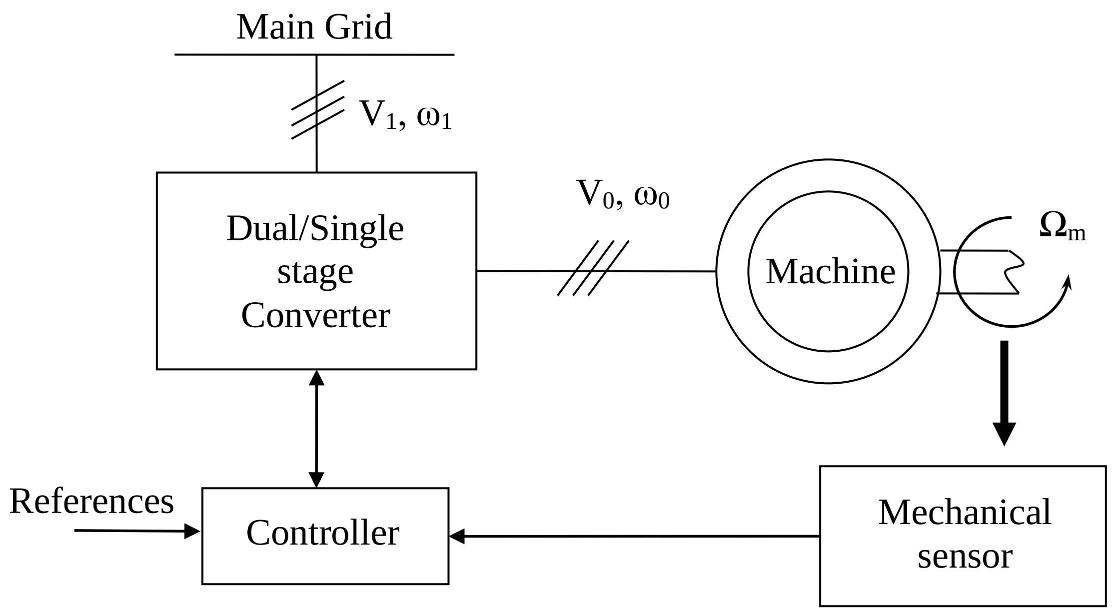
Summary

Chapter 1
DC Machine Drives

1.1 Electrical drive definition

A power drive system is a system which converts electric energy into mechanical energy, by making use of power electronic apparatuses, following an input function (and according to established programme). CEI 301-1



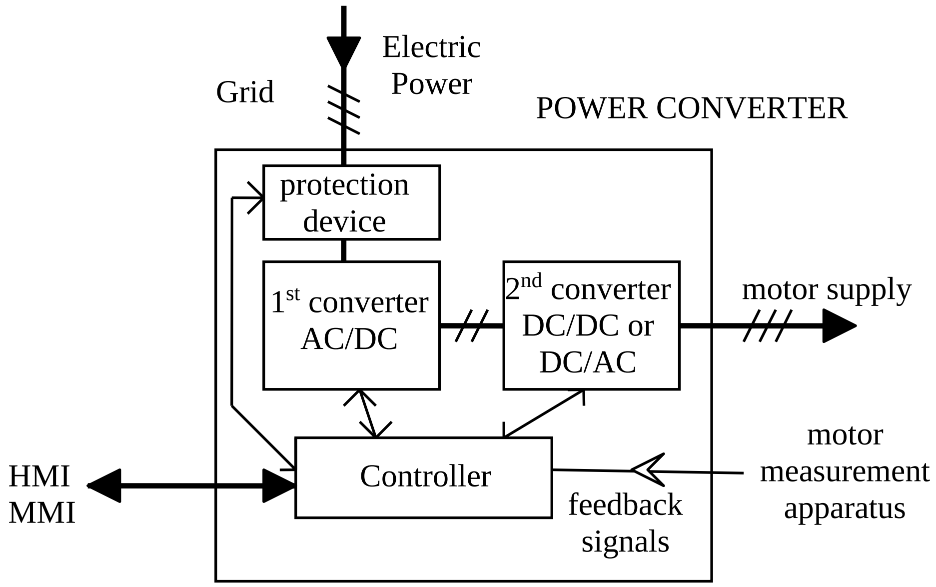
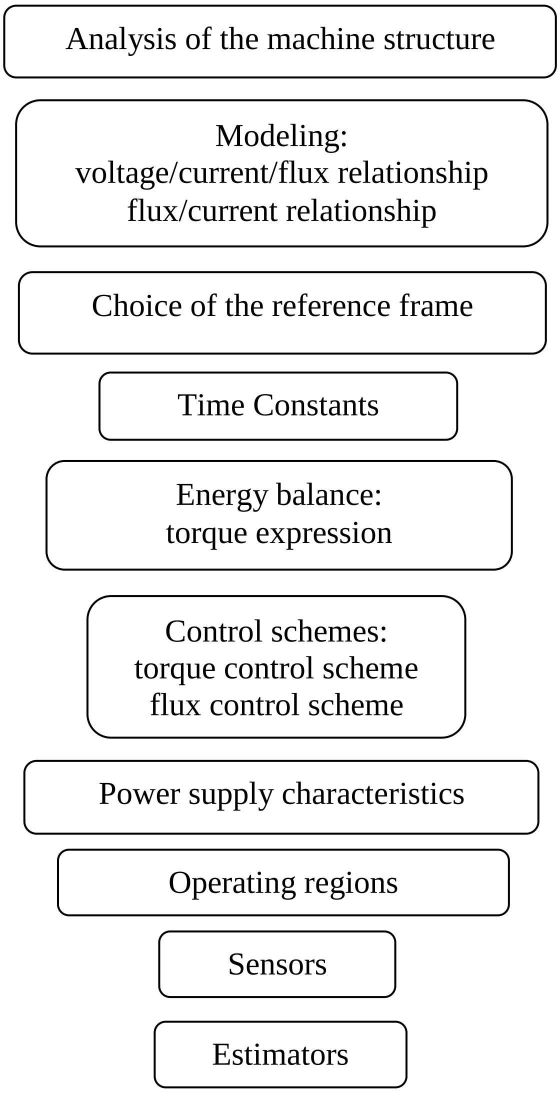


Figure 1-1 Double stage Power converter

1.2 Procedure

In order to study a drive, you have to adopt the following procedure:



1.3 Introduction

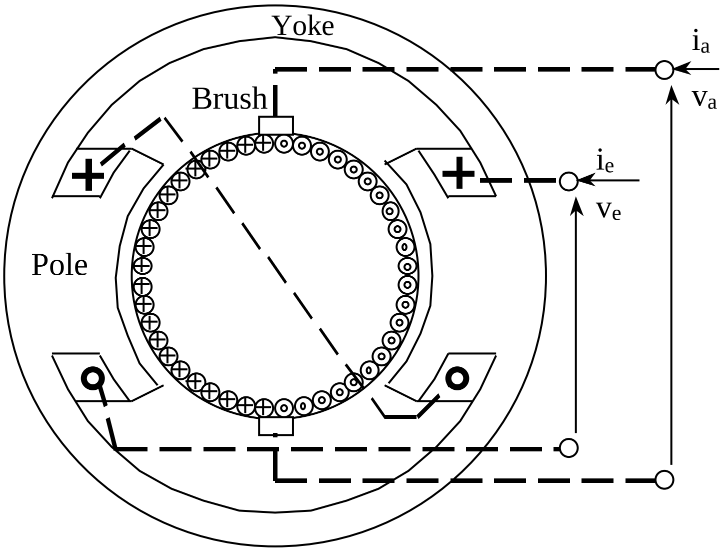


Figure 1-2. Two poles, separately excited, DC machine cross-section

Structurally, a d.c. machine consists of:

- an external stator with salient poles which acts as an inducer
- an internal rotor which acts as an armature.

Around the polar body it will have a coil into which a current is flowing; it is made by turns connected in series, forming the excitation or "main" field winding.

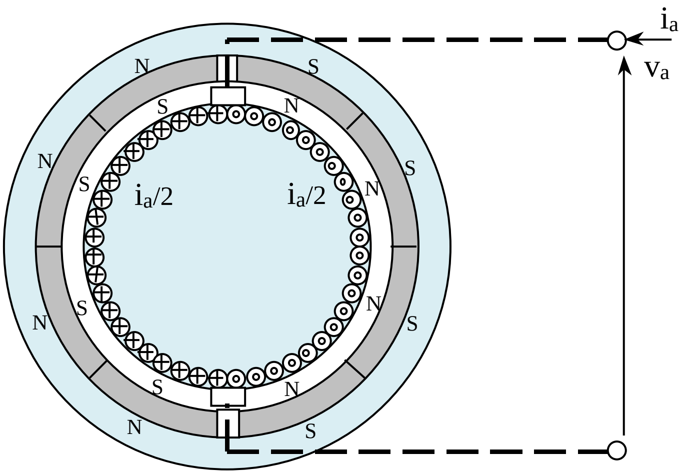


Figure 1-3. Two poles PM DC machine cross-section

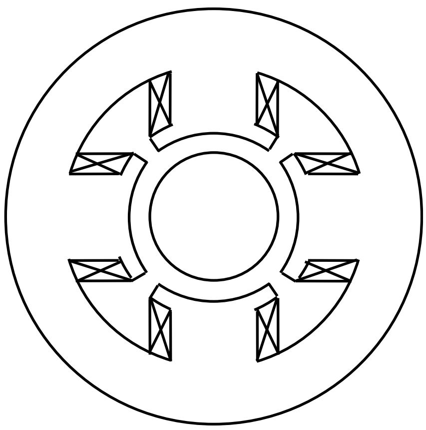


Figure 1-4. Four poles, separately excited, DC machine cross-section

The armature consists of a cylinder made of laminated ferromagnetic material with distributed slots on the periphery. In these slots a closed coil is wound. On the rotor another device is also mounted; it is called "commutator".

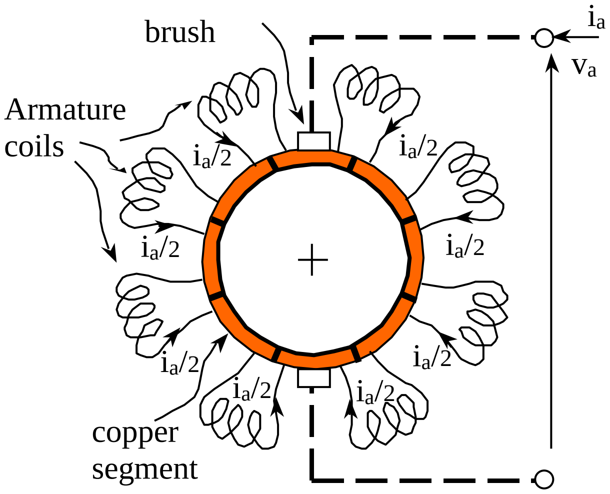


Figure 1-5. Commutator with 8 copper segments and 2 brushes

To understand the operation of the machine it should initially refer to a simplified armature winding made by a single turn rotating on itself, surrounded by a uniform magnetic field.

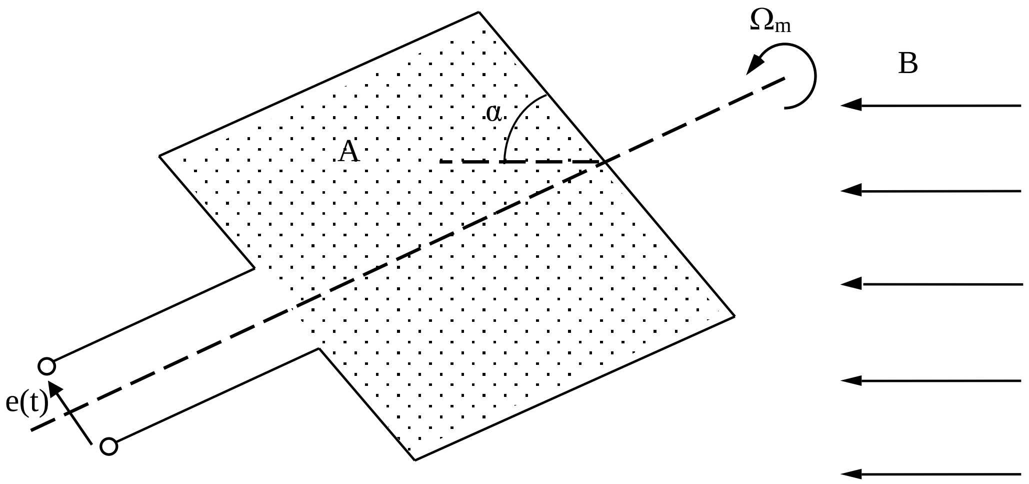


Figure 1-6. single turn armature winding

If we assume that the plane of the coil forms, at any given time, an angle α with the direction of the vector B (flux density), we have that the magnetic flux linked with the coil is (*A* refers to the surface of the coil itself):

$$\psi = B \cdot A \cdot \sin\alpha$$

If we now assume that the coil rotates with constant speed Ω_m, we have that, at its terminals, an induced electromotive force is measurable; its value is equal to:

$$e(t) = -\frac{d\psi}{dt} = -\frac{d}{dt}(B \cdot A \cdot \sin(\Omega_m t)) = -\Omega_m \cdot B \cdot A \cdot \cos(\Omega_m t)$$

where α is replaced with $\Omega_m t$, being the angular speed constant.

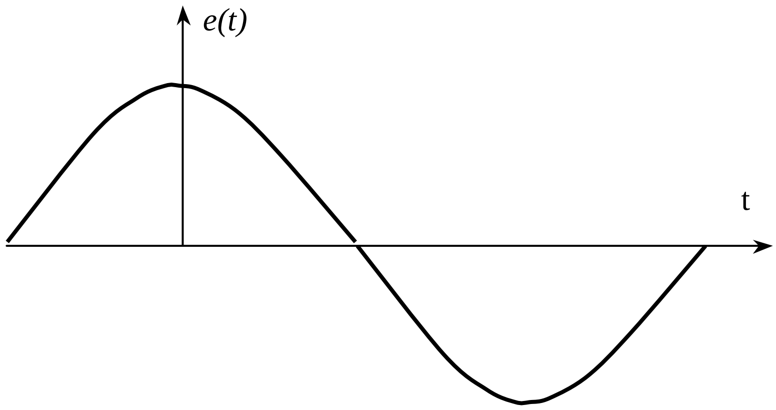


Figure 1-7. electromotive force waveform

If now we suppose to replace the terminals with two semi-rings and to include two electrical contacts (brushes) properly fixed in space, it is easy to see that the two contacts collect a rectified electromotive force consisting of two half sine wave.

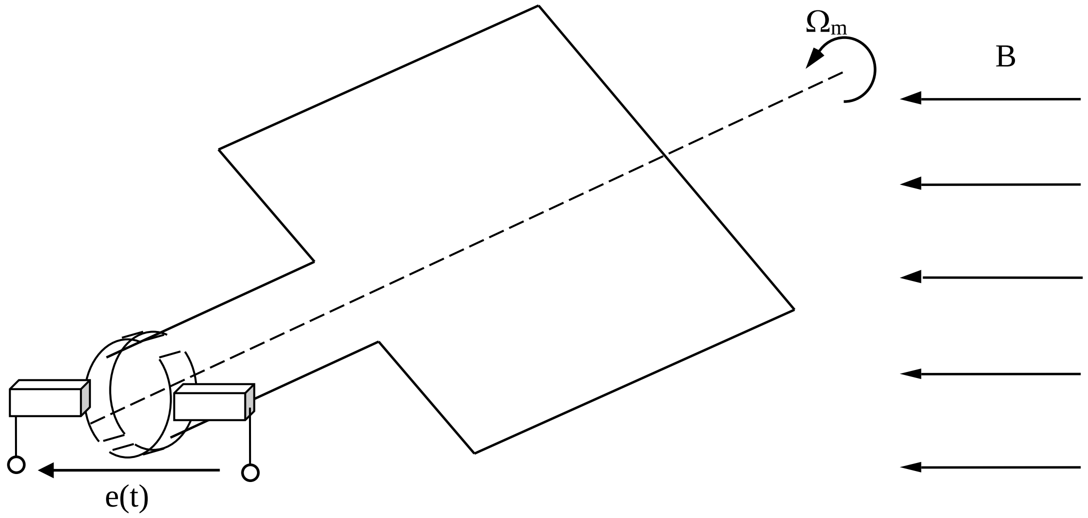


Figure 1-8. Semi-rings and brushes

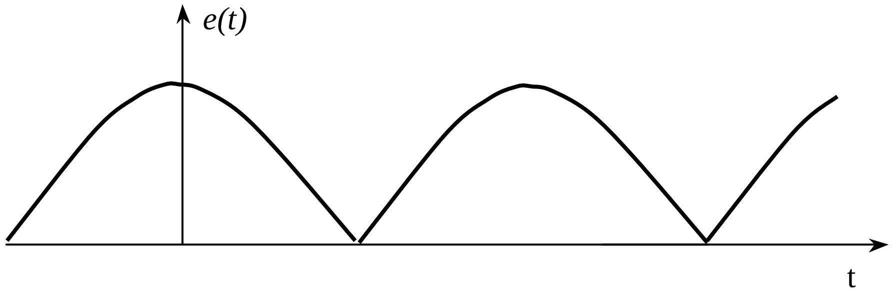


Figure 1-9. electromotive force waveform (two semi-rings and two brushes)

By inserting an increasing number of turns, you get a rectification effect, more and more effective so to reach a nearly constant value. At the same time the ring, consisting simply of two semi-rings, has to be made, as the turns increase, by thicker and thicker segments. Their task is to give the proper voltage of the coil on the electrical contacts (brushes). The set of segments that provides the electromotive force to the brushes is the so-called commutator.

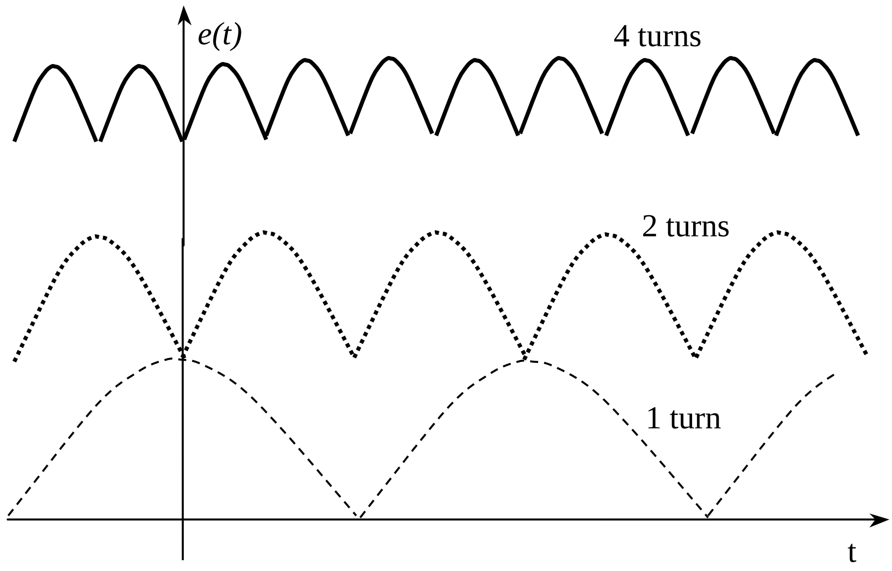


Figure 1-10. electromotive force waveform with two brushes and a commutator with: two semi-rings (1 turn), 4 segments (2 turns), 8 segments (4 turns)

To summarize: the principle of operation of the machine is based on the following points:

- there is a winding (excitation), realized on the stator, that generates a constant excitation magnetic field
- there is, also, a winding mounted on the rotor (armature), connected to a device with some segments, called commutator, which, through the brushes mounted on the inter-polar axis, realizes the rectification of ac electromotive forces into a rather constant voltage (dc voltage), proportional to the mechanical speed and to the flux ψ_{ae}, in some way linked with the armature winding, but supported by the only excitation current i_e.

1.4 Dynamic model of th DC machine

To achieve an electrical drive, the study of the dynamic behavior of the machine is relevant.

Figure 1-11 shows that the field lines due to the excitation take place mainly in ferromagnetic material, while those due to the armature current must pass through high air gaps with the result that the permeance (inverse of the reactance) of the magnetic circuit of the armature is much lower than the permeance of the excitation one. You should remind that the time constant L/R does not depend on the number of turns: the inductance is proportional to the number of turns squared, while the resistance is proportional to the length of the wire (which is proportional to the number of turns) and inversely proportional to the cross-section of the conductor (which, for the same area occupied by the conductors, is inversely proportional to the number of turns), then the resistance is proportional to the square of the number of turns. Therefore, it appears that the armature time constant τ_a (L_a/R_a) is much lower than the excitation time constant τ_e (L_e/R_e). Furthermore, due to the longer path of the excitation flux into the ferromagnetic material than the air path, the relationship between excitation flux and excitation current is not linear (due to the non-linearity of the ferromagnetic material), while the armature inductance L_a may be considered constant (the reluctance of armature flux path inside the ferromagnetic material is negligible respect to the reluctance of the air-gap path).

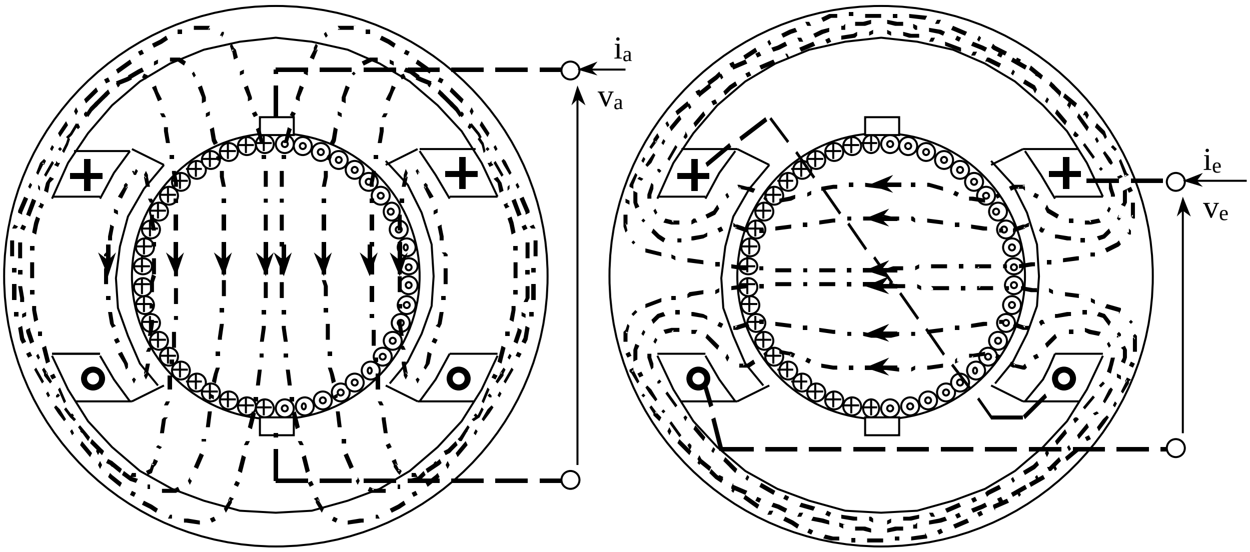


Figure 1-11. Development of magnetic field lines inside the machine

The dynamic equations are those typical of a winding: the applied voltage is equal to the sum of a resistive voltage drop and the derivative of the flux linked with the winding itself.

In the case of the armature winding, the situation is unexpected because the commutator presence causes, besides the two contributions just mentioned, a contribution (emf E) due to the fact that the armature conductors are moving inside a field due to the excitation current.

The dynamic equations are then (*p* is equivalent to *d/dt*):

$$v_a = R_a \cdot i_a + p\psi_a + E$$

$$v_e = R_e \cdot i_e + p\psi_e$$

with the following flux/current relationship (a linear relationship is only true for the armature):

$$\psi_a = L_a \cdot i_a$$

$$\psi_e = f(i_e) = L_e(i_e) \cdot i_e$$

$$\psi_{ae} = g(i_e) = L_{ae}(i_e) \cdot i_e$$

where ψ_a is the magnetic flux, linked with the armature winding, due to the armature current; ψ_e is the magnetic flux, linked to the excitation winding, due to the excitation current; ψ_{ae} is the magnetic flux, in some way linked with the armature winding, due to the excitation current ("in some way " because the two windings have magnetic axes orthogonal each other, so there is no mutual coupling between them, but the coils of the armature winding, which has a fixed magnetic axis due to the commutator and the brushes, are moving inside a magnetic flux due to the excitation current).

The armature and excitation dynamic equivalent circuits are shown in Figure 1-12.

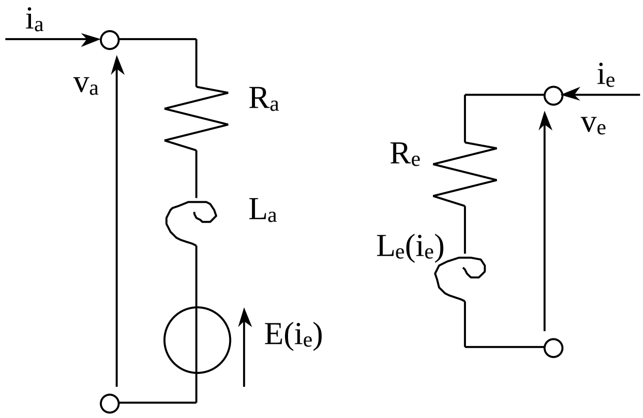


Figure 1-12. Dynamic equivalent circuit of a separately excited DC machine

The electromotive force E is proportional to the flux ψ_{ae}, linked with the armature winding, and to the rotation speed Ω_m of the machine; the relationship between E and the excitation current is non-linear, due to the non-linear relationship between ψ_{ae} and i_e:

$$E = k_e \cdot \psi_{ae}(i_e) \cdot \Omega_m$$

The torque expression may be obtained by an energy balance. As regards to the excitation circuit, you get:

$$v_e \cdot i_e = R_e \cdot i_e^2 + i_e p \psi_e$$

The term on the left is the electric power entering the system through the excitation terminals; at steady state and with constant voltages, it is constant. The first term on the right stands for the Joule losses, while the second one is the derivative of the internal magnetic energy stored in the excitation magnetic circuit (at steady state is equal to zero).

As regards to the armature circuit, it is:

$$v_a \cdot i_a = R_a \cdot i_a^2 + i_a p \psi_a + E \cdot i_a$$

The term on the left is the electric power entering the system through the armature terminals; at steady state and with constant voltages, it is constant. The first term on the right is equal to the Joule losses (in the armature resistance), while the second one is the derivative of the internal magnetic energy stored in the self-inductance L_a (at steady state is equal to zero). The third one represents the power transmitted between stator and rotor.

$$P_e = E \cdot i_a = k_e \cdot \psi_{ae} \cdot \Omega_m \cdot i_a$$

Thanks to the law of conservation of energy and neglecting any friction, it can be said that the transmitted electrical power is equal to the mechanical power and then you get:

$$P_m = T_e \cdot \Omega_m = P_e = k_e \cdot \psi_{ae} \cdot \Omega_m \cdot i_a$$

$$T_e = k_e \cdot \psi_{ae} \cdot i_a$$

which is the expression of torque in a dc machine.

In general, it is:

$$E = k_e \cdot \psi_{ae} \cdot \Omega_m$$

$$T_e = k_c \cdot \psi_{ae} \cdot i_a$$

with k_c equal to k_e (or very similar).

The mechanical system is represented by a system of differential and algebraic equations whose input is the electromagnetic torque T_e and whose output is the mechanical speed Ω_m.

1.5 Steady state model and basic equations

At steady state and with constant voltages, at its terminals the machine looks like an electromotive force which is connected in series with a resistance that takes into account the power losses by Joule effect in the armature conductors.

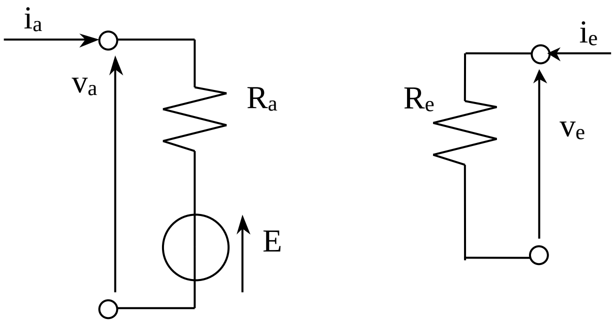


Figure 1-13. Steady state equivalent circuit of a DC machine

The armature and excitation steady state equivalent circuits are shown in Figure 1-13. The steady state equations are:

$$v_a = R_a \cdot i_a + E$$

$$v_e = R_e \cdot i_e$$

with the following flux/current relationship:

$$\psi_a = L_a \cdot i_a$$

$$\psi_e = f(i_e) = L_e(i_e) \cdot i_e$$

$$\psi_{ae} = g(i_e) = L_{ae}(i_e) \cdot i_e$$

The electromotive force E, as established above, is proportional to the flux ψ_{ae} linked with the armature windings, function of the excitation current i_e, and to the rotation speed Ω_m of the machine:

$$E = k_e \cdot \psi_{ae} \cdot \Omega_m$$

The torque is:

$$T_e = k_c \cdot \psi_{ae} \cdot i_a$$

Depending on how you create the excitation field and depending on how it is connected to the armature winding, you can have different types of dc machine:

- permanent magnet DC machine
- DC series-wound machine
- DC shunt-wound machine
- separately excited DC machine

In the context of electrical drives applications, the study of two types of machines is particularly important: permanent magnet and separately excited DC machine.

1.6 Permanent magnet DC machine

Main feature of this machine is the presence of permanent magnets on the stator that creates a constant flux density B and, accordingly, a constant excitation flux and a constant flux linked with the armature windings (Ψ_{apm} instead of ψ_{ae}). The mechanical torque is thus a function of the only armature current. You can write:

$$T_e = k_c \cdot \Psi_{apm} \cdot i_a = K_{cPM} \cdot i_a$$

Similarly, the electromotive force is a function of speed only:

$$E = k_e \cdot \Psi_{apm} \cdot \Omega_m = K_{ePM} \cdot \Omega_m$$

From these equations, we can easily obtain the torque versus mechanical speed characteristics (in steady state condition) for this type of machine.

Starting from the electrical relationship:

$$v_a = E + R_a \cdot i_a$$

$$i_a = \frac{v_a - E}{R_a}$$

we have:

$$T_e = K_{cPM} \cdot \frac{v_a - E}{R_a} = K_{cPM} \cdot \frac{v_a - K_{ePM}\Omega_m}{R_a}$$

This equation shows that the relationship between torque and speed is linear (with a constant value of the armature voltage), so you can get a family of curves, of equal slope, as a function of the supply voltage v_a. The slope of these curves, because of the typical values of the variables involved (the value of the armature resistance R_a is usually low), is very high.

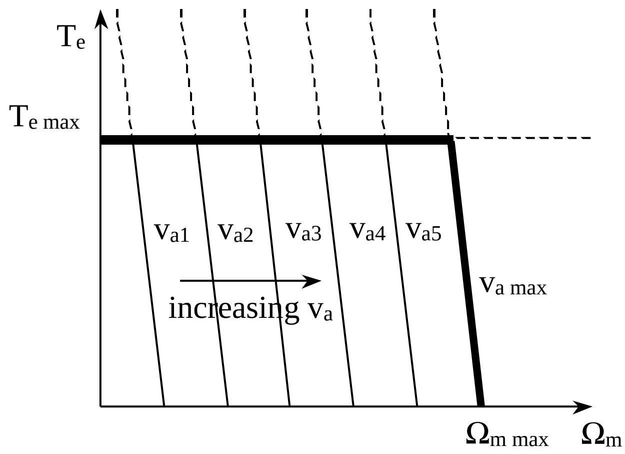


Figure 1-14. Torque/speed characteristics as a function of the armature voltage, and operating regions

The operating regions of the machine (i.e. the locus of operating points acceptable for the type of service requested) is limited (as well as you can see in Figure 1-14) by the following factors:

- the maximum permissible voltage (due to the power converter limitation and to insulation): determines the maximum mechanical speed
- the maximum allowable temperature (the coil temperature depends on the Joule losses, function of the armature current, the requested type of service and the cooling system) determines, given the service duty (continuous, intermittent,…) and the cooling system (natural air cooling, forced air cooling, water cooling, oil…), the maximum armature current and, consequently, the maximum allowable torque.

In the case of a permanent magnet machine, then, the maximum speed corresponds to the no load speed (zero load torque), obtained at the maximum value of the supply voltage.

1.7 Separately excited DC machine

The strength of this machine is the ability to control, separately, either the armature current and the excitation current (and therefore the flux ψ_{ae}).

The equations of the mechanical system are similar to the previous case, but now there is a new variable, the magnetic flux ψ_{ae}:

$$T_e = k_c \cdot \psi_{ae} \cdot \frac{v_a - E}{R_a} = k_c \cdot \psi_{ae} \cdot \frac{v_a - k_e \cdot \psi_{ae} \cdot \Omega_m}{R_a}$$

We define now "rated voltage" v_{an} as the highest voltage obtainable from the power electronic converter, without prejudice the insulation of the armature windings and maintaining a certain voltage margin to allow the armature current to change during transient.

In order to use the ferromagnetic material in a good manner, the machine should operate at the rated value of the flux ψ_{ae} (generally, the operating point corresponds to the knee of the characteristic flux/excitation current).

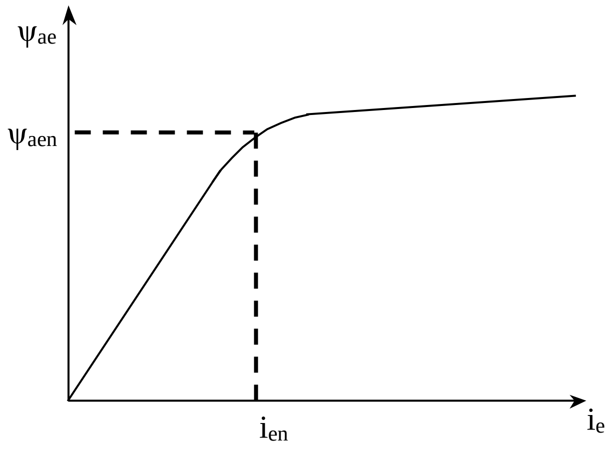


Figure 1-15. Non-linear relationship $\psi_{ae} = g(i_e)$; i_{en}= rated value of the excitation current, ψ_{aen}= rated value of the flux ψ_{ae}

With increasing speed, however, the electromotive force E, in these conditions, grows. Consequently, the armature voltage must grow because it differs from E only for the voltage drop over the armature resistance (in steady state condition). It is therefore clear that there is a speed at which the armature voltage reaches the rated voltage. This speed is referred to as base speed Ω_b. Above this speed, because the voltage can't grow anymore, E has to remain constant. To go beyond that speed, it is necessary to decrease the value of the flux ψ_{ae}. ($E = k_e \cdot \psi_{ae} \cdot \Omega_m$)

It is useful to divide the operating regions in two areas:

• speed lower than the base speed, that is to say, an applied voltage lower than the rated one

• speed greater than the base speed, that is to say, the applied voltage is equal to the rated one

As regards the first region you have nothing to add than that discussed for the permanent magnet machine as it operates by maintaining the flux ψ_{ae} constant and equal to its rated value ψ_{aen}.

Regarding the second area, the flux has to be adjusted so that, with increasing speed, the armature voltage remains constant and equal to the rated value v_{an}. In this case, acting at a constant value of the armature current, the electromotive force E remains constant ($E = v_a - R_a\, i_a$). The evolution of the flux ψ_{ae} is hyperbolic.

Since the maximum value of the armature current cannot be overcome (thermal effect), the torque must therefore decrease following the same trend.

Speed limits in this area are basically fixed by problems arising from operation of the commutator at high speed.

From a graphical point of view, changing the flux value, at the same armature voltage, causes a rotation of the mechanical characteristic: the intersection with the vertical axis (starting torque) is proportional to the flux $T_{e\_st} = k_c \cdot \psi_{ae} \cdot \frac{v_a}{R_a}$ while the intersection with the horizontal axis (no load mechanical speed) is inversely proportional $\Omega_{m0} = \frac{v_a}{k_e \cdot \psi_{ae}}$.

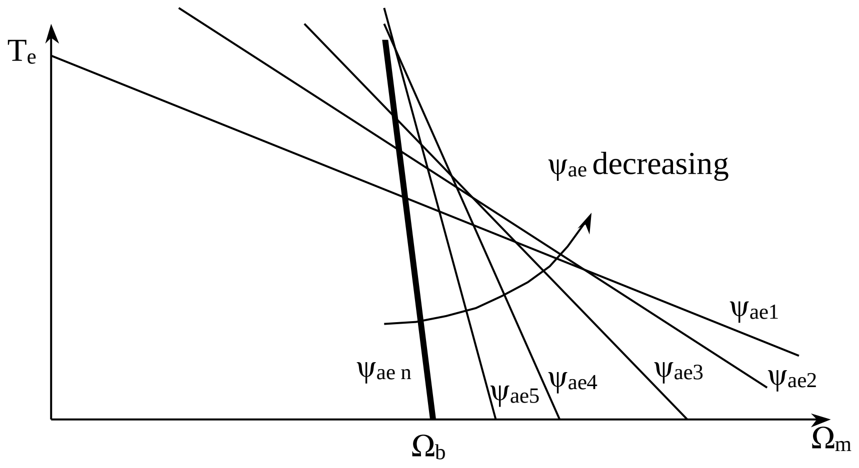


Figure 1-16. Mechanical characteristics (torque vs speed) as a reducing of the flux ψ_{ae} (weakening operation of the machine)

As regards the operating regions, please refer to diagram in Figure 1-17.

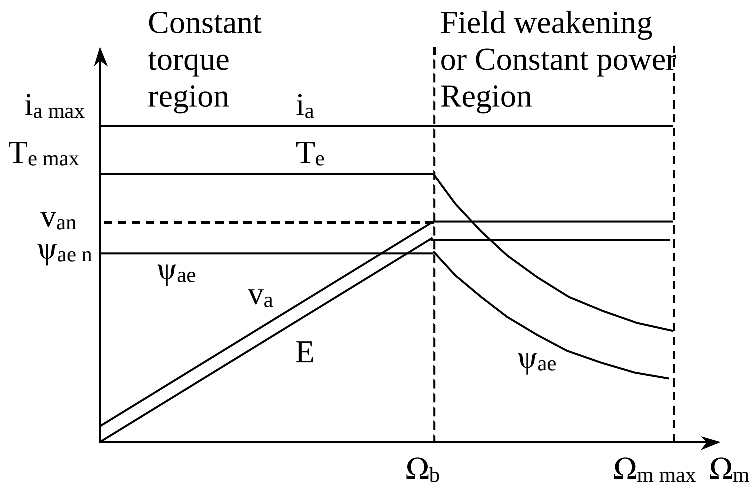


Figure 1-17. Operating regions

The armature current is considered constant and equal to the maximum allowed by the cooling system and the type of service (load duty). If the efficiency of the cooling system depends on the mechanical speed (as in the case of fan keyed on the machine shaft), the maximum allowable armature current must decrease with decreasing speed (see Figure 1-18). In electrical drive applications, however, in which you want to operate with maximum torque up to very low speed for a long time, the cooling system has to be independent by the speed.

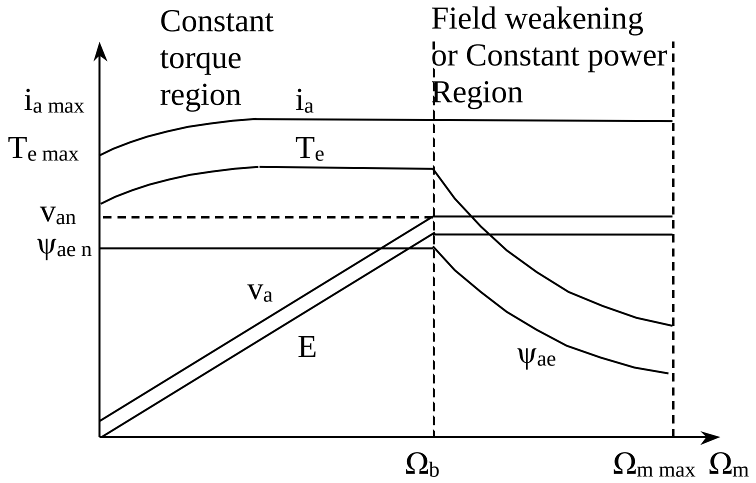


Figure 1-18. Operating regions with a cooling system whose efficiency depends on the mechanical speed

The armature voltage is not zero at zero speed as a voltage is needed in order to supply the current in the armature resistance.

The trend of the power is similar to the trend of the armature voltage (if the current is constant). The weakening zone is also called "constant power region". The area on the left, characterized by lower speeds than the base speed, is called "constant torque region".

The operating regions are limited by the maximum speed allowed by the mechanical system (mainly by the commutator).

The sequence with which to draw the curves of the operating regions is shown in Figure 1-19.

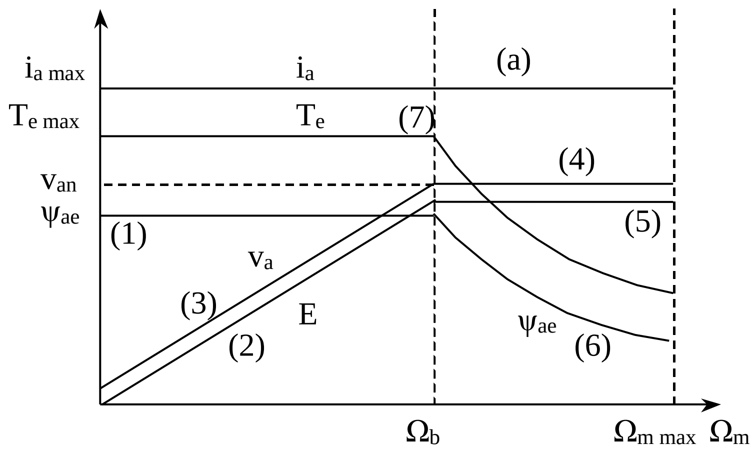


Figure 1-19. Sequence

On the one hand there is a limit on the thermal current (curve (a)); on the other hand the good use of the ferromagnetic material in the machine defines the value of the flux ψ_{ae} (1). Next step is the characteristic E (2) (proportional to the machine speed if the flux is constant) and the voltage v_a (3) (slightly greater than E due to the resistive voltage drop) till the value of the rated voltage v_{an}. This point defines the base speed Ω_b. Then the voltage has to be limited to the nominal voltage (4), so it is necessary to maintain constant E (5), weakening the flux ψ_{ae} of the machine (6). The torque is proportional to the product of the armature current and flux ψ_{ae} (7).

Regarding the armature current, the line (a) represents the upper limit: then the armature current can assume a value within the entire area bounded by that line. The flux, the armature voltage, E are lines that represent the possible points of steady state operation of such quantities. The torque is an area (such as the armature current).

1.8 Control scheme

Since the goal of an electric drive is to achieve a given torque (or, at least, to pursue a given reference speed), power control can operate both on the armature current and on the flux ψ_{ae} (and, therefore, the excitation current i_e). Given the large difference between the time constants of armature and excitation, the torque control is usually achieved by controlling the armature current. The flux ψ_{ae} follows the ideal operating regions (in order to exploit at best the ferromagnetic material). In the case of permanent magnet machine the problem does not arise because the flux ψ_{ae} can be considered constant e not controllable.

Having a power supply that can provide "instantly" the current request (this solution is difficult to realize), the control scheme is simple and is described in Figure 1-20, which shows the outer control closed loop of the mechanical speed. The transfer function **F**(s) of the linearized mechanical load is included in the scheme.

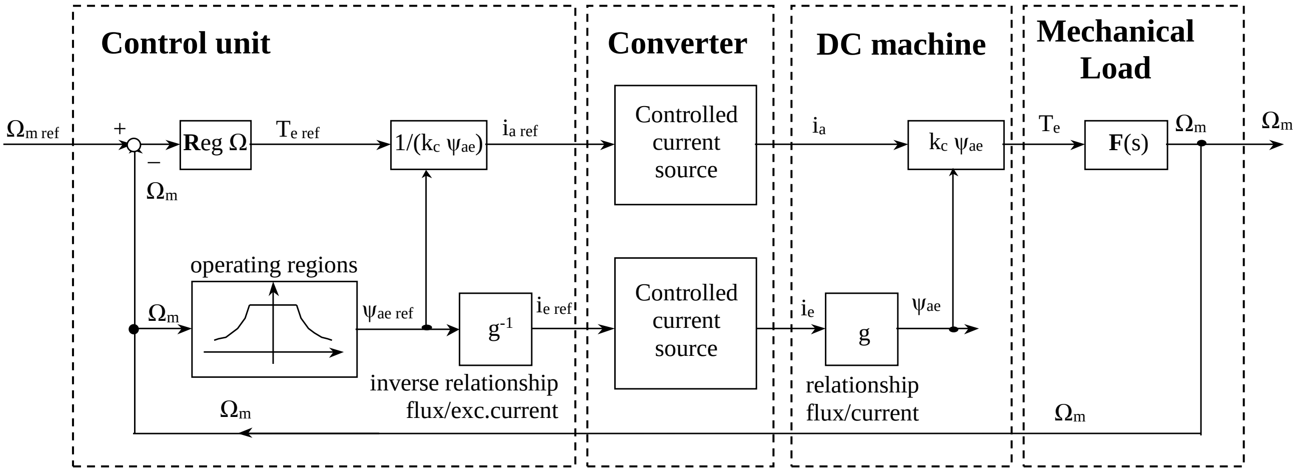


Figure 1-20. Speed control scheme of a separately excited DC machine (controlled current sources)

Figure 1-21 shows the corresponding diagram in the case of excitation due to permanent magnets. The coefficient $1/K_{cPM}$ is often incorporated into the speed controller gain.

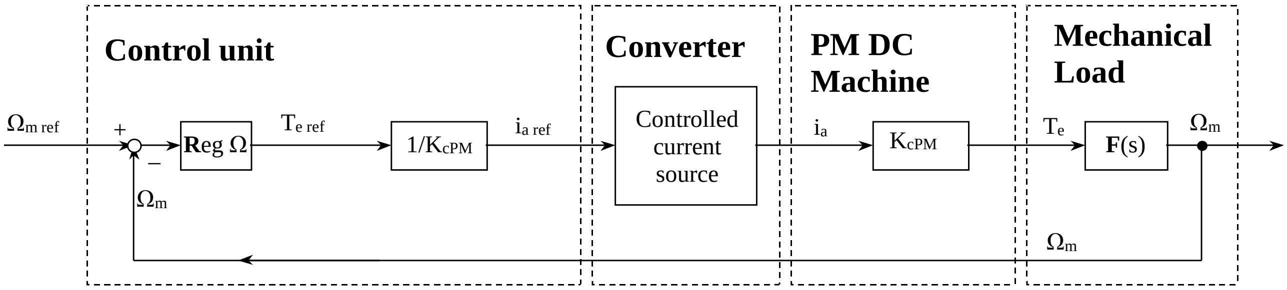


Figure 1-21. Speed control scheme of a PM DC machine (controlled current source)

In Figure 1-20 and Figure 1-21, the power supply device is represented as a device with a fast current control that allows the motor currents to follow closely the reference values. In effect, using inverter transistors (or IGBT) with high PWM switching frequency (typically higher than 10 kHz) it can be obtained that the currents overlap its references (apart from the unavoidable ripple produced by the PWM). Taking into account the dynamics of the stator, the control scheme changes, as shown in Figure 1-22 and Figure 1-23. In this case the power unit has the task of establishing a reference voltage, while the current control is now performed by the controller (**R**eg i) (more commonly used solution).

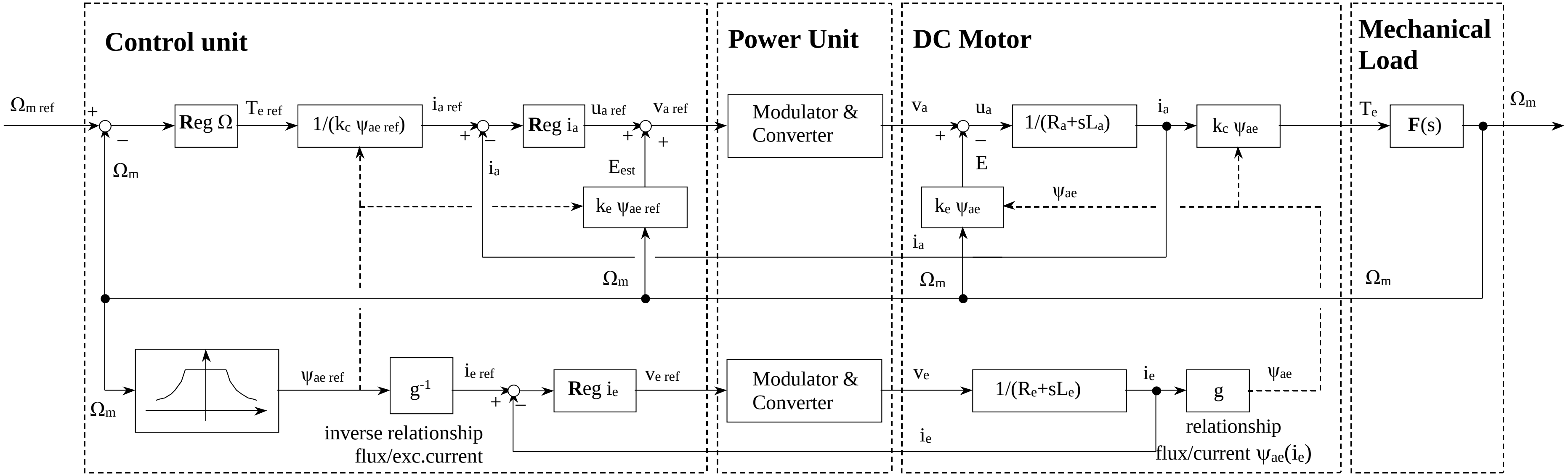


Figure 1-22. Speed control scheme of a separately excited DC machine (controlled voltage sources)

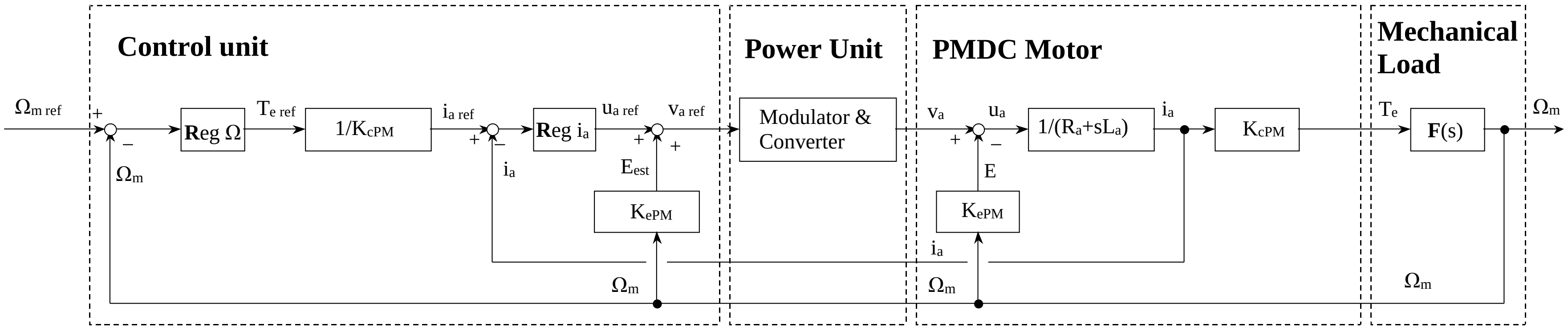


Figure 1-23. Speed control scheme of a PM DC machine (controlled voltage source)

The structure of the current controller is based on the dynamic equations of the machine:

$$v_a = R_a \cdot i_a + L_a \cdot pi_a + k_e \cdot \Omega_m \cdot \psi_{ae}(i_e)$$

$$v_e = R_e \cdot i_e + p\psi_e(i_e) \approx R_e \cdot i_e + L_e \cdot pi_e$$

You notice immediately that the expression of the armature voltage is made by two terms: the voltage $R_a i_a + L_a p i_a$ (which will be denoted by u_a) and the term $k_e \Omega_m \psi_{ae}$ which represents the electromotive force E, proportional to the mechanical speed and to the flux ψ_{ae}. This term represents a kind of coupling between the excitation circuit and armature circuit (not vice versa). It is also a function of the mechanical speed. It can be considered and treated as a disturbance (easily compensated if you know the mechanical speed and the parameters of the machine). Therefore, it is natural that the armature current controller output is the voltage $u_{a\ ref}$, the only one that actually acts on the current i_a. The reference value of the armature voltage $v_{a\ ref}$ is obtained by the sum of $u_{a\ ref}$ and the estimation of E.

It is good evidence that the excitation differential equation is non-linear. To the synthesis of the controller, it should be linearized or, at least, it is necessary to consider linear the relationship flux/current.

If in the control scheme the compensation of E is omitted, the system, at steady state, however, would be able to follow the current reference, because the action of the integral part of the controller is able to substitute the compensation of the disturbance. There might be problems at the start of the control system if the mechanical speed is different by zero (so-called "starting on the fly"). The integral part of the PI controller gives, initially, a null output, so the output of the current regulator (due to the only proportional term) could be lower than E. The armature current could assume, in the first instants, a negative value because the term (v_a-E) is negative (and therefore a negative torque or braking) regardless of the value of the reference current. For a start on the fly it is necessary to compensate the electromotive force E.

1.9 Voltage regulator

To exploit the operating conditions of the machine and to become independent from any errors on the parameters of the machine itself, the value of the reference flux, instead of being calculated with an open-loop approach, through the use of the operating regions and knowing the mechanical speed, can be obtained as output of a controller of the armature voltage, suitably saturated and equipped with anti-windup property . The estimation of the armature voltage is given by the sum of the estimated emf E and the resistive voltage drop $R_a i_a$. If this value is lower than the maximum voltage that the power supply can provide (without prejudice to the characteristics of conductor insulation and maintaining a certain margin for the dynamic control of the current), the controller saturates to the rated value of the flux ψ_{ae} (constant torque region). As soon as the estimated voltage reaches the maximum value, the controller begins to work, lowering appropriately, as the speed increases, the flux ψ_{ae} so as to act on the emf E, and consequently on the armature voltage (field weakening region).

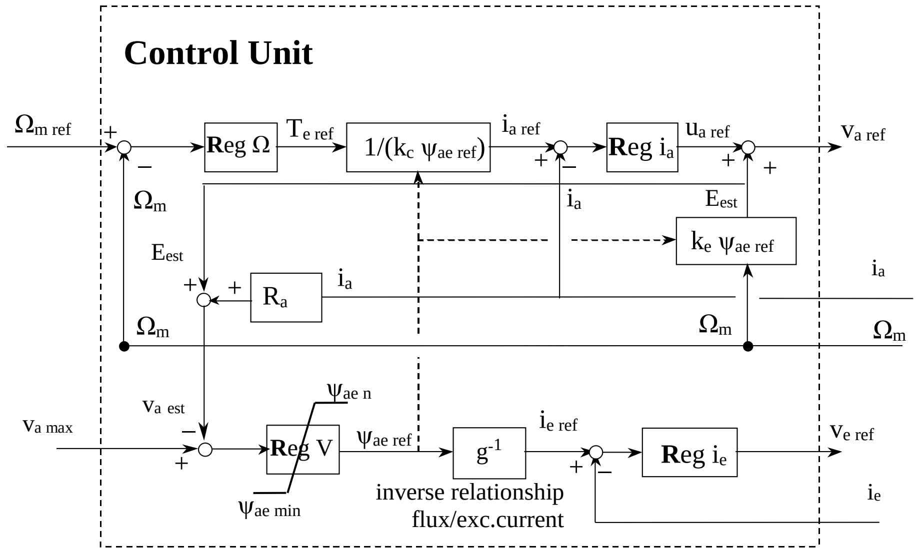


Figure 1-24. Control unit with a voltage regulator instead of the use of the operating regions

1.10 Current regulators design

If the compensation terms are well designed and if it is reasonable to consider the power converter as a pure unity gain, the controllers can be easily designed taking into account that the transfer function has an expression like *R+sL* (for the excitation controller, the dynamic equations are linearized around the operating point, using the differential value of the inductance).

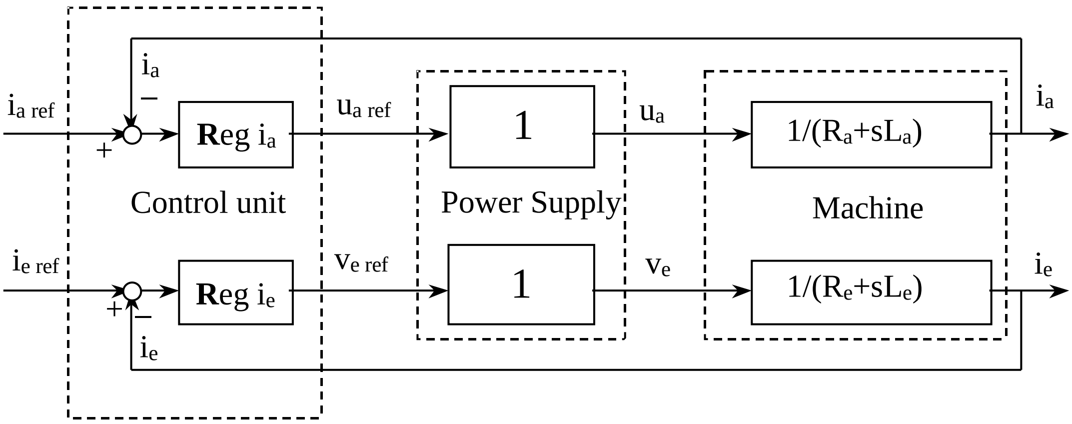


Figure 1-25. Scheme for the current controllers design

For a better representation of the power supply, the unity gain may be replaced by a delay whose value is tied to the switching time of the converter. If the modulator is based on a PWM technique, for example, the delay between the change of the reference and its implementation varies from 0 to the entire period of switching. On average, we can assume a delay of half the switching period.

1.11 Speed regulator design

In order to design the speed controller, you need to know the transfer function of the overall system (current amplifier, machine, mechanical load). The output of the speed controller is the target value of the electromagnetic torque (which is linked to the armature current). The scheme is shown in Figure 1-26.

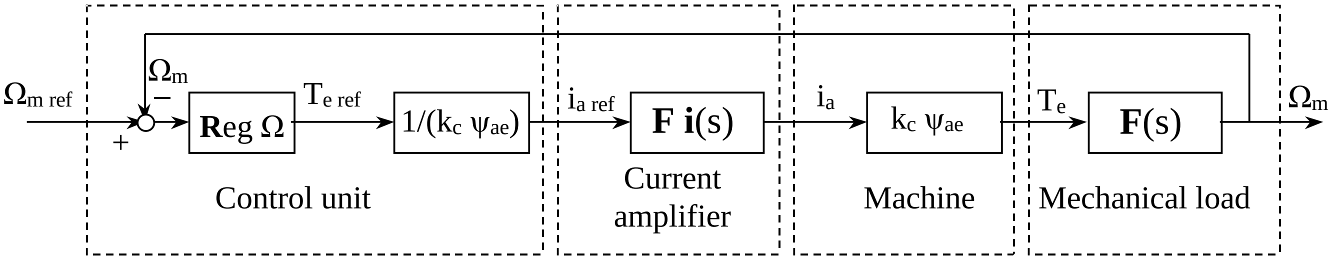


Figure 1-26. Scheme for the speed regulator design

You can follow two paths. If the bandwidth of the current regulator is very high compared to that required to the speed controller, you may consider the current amplifier as ideal (unity gain). Otherwise, once the current controller is designed, the transfer function of the current amplifier **Fi(s)** is known. Therefore the speed controller has to be designed considering a system characterized by a transfer function equal to the product of **F i(s)** with the transfer function of the mechanical load **F(s)**. In the event that the control system does not provide compensation for the flux ψ_{ae} variations (as in Figure 1-27), the performance of the control system could vary from the constant torque region and the weakening zone.

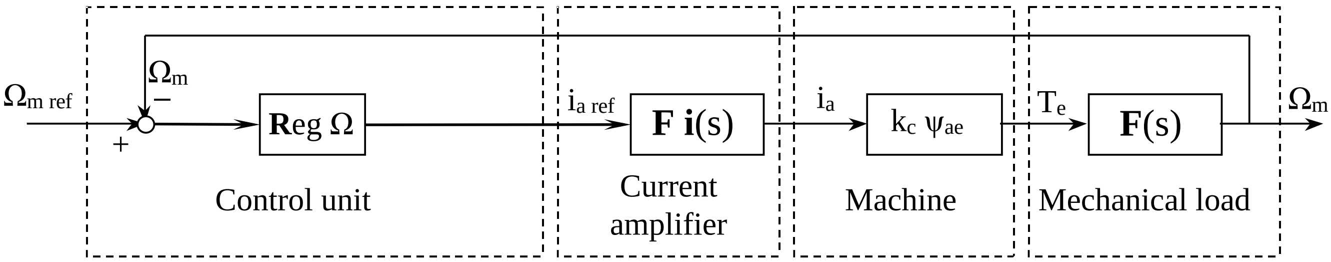


Figure 1-27. Scheme of the mechanical speed regulator without any adaptive term