 0dB axis with a slope -20dB/decade to guarantee $\varphi_m \cong 90°$ and asymptotic stability

- Bandwidth is $|L(j\omega_c)| = 1 \;\; \Rightarrow \;\; \omega_c = \sqrt{k_p^2 - R^2}/L$

Property of P-Action #1

Step Response

- **Data**: $R = 0.025$ and $L = 0.1H \;\rightarrow\; \tau_G = 4s$ and $T_A^{(G)} = 5\,\tau_G = 20s$
- **Desired characteristics:** $T_A^{(F)} = 1s \;\rightarrow\; \tau_F = T_A^{(F)}/\,5 = 0.2s$



1. dominant pole

$$\omega_c = 2\pi/\tau_F = 31.4\,\mathrm{rad/s}$$

2. proportional gain

$$k_p = \sqrt{R^2 + \omega_c^2 L^2} = 3.17\ (\Omega)$$

increasing k_p the reference-tracking is better but we still have an offset (it could be 0 only if $k_p \rightarrow \infty$)

- The offset is computed through the **final value theorem**

$$\textbf{(f.v.t.)} \qquad \lim_{s\to 0} s\,F(s)\,\frac{1}{s} = \lim_{s\to 0}\frac{k_p}{k_p + R + sL} = \frac{k_p}{k_p + R}$$

PI-Action

Current Control of an RL load

- Consider a PI controller $R(s) \rightarrow u(s) = (k_p + k_i/s)\, e(s)$

$$L(s) = \frac{sk_p + k_i}{s\,(R + sL)} \qquad F(s) = \frac{sk_p + k_i}{k_i + s\,(k_p + R) + s^2 L}$$

- Exploit the relationship between $L(s)$ and $F(s)$

$$|F(s)| = \frac{|L(s)|}{|1 + L(s)|} \quad \begin{cases} 1 & \forall\omega : \ |L(s)| \gg 1 \quad \rightarrow \ \omega \ll \omega_c \\ |L(s)| & \forall\omega : \ |L(s)| \ll 1 \quad \rightarrow \ \omega \gg \omega_c \end{cases}$$

- It can be demonstrated

$$|F(j\omega_c)| = \frac{|L(j\omega_c)|}{|1 + L(j\omega_c)|} = \frac{1}{2\sin\left(\varphi_m/2\right)} \quad \underset{(\mathbf{mps})}{\xrightarrow{\varphi_m \cong 90^\circ}} \quad |F(j\omega_c)|^2 = \frac{1}{2}$$

explicit characteristic equation from $F(j\omega)$

$$|F(j\omega_c)|^2 = \frac{1}{2} \quad \rightarrow \quad k_i^2 + 2\omega_c^2 k_i L + \omega_c^2 k_p^2 - \omega_c^2\left(R^2 + 2k_p R\right) - \omega_c^4 L^2 = 0$$

Property of PI-Action #1

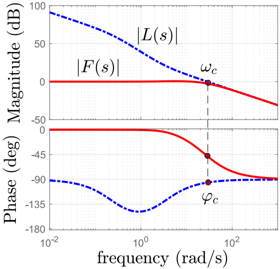
Current Control of an RL load

1. The solution of the 2nd order eq. leads to

$$\boldsymbol{k_i} = \underbrace{-\omega_c^2 L \pm \omega_c \sqrt{2\omega_c^2 L^2 - k_p^2 + R^2 + 2k_p R}}_{>0}$$

2. Note that k_i must be $k_i > 0 \cap k_i \in \mathbb{R}$, then

$$R - \sqrt{2R^2 + \omega_c L^2} < k_p < \underbrace{R + \sqrt{2R^2 + \omega_c L^2}}_{k_{p,\max}}$$



$$k_{p,\max} = R + \sqrt{2R^2 + \omega_c L^2}$$

- a **practical rule** is $\boldsymbol{k_p} = 0.9 k_{p,\max}$

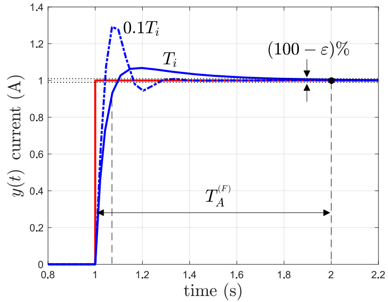
$$\varphi_m = \pi - |\angle L(j\omega_c)| \rightarrow 90^\circ$$

$$\angle L(j\omega_c) = \frac{\pi}{2} - \operatorname{atan}\left(\frac{k_i}{\omega_c k_p}\right) - \operatorname{atan}\left(\frac{L}{R}\right)$$

Property of PI-Action #2

Current Control of an RL load

- An alternative form $R(s) = k_p\,(1 + sT_i)\,/sT_i$ where $T_i = k_p/k_i$
- **Desired characteristics**: $T_A^{(F)} = 1s \;\rightarrow\; \tau_F = T_A^{(F)}/5 = 0.2s$



1. proportional/integral gains

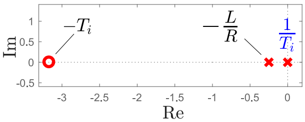
   $k_p = 3.17\ (\Omega) \quad k_i = 9.03\ (\Omega s)$

1. dominant pole

   $$\omega_c = 2\pi/\tau_F = 31.4\,\text{rad/s}$$

   that is related to T_i as

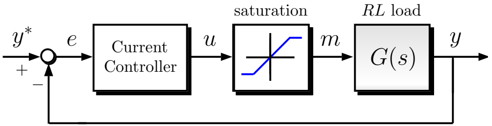
   $$\omega_c = 2\pi/T_i \;\rightarrow\; T_i = 2\pi/\omega_c$$

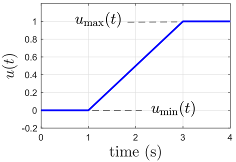


- higher is the integral action, faster is the control but with an oscillating response

Nonlinear Effects

Actuator Saturation





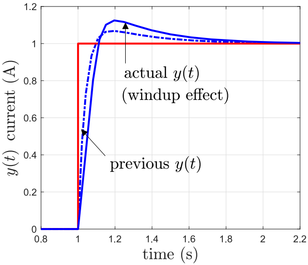
- A linear controller is simple to design, but the performances are good as long as dynamics remain *close* to linear theory
- Nonlinear effects require care, such as **actuator saturation**
  - Saturation can be defined as the static nonlinearity

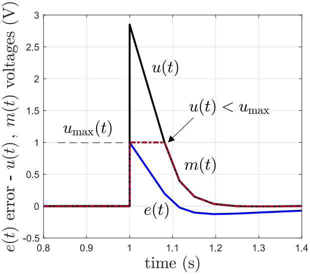
$$\mathrm{sat}\left(u(t)\right) \begin{cases} u_{\min}(t) & \text{if} \quad u(t) < u_{\min}(t) \\ u(t) & \text{if} \quad u_{\min}(t) < u(t) < u_{\max}(t) \\ u_{\max}(t) & \text{if} \quad u(t) > u_{\max}(t) \end{cases}$$

$u_{\min}(t)$ and $u_{\max}(t)$ are min and max allowed actuation signals (e.g. $\pm 1\,V$ for a voltage supply)

The Windup Problem

Integrating Error (Windup)



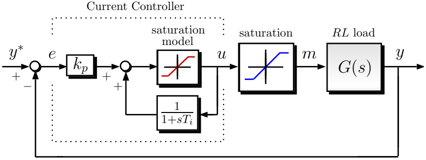


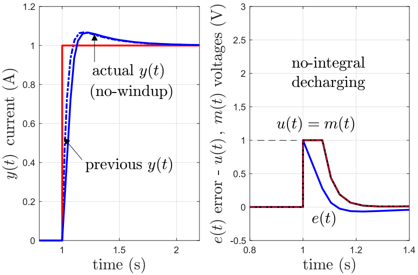
- The output $y(t)$ takes a longer time to achieve steady-state due to "*windup*" of the integrator (higher peak response)
- If $e(t)$ increase, $u(t)$ increase even if $m(t)$ is saturated to u_{max}, the PI **keeps integrating the tracking error** $e(s) \rightarrow$ producing $u(t) \neq m(t)$
- If $e(t)$ change sign we have to wait until $u(t) < u_{max}$ before to come back into linear region $m(t) = u(t)$, this means wait for *integral discharge*

Anti-Windup Techniques

Saturation Model

- **Practical issue?** A controller without any information about saturation
- There are several techniques that have in common the idea to augment the controller with *saturation information* (or dynamical model)





- This scheme has no effect when the actuator is not saturating, while keeps same behavior between $u(t)$, $m(t)$ when $e(t)$ change sign
- Integrator windup is avoided thanks to back-info

Exercise

Current Control of an RL load

An RL load, $R = 1\Omega$ and $L = 1mH$, is fed by a voltage supply $(\pm 30V)$. Design a PI controller in order to follow:

1. a step-command from 0 to $10A$ in $1s$
2. a sinusoidal-command of $10A$ at $5Hz$

Hint:

- Write a MATLAB script to compute k_p and k_i given R, L, and ω_c
- Test different ω_c to verify the previous choice